Sell Probabilistic Goods? A Behavioral Explanation for Opaque Selling

Tingliang Huang
Department of Management Science and Innovation, University College London, London WC1E 6BT, United Kingdom, t.huang@ucl.ac.uk

Yimin Yu
Department of Management Sciences, College of Business, City University of Hong Kong, Kowloon, Hong Kong SAR, yiminyu@cityu.edu.hk

Probabilistic or opaque selling, whereby the seller hides the exact identity of the product until after the buyer makes the payment, has been used in practice and received considerable attention in the literature. Under what conditions, and why, is probabilistic selling attractive to firms? The extant literature has offered the following explanations: to price discriminate heterogeneous consumers, to reduce supply-demand mismatches, and to soften price competition. In this paper, we provide a new explanation: to exploit consumer bounded rationality in the sense of anecdotal reasoning. We build a simple model where the firm is a monopoly, consumers are homogeneous, and there is no demand uncertainty or capacity constraint. This model allows us to isolate the impact of consumer bounded rationality on the adoption of opaque selling. We find that while it is never optimal to use opaque selling when consumers have rational expectations, it can be optimal when consumers are boundedly rational. We show that opaque selling may soften price competition and increase the industry profits due to consumer bounded rationality. Our findings underscore the importance of consumer bounded rationality and show that opaque selling might be even more attractive than previously thought.

Key words: probabilistic goods, opaque selling, pricing, anecdotal reasoning, law of small numbers, bounded rationality

1. Introduction

Probabilistic or opaque selling, whereby the seller hides the exact identity of the product until after the buyer makes the payment, is a creative way of selling products and services (Fay and Xie (2008)). This strategy was recently introduced by Hotwire and Priceline to sell airline tickets, hotel rooms, and rental cars. Many online travel agency websites have since introduced their own variants of opaque selling: Booking – Hidden Hotel, GTAHotels.com – Mystery Hotel, Hotel.de – Hotel Roulette, Easyclicktravel.com – Off The Record, HotelDirect.co.uk – Hidden Gem' Hotels, GetGoing.com – Pick Two, Get One, to name just a few.[1] According to MarketMetrix, the two

[1] See http://www.xotels.com/en/revenue-management/revenue-management-book/opaque-pricing for more examples; accessed on December 18, 2013.

biggest opaque sellers, Priceline and Hotwire combined account for 6.7% of worldwide online hotel bookings in 2006, comparable to the transparent sellers, Expedia.com (10.4%), Travelocity.com (6.8%) and Orbitz.com (4.9%).[2] According to TravelClick, the opaque channel accounted for 6% of all hotel reservations for major brands in 2012, up 2% from 2010.[3]

This selling strategy actually goes beyond the travel industry. Fukubukuro, a Japanese New Year's Day tradition, uses opaque selling in shops across Japan each year with retailers packaging an unknown collection of things into "lucky bags" (or "mystery bags") and offering them to consumers. When Apple opened its Union Square retail store in San Francisco in 2004, it sold lucky bags with the chance to win the then-new iPod mini. The bags were said to contain several unknown items. When you go to the L'Astrance restaurant in Paris, there is only one menu, and you do not know the specific dishes when you order. Also, the menu might be different for different tables, so you can only *guess* what you will get by looking at the other guests.[4] Other examples in non-travel industries include: Priceline offered opaque home financing and equity loans; in the consignment businesses, apparel manufacturers conceal their brand names by snipping off the tags; package-goods manufacturers of national brands serve as the anonymous suppliers of private labels (Fay (2008b)). The examples of probabilistic selling are abundant across different industries. Indeed, we believe that this novel selling strategy has a great potential to be used by more firms and organizations[5] in the future given that its advantage does not appear to depend on industry-specific characteristics (see, e.g., Fay and Xie (2008)). We refer the reader to the US patent 8117063, Xie and Fay (2012), for the system and methods on how to create probabilistic products and facilitate probabilistic selling in practice.

This novel business model has received much attention and interest from both practitioners and academics. Inter alia, we are all interested in this question: Why is probabilistic selling attractive to firms? The extant literature has offered the following primary explanations: to price discriminate heterogeneous consumers with different preferences (Jiang (2007) and Fay and Xie (2008)), to reduce mismatches between uncertain demand and capacity (Fay and Xie (2008) and Jerath et al. (2010)), and to soften price competition (Shapiro and Shi (2008) and Fay (2008b)). From different angles, these studies uncover the fundamental factors that make probabilistic selling profitable, and

[2] Source: MarketMetrics.com, http://marketmetrix.com/en/default.aspx?s=research&p=research6. See Shapiro and Shi (2008) for more discussions.

[3] Source: http://www.secrethotels.eu/secret-hotels-win-win-for-hotels-and-travellers/, accessed on December 15, 2013.

[4] Source: http://foodsnobblog.wordpress.com/2009/01/07/lastrance-paris/, accessed on December 18, 2013.

[5] For example, Xie and Fay (2012) point out that the opaque product could even be "a set of class registrations for university students, each registration being for a particular class at different times and/or taught by different professors, or for different classes taught at the same time and/or by the same professor" (p. 11).

convincingly demonstrate that probabilistic selling is advantageous to the seller without requiring specific industry characteristics.

Consumers may welcome this business model because they may get their preferred products at lower prices than otherwise. According to MarketMetrix, Hotwire was ranked the highest in customer satisfaction on hotel travel websites from 2006 to 2010.[6] However, due to the opacity or ambiguity of the product, customers also face difficult decisions to make for a given price. When deciding whether or not to purchase, consumers have to trade-off between the *certain* price paid and the *uncertain* value of the product they are about to be offered. Indeed, there are even some communities and information board websites with tips and discussions by consumers on how to get the best deals on these opaque websites, for example, biddingfortravel.yuku.com, betterbidding.com, and bidontravel.com. On these websites, consumers share their experiences with each other, so that they can learn better about the sellers' product offering strategies. The way they learn is primarily through anecdotes or samples which are essentially *realizations* of the sellers' *random* product offerings previously. This suggests that consumers typically find it difficult to infer the sellers' strategies, and have to rely on (other consumers' or their own) experiences and anecdotes to make their own purchasing decisions. An interesting and important question to be answered is: How would this kind of anecdotal learning behavior affect the profitability of opaque selling? To answer this question, we have to adopt a model that captures the dynamics of anecdotal learning and reasoning behavior, and do not *a priori* assume that consumers perfectly know the likelihood of each product to be offered (i.e., "rational expectations").

A common feature in the existing literature is that "rational expectations" are assumed in studying the game between the firm and its consumers in the context of opaque/probabilistic selling (see, e.g., Fay and Xie (2008), Fay and Xie (2010), Jerath et al. (2010), Jerath et al. (2009) and references therein). This assumption suggests that the firm and its consumers alike, have "common knowledge of the model." In particular, consumers can *perfectly* anticipate the firm's product offering strategy (i.e., the likelihood of each component product to be offered) in equilibrium. This assumption is justified in settings where adaptive learning in an infinite-time horizon is possible. In such settings, consumers have frequent repeated interactions with the firm, and thus can perfectly learn the firm's product offering strategy eventually.

However, in other settings in reality, such infinite learning opportunities might not be available or possible, as evidenced by the existence of these "decoder" websites that use feedback/experiences

[6] Source: http://corp.marketmetrix.com/, accessed on December 13, 2013.

from previous purchasers to help construct a profile of the opaque products/services. Hence, consumers only have scarce opportunities to learn the firm's product offering strategy, and have to make decisions based on limited information. To quote Camerer and Loewenstein (2004), "many important aspects of economic life are like the first few periods of an experiment rather than the last" (p. 8-9). In this paper, we are interested in such settings. We provide a study that is complementary to the existing literature by assuming that consumers may not have sufficient information or enough opportunities to learn the seller's opaque product offering strategy (i.e., how likely each component product is offered).

To model the settings where the firm has perfect information while consumers only have scarce opportunities to learn the firm's strategy, we need a new modeling framework that goes beyond rational expectations. We resort to the *anecdotal* reasoning framework in the economics literature to capture consumer bounded rationality (see Osborne and Rubinstein (1998) and Spiegler (2006a,b)), as opposed to *probabilistic* reasoning (i.e., knowing the probability of each outcome and computing the expectations). We study the general $S(K)$ model where the parameter K denotes the size of samples drawn, which can be viewed as the level of consumer rationality. When $K = \infty$, we obtain the rational expectations model.

To isolate the impact of bounded rationality, we intentionally choose to use a simple model where the existing literature based on rational expectations would recommend *not* to use the opaque selling strategy. Our research question is: What is the impact of consumer bounded rationality on the firm's decision in adopting opaque selling? Interestingly, we find that bounded rationality alone can justify the adoption of the opaque selling strategy. In other words, in the absence of all the reasons provided in the literature for adopting the opaque selling strategy, the firm may still want to use this strategy if consumers are boundedly rational in the sense of anecdotal reasoning. Intuitively, the firm can adopt opaque selling simply for the purpose of *exploiting* or taking advantage of consumer bounded rationality. The main managerial insight is that consumer bounded rationality is important for firms that are considering to adopt the opaque selling strategy, in addition to other factors studied in the literature. In particular, the presence of the "decoder" websites (e.g., biddingfortravel.yuku.com, betterbidding.com, and bidontravel.com) does not necessarily hurt the profitability of using opaque selling. On the contrary and surprisingly, it can be precisely the anecdotal learning and reasoning behavior of consumers that motivates the firm to use opaque selling. We also conduct a series of robustness checks with respect to rational priors, repeat customers, and customer heterogeneity. In a competitive market, we show that opaque selling may *soften*

price competition and increase the industry profits even in the absence of consumer heterogeneity. In addition, our assumption of asymmetric costs, and the prediction of offering products with nonequal probabilities complement the extant literature. Our findings underscore the importance of consumer bounded rationality and show that opaque selling might be even more attractive than previously thought.

The remainder of the paper is organized as follows. We review relevant literature in Section 2. Section 3 presents the basic model and robustness checks. Section 4 extends the model to a competitive market. Section 5 provides concluding remarks. All technical proofs are relegated to the Appendix.

2. Related Literature

The recent body of marketing, economics, and operations management literature on opaque/probabilistic selling starts from the work of Jiang (2007), Fay and Xie (2008), Fay and Xie (2010), and Jerath et al. (2010). Jerath et al. (2009) provide an excellent review chapter. In our model, a price for the opaque product is posted and consumers decide whether to purchase or not, which is consistent with the practice of Hotwire. Alternatively, firms can ask consumers to bid the prices they are willing to pay and their bids can be accepted or rejected (e.g., Priceline). In this paper, we only focus on the literature that uses *posted prices*.

We first review the studies in the monopoly setting. Jiang (2007) uses a Hotelling model to study the setting in which an opaque ticket (which is a morning or night flight) is sold. The main assumption made there is that, although the flight information is not revealed for the opaque tickets, consumers expect an *equal* probability of obtaining each ticket independent of the actual allocation made by the firm. This is a common assumption in this stream of literature. The key implicit justification for this assumption is that, consumers follow probabilistic reasoning and they have perfect information that allows them to rationally expect the firm's strategies. In this paper, we shall show that bounded rationality alone can lead to non-equal product offering strategies, in the absence of capacity constraints and demand uncertainty. In contrast to Jiang (2007), we assume that consumers are homogeneous in our basic model (yet we also include consumer heterogeneity as a robustness check), and we focus on the role of consumer bounded rationality on the profitability of opaque selling.

Fay and Xie (2008) refer to opaque selling as "probabilistic selling." They consider a monopolist offering two products with consumers distributed on a Hotelling line as in Jiang (2007). The seller can use two selling strategies: traditional selling and probabilistic selling. An important feature

of their model is that rational expectations are assumed: the consumers are rational and forward-looking, i.e., their expectations are correct in equilibrium. Fay and Xie (2008) find that probabilistic selling strictly improves the firm's profit if production costs are sufficiently low. However, the advantage from probabilistic selling depends strongly on the magnitude of travel costs (which can be viewed as a measure of consumer heterogeneity). Fay and Xie (2008) show that profit advantage from opaque selling is highest when the horizontal differentiation of the products is at the intermediate level, and advantages of opaque selling do not depend much on standard assumptions behind the classical Hotelling model. Consistent with Jiang (2007), they show that it is generally optimal to assign an equal probability to each component product as the probabilistic good. As discussed in Jerath et al. (2009), such a symmetric assignment is largely driven by the absence of capacity constraints and demand uncertainty. Complementary to this literature, we show that rational expectations also play an important role. We find that, with boundedly rational consumers, the component product assignment may be asymmetric in both monopoly and competitive settings. Fay and Xie (2008) also show that offering probabilistic goods can reduce the seller's information disadvantage and lessen the negative effect of demand uncertainty on profit by reducing the mismatch between capacity and demand. Fay and Xie (2010) compare probabilistic selling with another related selling strategy: advance selling. Using a general economic model, they show that the seller can address unobservable buyer heterogeneity by inducing sales involving buyer uncertainty via two different mechanisms: homogenizing heterogeneous consumers and separating heterogeneous consumers.

There are only a few studies of opaque selling in competitive settings. Shapiro and Shi (2008) show that opaque selling enables sellers to price discriminate between those consumers who are sensitive to product/service characteristics and those who are not. Sellers can profit from such discrimination despite the fact that the opaque feature virtually erases product differentiation and thus intensifies competition. Interestingly, we show that, in the absence of consumer heterogeneity for price discrimination, opaque selling may still soften competition in the presence of consumer bounded rationality. Fay (2008b) constructs a model of opaque selling in which channel considerations are investigated by considering a wide variety of contracts between service providers and an opaque intermediary. An important assumption in the model is that, although the consumers cannot observe the number of tickets initially allocated by each firm to the opaque channel, when purchasing an opaque ticket they expect to obtain it from either firm with equal probability. This assumption is the same as Jiang (2007) and Fay and Xie (2008). Fay (2008b) finds that a monopolist can improve profits by introducing an opaque good at a small discount and by raising the

prices of the traditional goods. However, if there is competition between the selling firms, then the dynamics are different. Fay (2008b) finds that, with sufficient brand loyalty, opaque sales help reduce price rivalry and increase firms' profits. In contrast, in our model, there is no consumer heterogeneity, yet we show that opaque selling may still soften price competition when consumers are boundedly rational. Jerath et al. (2010) analyze opaque selling in a two-period model with demand uncertainty and limited capacity. A key assumption made in this paper is that consumers develop rational expectations about future product availability on which they base their decisions, and these expectations are consistent in equilibrium. In contract, we study consumer boundedly rational expectations.

Our work also belongs to a growing body of economics and marketing literature that studies market interaction between rational firms and boundedly rational customers with cognitive imperfections or rational consumers with limited information: bounded ability to grasp inter-temporal patterns in Piccione and Rubinstein (2003), biased beliefs concerning future tastes in Eliaz and Spiegler (2006), limited memory in Chen et al. (2010), and expectation formation through learning in Fay (2008a), to name just a few. The common argument for this literature is that the firm and customers often differ in their ability to understand the market. In reality, the firm interacts more frequently with the market, and pays much closer attention to it, than its customers. As a result, the firm has more opportunities to learn the market. Therefore, this stream of literature argues that the assumption that all agents, the firm and its customers alike, have "common knowledge of the model" might not be plausible in many situations.

Our study shares a common theme and is complementary to the recent theoretical, empirical, and experimental studies on word of mouth and observational learning in economics and marketing settings. Banerjee (1992) and Banerjee (1993) present two models that suggest that people may place significant weight on the opinions of others that leads to "herding." Godes and Mayzlin (2004) present an empirical study to measure word of mouth using data from television shows. Chen et al. (2011) recently designed field studies to investigate how consumers' purchase decisions can be influenced by others' opinions, or word of mouth, and/or others' actions, or observational learning. They were able to disentangle the competing effects and show how these two social influences might differ from and interact with each other.

Our anecdotal reasoning framework follows the recent economics literature on modeling boundedly rational expectations and anecdotal reasoning. Osborne and Rubinstein (1998) formulate the $S(1)$ procedure in the context of strategic-form games, in which all players behave according to this procedure. They focus on developing a novel equilibrium concept. In contrast, in our study, the

firm is fully rational, and only the customers employ the anecdotal reasoning procedure. The $S(1)$ framework has been applied in a variety of economic settings; see, for example, Spiegler (2006a), Spiegler (2006b), (Spiegler 2011, Chapters 6-7), Szech (2011), and references therein. There are only a few studies parallel to ours applying the anecdotal reasoning framework to marketing or operations settings. For example, Huang and Chen (2012) study a recently observed practice of advance selling without showing the price, and investigate how consumer bounded rationality and limited capacity may play a role. The opaque selling setting studied in this paper is quite different from theirs: They study a monopolist firm that sells a single product in two periods. There is a pre-order period where product availability is guaranteed but price is not shown to consumers, and a spot selling period with the possibility of stockouts that is unknown to consumers and a known regular price. They intend to understand the role of consumer bounded rationality in inferring both pre-order price and product fill rate (e.g., how likely a stockout occurs). In contrast, in our setting, there are two component products, prices are posted and there is no capacity constraint. We focus on consumer bounded rationality in determining the firm's optimal strategy in assigning multiple component products in the important context of opaque selling. In terms of modeling bounded rationality, they follow the economics literature (e.g., Spiegler (2006a)) by mainly focusing on the $S(1)$ framework. In this paper, however, we focus on a general $S(K)$ model that generalizes the commonly used rational expectations framework in the extant literature.

3. The Model

In this section, we first specify the basic model setup. Then, we characterize the firm's optimal pricing and selling strategies. Finally, we examine the robustness of our findings by relaxing several of the assumptions of the basic model.

3.1. Model Setup

We consider the simplest possible model to isolate and demonstrate the impact of consumer bounded rationality on the firm's opaque selling strategy. A monopolist firm sells a product of two different "versions" to a population of homogeneous consumers. There is no demand uncertainty, and the number of consumers is normalized to 1. The two versions of the product, product H and product L, have different values for the consumers: v_H and v_L. We assume that $v_H > v_L$. Following the terminology in Fay and Xie (2008), we can view the two versions as two different "component products." Given that all the consumers are homogeneous, the product differentiation between (component) product H and (component) product L can be treated as either vertical or horizontal. The marginal cost for offering product m is c_m, $m = H, L$, and it is natural to assume $c_H > c_L$.

Notice that we do not make assumptions regarding the relative magnitude of the profit margins $v_H - c_H$ and $v_L - c_L$.[7]

Probabilistic selling. Suppose that the firm uses the probabilistic/opaque selling strategy. The firm guarantees one of the two products, but hides the identity of the product that the consumer will actually obtain until after the purchase is completed. The decision of adopting the opaque selling strategy and the price of the opaque goods needs to be made before any of the selling seasons. If the firm does not use opaque selling, then it sells the *transparent* products H and L, or one of them to consumers. In our simple setting, the firm can freely choose one of the following three options: (i) sell transparent products only, i.e., traditional selling, (ii) sell probabilistic products only, and (iii) sell both transparent and opaque products at the same time. We shall show how consumer bounded rationality can induce the firm to sell opaque goods, in the absence of all the motivations proposed in the existing literature.

Information structure. The firm's opaque selling strategy and pricing decisions are all observable to consumers. However, if the firm uses opaque selling, the chance or probability of offering product H, $\xi \in [0,1]$, is *not known* to consumers. All the other parameters are common knowledge. In the extant literature, it is typically assumed that each consumer has *rational expectations* about the probability of offering each component product. Rational expectations can be justified when consumers have ample opportunities to repeatedly and adaptively learn the firm's strategies over an infinitely long time. The salient and intended feature of our model here is to relax this assumption. Our model applies to settings where consumers may not have opportunities that allow them to form rational expectations. In particular, we shall adopt the *anecdotal reasoning* framework proposed in the recent economics literature (see, e.g., Osborne and Rubinstein (1998), Spiegler (2006a), Spiegler (2006b), (Spiegler 2011, Chapters 6-7), Szech (2011) and references therein). In this framework, consumers use past experiences, word-of-mouth, and anecdotal reasoning to make their purchasing decisions. As discussed in the introduction, in the flight or hotel booking settings, some communities and information board websites, such as biddingfortravel.yuku.com, betterbidding.com, and bidontravel.com, provide such information to consumers. The specifics of our model would be clearly stated shortly.

Sequence of events. To model consumer bounded rationality (or equivalently the limited/scarce learning opportunities for consumers), we consider the setting where the firm sells the product to different generations of *new* consumers.[8] There are infinitely many discrete time

[7] As we shall show, their relative magnitude determines which transparent selling is more profitable, selling product H or selling product L.

[8] Our framework can be used to model repeat customers as well, as we will show in Section 3.3.

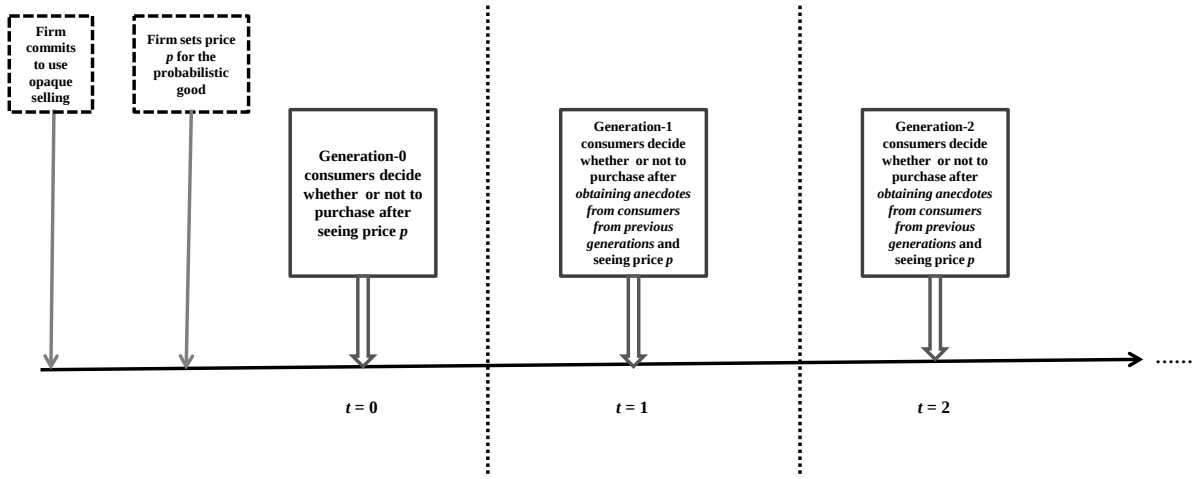
"stages" indexed by $t = 0, 1, 2, 3, ...$ The firm first commits to using the opaque selling strategy and setting price p before all the selling seasons. Recall that we denote $\xi = \mathbb{P}(i = H) \in [0, 1]$ as the probability of offering product H in its opaque selling, where i denotes consumer i's product "realization" from the firm. Boundedly rational consumers do not exactly know ξ. Motivated by Spiegler (2006a), we use the following dynamics to model the anecdotal learning and reasoning process: In stage 0, generation-0 new consumers enter into the market and randomize with equal probabilities[9] to purchase or not purchase given that they have *no* information about ξ. After their purchasing decisions, each consumer may obtain her individual product realization. Then, they leave the market. In stage 1, generation-1 consumers enter into the market. Before making their purchasing decisions, each consumer has the opportunity to communicate with some of the generation-0 consumers so that each consumer can obtain K "samples" or "anecdotes" of the product realization in stage 0, $K = 1, 2, 3, ...$ In general, in stage $t = 1, 2, 3, ...,$ each generation-t consumers can sample from some of the generation-(t') consumers about the realized product before making her purchasing decision in period t, for all $t' < t$. Each generation-t consumer decides to purchase the product or not based on her K "samples." However, her *own* product offered by the firm at stage t is an *independent* realization from ξ. Hence, the product that she actually obtains is probably different from her samples. In fact, one can show that, with such limited information, it is rational for each consumer to rely on her samples/anecdotes. Therefore, bounded rationality in the sense of anecdotal reasoning can be equivalently interpreted as full rationality with limited or scarce information in our setting (see Spiegler (2011) for more discussions).

Figure 1 depicts the sequence of events for $t = 0, 1, 2$ for illustration purpose. We shall focus on the long-run steady-state, i.e., for $t = \infty$. Hence, our multi-stage model reduces to a single-stage model where this single-stage dynamics represent the anecdotal reasoning we are interested to capture. In other words, the dynamic model illustrated in Figure 1 provides a justification of the (steady-state) one-period model we shall focus on. The distinctive feature and advantage of this (steady-state) one-period model is that it captures the dynamics of consumer anecdotal learning and reasoning in a simple fashion.

Remark 1. Our model is motivated by the literature (e.g., Jiang (2007), Fay and Xie (2008), Fay and Xie (2010), Jerath et al. (2010), Jerath et al. (2009)). However, there are several notable differences between our model and the literature: First, we assume consumers are homogeneous

[9] Assuming any other strictly positive probability will not affect our results.

Figure 1 Sequence of events



while they typically adopt a Hotelling model to study consumer heterogeneity.[10] Second, the market size, i.e., the demand, is deterministic and normalized to 1 for convenience. Third, capacity is ample so that the firm does not make the quantity decision. Fourth, we investigate the role of asymmetric costs while they are assumed to be symmetric in the extant literature. Finally and importantly, while the literature assumes rational expectations, we relax this assumption by focusing on boundedly rational expectations in the sense of anecdotal reasoning. These model differences allow us to separate and focus on the impact of *boundedly rational expectations* on the firm's opaque selling strategy.

Benchmark. Before analyzing the model of boundedly rational consumers, let us first discuss the benchmark case where consumers have rational expectations as studied in the literature, meaning they *perfectly know or anticipate* the firm's strategy ξ. In this case, we show that the firm does not have any incentives to use opaque selling:

Proposition 1. *If consumers have rational expectations, the firm does not use the opaque selling strategy in equilibrium.*

The proofs of all the propositions and lemmas are relegated to Appendix A. If consumers have rational expectations, the firm earns the maximum profit $\Pi_R^* = \max\{v_H - c_H, v_L - c_L\}$ by only selling either product H with price $p_R^* = v_H$ or only product L with price $p_R^* = v_L$. Hence, opaque selling is never (strictly) optimal in this setting.

[10] Not adopting the Hotelling model serves our purpose: we would like to eliminate consumer heterogeneity as a possible motivation for using opaque selling, and thus focus on consumer bounded rationality. Nevertheless, we also extend our model to include consumer heterogeneity in Section 3.3 and the online supplement as a robustness check.

Proposition 1 is completely in line with the extant literature: without heterogeneous consumers, demand uncertainty, capacity issues, or competition, opaque selling is not recommended. Hence, probabilistic selling is not profitable if consumers use probabilistic reasoning in this setting.

In this paper, we shall assume that consumers are boundedly rational in the sense of anecdotal reasoning. This framework was first developed by Osborne and Rubinstein (1998) where all agents are boundedly rational, later modified and elaborated by Spiegler (2006a,b) to allow for a rational agent in the game. We shall show that consumer bounded rationality alone can provide a new rationale for adopting opaque selling.

Before analyzing the model, we first discuss the specifics of the anecdotal-reasoning framework, which generalizes what is coined as $S(1)$ in the recent economics literature (see, e.g., Spiegler (2006a,b)):

The $S(K)$ framework. Each individual consumer obtains K anecdotes or samples about a product realization that occurred in some previous periods, $K = 1, 2, 3, ...$ These samples can be obtained through multiple consumers from the previous generations. For example, the decoder websites aforementioned may facilitate the sample/anecdote dissemination. We use the indicator random variable $I_{\{i,j\}} = I_{\{j=H\}}$ to denote whether consumer i gets a sample j indicating the "realized" product was product H or not: $I_{\{i,j\}} = 1$ if product H was offered; $I_{\{i,j\}} = 0$ otherwise. It is clear that $\mathbb{E}I_{\{i,j\}} = \xi$ since we focus on the *steady-state* dynamics of the system described in Figure 1. Each consumer's samples are totally independent of other consumers', and thus consumer samples can be different from each other. Each consumer relies on her samples/anecdotes to make her purchasing decision.

We now analytically formalize the $S(K)$ model. Denote

$$\xi_i(K) \equiv \frac{1}{K} \sum_{j=1}^{K} I_{\{i,j\}} \tag{1}$$

as the mean of the samples that consumer i obtains, where $I_{\{i,j\}}$ is the indicator random variable for product H offered or not for the j-th sample. We assume that each consumer "combines" multiple samples by simply taking the sample average.[11] It is clear that $\lim_{K\to\infty} \xi_i(K) = \xi$, i.e., the sample average becomes the population mean, then we are back to the rational-expectations case (recall Proposition 1). Hence, the rational-expectations model can be viewed as a special case of the anecdotal reasoning framework when $K = \infty$.

[11] How a consumer combines different samples to make her purchasing decision is an interesting question. For example, she may use a conservative rule by relying on the "worst" sample or an optimistic rule by believing in the "best" sample only. In this paper, we adopt the commonly-used statistical rule of taking the sample average.

Under the anecdotal reasoning based on K samples, consumer i purchases if and only if

$$\xi_i(K)v_H + (1-\xi_i(K))v_L \geq p. \tag{2}$$

If $K=\infty$, inequality (2) is precisely the condition for a rational consumer to purchase the opaque product. We are interested in cases where $K<\infty$, i.e., there are a limited number of samples for each consumer. Equivalently, we have referred to those consumers as boundedly rational consumers throughout the paper, to distinguish from rational consumers who perfectly know ξ. The parameter K can be defined as the level of consumer rationality. The larger K is, the more "accurate" consumer expectations are: when $K=1$, we obtain the $S(1)$ model used in the economics literature; when $K=\infty$, we are back to the benchmark case where consumers have rational expectations.

3.2. Optimal Pricing and Selling Strategies

We now investigate the firm's optimal pricing and selling strategies. Let $\gamma_K(\xi,p)$ be the fraction of the consumers that purchase, then we have the following result:

LEMMA 1. *The fraction of the consumers that purchase is*

$$\gamma_K(\xi,p) = 1 - \Sigma_{n=0}^{\lfloor \frac{K(p-v_L)}{(v_H-v_L)}-\epsilon \rfloor} B(n;K,\xi) \tag{3}$$

for an arbitrarily small $\epsilon>0$, where $B(n;K,\xi)=\binom{K}{n}\xi^n(1-\xi)^{K-n}$ is the probability mass function of the binomial distribution with parameters K and ξ.

The firm chooses (ξ,p) to maximize its expected profit

$$\Pi_K(\xi,p) = \gamma_K(\xi,p)\left[p-\xi c_H-(1-\xi)c_L\right] = \left[1-\Sigma_{n=0}^{\lfloor \frac{K(p-v_L)}{(v_H-v_L)}-\epsilon \rfloor} B(n;K,\xi)\right]\left[p-\xi c_H-(1-\xi)c_L\right], \tag{4}$$

subject to $\xi\in[0,1]$ and $p\in[v_L,v_H]$.

REMARK 2. This profit in equation (4) is the firm profit in the steady state. Formally, as depicted in Figure 1, let us denote $\Pi_K^{(t)}(\xi,p)$ as the profit in period t, for $t=0,1,2,...$ Then, we can define the long-run average profit

$$\widehat{\Pi}_K(\xi,p) \equiv \lim_{T\to\infty}\frac{1}{T}\Sigma_{t=0}^{T}\Pi_K^{(t)}(\xi,p).$$

Given that the steady-state profit $\Pi_K(\xi,p)=\lim_{t\to\infty}\Pi_K^{(t)}(\xi,p)$, we can show that $\Pi_K(\xi,p)=\widehat{\Pi}_K(\xi,p)$ by noticing that the system goes to the steady state from $t=1$ on. Hence, although the profit in the initial period $t=0$ might be low, it does not impact the long-run average profit $\Pi_K(\xi,p)$.

Solving this two-variable constrained optimization problem is generally challenging; however, we have the following result to characterize the optimal selling strategies:

PROPOSITION 2. *(i) The optimal price* $p_K^* \in \{p_j = v_L + \frac{j}{K}(v_H - v_L), j = 0, 1, 2, ..., K\}$, *and the optimal product offering strategy* $\xi_K^* \in \arg\max_{\xi \in [0,1]} \Pi_K(\xi, p_K^*)$.
(ii) The optimal policy (p_K^*, ξ_K^*) *can be found using the following algorithm: Compute* $\xi(p_j) \in (0,1)$ *that solves the polynomial equation of degree* K*:*

$$(K-j+1)\binom{K}{j-1}(1-\xi(p_j))^{K-j}(\xi(p_j))^{j-1}[p_j - \xi(p_j)c_H - (1-\xi(p_j))c_L]$$
$$-(c_H - c_L)\left[1-(K-j+1)\binom{K}{j-1}\int_0^{1-\xi(p_j)} t^{K-j}(1-t)^{j-1}dt\right] = 0$$

for each p_j, $j = 1, 2, ..., K-1$*; then compute the* $K+1$ *candidate profits* $\Pi_K(0, v_L)$, $\Pi_K(1, v_H)$ *and* $\Pi_K(\xi(p_j), p_j)$ *for* $j = 1, 2, ..., K-1$*; the optimal policy* $p_K^* = p_j$ *and* $\xi_K^* = \xi(p_j)$ *is the pair that achieves the maximum among them.*

Part (i) of Proposition 2 shows that solving the two-variable optimization problem in (4) can be done by restricting our search of the optimal price over a *finite* number of candidates, which allows us to solve a simpler optimization problem. Furthermore, Part (ii) provides an implementable computing algorithm to find the optimal pricing and product offering policy. This algorithm is easily implementable in typical computing or optimization software due to the fact that the first-order conditions are all *polynomial* equations. According to the fundamental theorem of algebra (see, for example, Jacobson (2012)), every non-zero, single-variable, degree K polynomial with (possibly complex) coefficients has, counted with multiplicity, exactly K (possibly complex) solutions. However, it remains unclear whether opaque selling is optimal, i.e., whether $\xi_K^* \in (0,1)$ or not. Providing necessary conditions analytically for the optimality of opaque selling for a general K is impossible due to the Abel's impossibility theorem (Jacobson (2012)). We refer the reader to Section 1 of the online supplement for details. To demonstrate how the algorithm works, we present a numerical example in Section 2 of the online supplement, where we show that the optimal price can be strictly lower than v_H under opaque selling.

It turns out we can provide simple *sufficient* conditions for opaque selling being optimal.

PROPOSITION 3. *If either of the two sufficient conditions holds: (i)* $v_H \in \left[v_L + c_H - c_L, \frac{(K+1)c_H - c_L}{K}\right)$*; (ii)* $v_L \in \left(v_H + c_L - c_H, c_L + \frac{K^K(v_H - c_L)^{K+1}}{(K+1)^{K+1}(c_H - c_L)^K}\right)$ *and* $v_H < \frac{(K+1)c_H - c_L}{K}$*, then it is optimal for the firm to use opaque selling. If neither of the two conditions holds, opaque selling may not be optimal.*

It is also clear that, when $K = \infty$, the condition of Proposition 3 can never be satisfied, which is consistent with Proposition 1 that, with rational expectations, opaque selling is not recommended.

REMARK 3. We would like to point out that there are other sufficient conditions for the optimality of opaque selling for a given K. Proposition 3 only gives the simplest one among them. To illustrate how one may find other sufficient conditions, we discuss the case when $K=2$; the same logic applies to any $K>2$. We have shown that the optimal price under opaque selling can only be either $p_1=\frac{v_H+v_L}{2}$ or $p_2=v_H$. Proposition 3 has provided a sufficient condition assuming that $p_2=v_H$ is the price charged. We now assume that firm charges the price $p_1=\frac{v_H+v_L}{2}$, in which case, the firm's expected profit function is

$$\begin{aligned}\Pi_K(\xi,p_1) &= [1-(1-\xi)^2]\left[\tfrac{v_H+v_L}{2}-\xi c_H-(1-\xi)c_L\right]\\ &= (c_H-c_L)\xi^3-\left[\tfrac{v_H+v_L}{2}+2(c_H-c_L)-c_L\right]\xi^2+(v_H+v_L-2c_L)\xi,\end{aligned}$$

for $\xi\in[0,1]$. The firm chooses $\xi\in[0,1]$ to maximize its profit. We have the first-order condition:

$$\frac{\partial\Pi_K(\xi,p_1)}{\partial\xi}=3(c_H-c_L)\xi^2-(v_H+v_L+4c_H-6c_L)\xi+v_H+v_L-2c_L=0,$$

which has two roots $\xi_1=\frac{v_H+v_L+4c_H-6c_L+\sqrt{(v_H+v_L+4c_H-6c_L)^2-12(c_H-c_L)(v_H+v_L-2c_L)}}{6(c_H-c_L)}$ and $\xi_2=\frac{v_H+v_L+4c_H-6c_L-\sqrt{(v_H+v_L+4c_H-6c_L)^2-12(c_H-c_L)(v_H+v_L-2c_L)}}{6(c_H-c_L)}$. If any one of the roots ξ_j are in the interval $(0,1)$ and yields $\Pi_K(\xi_j,p_1)>\max\{v_H-c_H,v_L-c_L\}$ for $j\in\{1,2\}$, then opaque selling is (strictly) optimal. Moreover, if $v_H+v_L\geq 2c_H$, then the second-order derivative $\frac{\partial^2\Pi_K(\xi,p_1)}{\partial\xi^2}=6(c_H-c_L)\xi-(v_H+v_L+4c_H-6c_L)\leq 0$ for $\xi\in[0,1]$, in which case any local optima $\xi_j\in(0,1)$ is globally optimal. However, this sufficient condition is not as clean as the one provided in Proposition 3. We refer the reader to Appendix B for some additional sufficient conditions.

What is the intuition for the result that opaque selling can be profitable due to consumer bounded rationality? Notice that some consumers have *overestimated* the true probability of receiving product H, and others have underestimated this probability. Because of this overestimation, the firm can charge a higher price or incur a lower cost for its opaque product than it could if all consumers had rational expectations. Meanwhile, charging a higher price or decreasing the probability of offering product H also lowers the customer demand quantity. Hence, when determining its optimal selling strategy ξ, the firm has a tradeoff between a *higher profit margin* and a *lower demand quantity*. This intricate tradeoff is nonlinear–polynomial in ξ, to be exact; recall the firm profit function in equation (4). It is not *a priori* clear whether opaque selling is profitable. The sufficient conditions in Proposition 3 tell us when opaque selling is indeed optimal. When neither of these sufficient conditions is satisfied, then it is indeed possible that the demand quantity reduction outweighs the profit margin increment, in which case probabilistic selling is not profitable.

The key message is that, bounded rationality in the sense of anecdotal reasoning alone provides strict incentives for the firm to use opaque selling. The intuitive explanation is that, consumers over-infer from their limited information. By creating product uncertainty or obfuscating its product offering, the firm may be able to take advantage of the consumers who overestimated the probability in getting product H without turning away too many consumers who underestimated.

Next, we would like to point out a couple of noteworthy findings that complement the existing literature (e.g., Fay and Xie (2008)). First, it is interesting that, our prediction of the product offering strategy is not necessarily one half which is typically recommended in the literature in the absence of capacity constraints and the absence of demand uncertainty (see, e.g., Fay and Xie (2008), and Jerath et al. (2010)).[12] This finding contributes to the existing literature, and is driven by consumer bounded rationality (in the sense of anecdotal reasoning).

Secondly, our study also complements Fay and Xie (2008) in that asymmetric costs are essential in our model while they assumed symmetric costs. As per Proposition 3, cost asymmetry can actually help the firm as $c_H > c_L$ is necessary for either of the two sufficient conditions to hold. On p. 685, Fay and Xie (2008) offered an insightful discussion that asymmetric costs may undermine the profitability of probabilistic selling since the firm may face the "uncertainty credibility" problem. It is intriguing that our model shows that asymmetric costs do not necessarily hurt the firm's profit. This seemingly striking difference can be explained by exploiting the uncertainty credibility argument. Recall that, our simple model represents the steady-state of the consumer-learning dynamics depicted in Figure 1. Anecdotal reasoning consumers believe in observed anecdotes/samples, and they make their purchasing decisions based on their observations. This provides an incentive for the firm to commit to *obfuscating* its product offering strategy: leaving strictly positive chances for the consumers to observe both favorable (i.e., product H) and unfavorable outcomes (i.e., product L). Hence, the independent information provided by some websites (for example, bidding-fortravel.yuku.com, betterbidding.com, and bidontravel.com) has at least two benefits for the firm: (i) it helps the firm overcome the commitment and credibility problem (as argued in Fay and Xie (2008)); (ii) it provides an important source for the anecdotes/samples that consumers can rely on to make their purchasing decisions. The interesting point is that, it can be precisely the consumers' anecdotal reasoning behavior that makes probabilistic selling profitable.

Finally, we would like to comment on the asymptotic behavior of the optimal product offering strategy ξ_K^* as K approaches infinity. Let $p_{j^*} = p_K^*$, we have:

[12] We refer the reader to the online supplement for a numerical study.

LEMMA 2. *When K is sufficiently large, we have $\xi_K^* \geq \frac{j^*}{K}$.*

Lemma 2 tells us that, the firm will offer product H with a strictly positive probability that is bounded from below, as K becomes large. This is consistent with the result that, when $K=\infty$, we have either (i) $j^*=K$, and $\xi_\infty^*=1$ if $v_H-c_H \geq v_L-c_L$, or (ii) $j^*=0$, and $\xi_\infty^*=0$ if $v_H-c_H < v_L-c_L$.

3.3. Model Robustness

In the basic model discussed above, we have shown that opaque selling can be optimal because of consumer bounded rationality. Intuitively, this is because the firm can take advantage of consumers who have overestimated the probability of obtaining the favorable product due to anecdotal reasoning without turning away too many consumers who underestimated. Mathematically, it comes from the fact that the firm's profit is a polynomial function of its product assignment probability, which oftentimes has interior maxima. In the online supplement, we have conducted several extensions and robustness checks of this main result, by incorporating rational priors, repeat customers, and consumer heterogeneity, respectively. For brevity, we only present the main results in this section, and refer the reader to the online supplement for details.

3.3.1. Rational Prior. In the basic model, we assumed that consumers do not have any prior information before obtaining any samples. Hence, they make decisions based on their anecdotes only. In some cases, before obtaining those samples/anecdotes, a consumer may have formed some prior belief from other sources. After obtaining some anecdotes, the consumer can combine the prior belief with these anecdotes. Would our previous finding be robust to the presence of prior beliefs/priors?

Suppose that each consumer comes to the market with a prior belief ξ_0 for the probability that product H will be offered by the firm. In general, this prior belief may be any value between zero and one. In this section, we assume that this prior belief happens to be rational/correct, i.e., $\xi_0=\xi$. After obtaining K samples, each consumer updates her belief by simply taking the average between them. Hence, the updated belief for consumer i becomes

$$\xi_i(K) \equiv \frac{1}{K+1}\left(\xi + \sum_{j=1}^{K} I_{\{i,j\}}\right).$$

It is still true that $\lim_{K\to\infty}\xi_i(K)=\xi$ for each consumer i, i.e., the sample average becomes the population mean, then we recover the rational-expectations model.

Under the anecdotal reasoning based on the prior rational belief ξ and K samples, consumer i purchases if and only if $\xi_i(K)v_H+(1-\xi_i(K))v_L \geq p$, which is similar as in the basic model. In the online supplement, we have investigated the firm's optimal selling strategy. In Proposition A-1 of

the online supplement, we show that the optimal policy (p_K^*, ξ_K^*) for the $S(K)$ model with rational priors can be found using the following algorithm: For each $j = 0, 1, 2, ..., K$, compute $\xi(j) \in [0, 1]$ that maximizes the single-variable polynomial function of degree $K+1$:

$$\widehat{\Pi}_K(\xi, j) = \Pi_K(\xi, p) = \left[1 - \Sigma_{n=0}^{j-1} B\left(n; K, \xi\right)\right] \left[v_L + \frac{\xi + j}{K+1}(v_H - v_L) - \xi c_H - (1-\xi)c_L\right],$$

where $\Sigma_{n=0}^{j-1} B\left(n; K, \xi\right) = 0$ if $j = 0$; then compute the $K+1$ candidate profits $\widehat{\Pi}_K(\xi(p_j), j)$ for $j = 0, 1, 2, ..., K$; the optimal policy $p_K^* = v_L + \frac{\xi_K^* + j}{K+1}(v_H - v_L)$ and $\xi_K^* = \xi(j)$ is the pair that achieves the maximum among them. Based on this result, we are able to provide similar sufficient conditions as in the basic model for opaque selling being optimal, in the presence of a rational prior.

3.3.2. Repeat Customers. In the basic model, we assumed that all customers in each period are *new* customers. They rely on anecdotes obtained from the customers who purchased in the previous periods. In this section, we would like to show that, our model can be used to model *repeat* customers, who may use their own previous experiences (in some previous periods) to estimate the probability ξ that product H will be offered by the firm.

Suppose that in each period $t-1$, a population of homogeneous customers come to the market. The customers have a common prior $\xi_0 \in [0, 1]$ for the probability of receiving product H from the firm. The prior can be biased. Based on this prior, they make their purchasing decisions at the beginning of period $t-1$, for $t = 1, 3, 5, ...$ After their purchasing decisions are made, each customer obtains a product realization. Each customer will use her purchasing experience as a new anecdote before making her purchasing decision in period t. Hence, each customer i has the updated estimate $\xi_{i,1} = \frac{1}{2}(\xi_0 + I_i)$, where $I_i \in \{0, 1\}$ is the indicator function for whether or not product H was received during period $t-1$. Hence, in period t, all customers are *repeat* customers who have purchased in period $t-1$.[13] We relegate the detailed analysis to the online supplement.

We show that opaque selling may still be optimal when selling to repeat customers. As long as repeat customers have scarce information that does not allow them to perfectly infer the firm's strategy, there is a room for the firm to benefit from those customers who overestimated the chance of receiving the favorable product. Admittedly, depending on the parameters, it is also possible that the firm has to lower its price so significantly that opaque selling becomes suboptimal.

[13] We focus on the setting where customers only repeat once, just for the purpose of demonstrating the main idea. However, one can easily extend our simple model to a setting where customers may purchase repeatedly for multiple times during multiple periods.

3.3.3. Consumer Heterogeneity. In the basic model, we have made a strong assumption that customers are all homogeneous, to isolate the impact of bounded rationality. We would like to conduct a robustness check with respect to this assumption, considering that consumer heterogeneity appears important in many real settings.

We assume that consumer valuations over product H are different.[14] We assume that v_H follows a cumulative distribution function F_H over the interval $[\underline{v}_H, \overline{v}_H]$, where $\underline{v}_H > v_L$. For simplicity, we further assume that F_H is a uniform distribution. In other words, we can think of consumers locating uniformly on the interval $[\underline{v}_H, \overline{v}_H]$ where the location indicates the valuation of product H. We have analyzed this setting in the online supplement, and find that our main finding remains. As a matter of fact, the existing literature has already shown that consumer heterogeneity alone provides incentives for the firm to use opaque selling.

In conclusion, the findings from analyzing these model extensions demonstrate the robustness of the results derived from the basic model.

However, we have assumed that the firm is a monopoly. How would our main result from the basic model change when the consumers have an additional option of buying from a transparent (i.e., non-opaque) competitor? For instance, Hotwire sells its hotels probabilistically, while its competitor (such as Hotels.com or Travelocity) lists a range of hotels transparently. Given that this type of competition is frequently observed in practice, we would like to explore any public policy implications. It is important to understand whether or not competition can exert external discipline on the opaque-selling firm. In the next section, we examine the impact of competition on the profit advantage of probabilistic selling.

4. Competition

Suppose that there are two firms in the market: Firm O that can sell component product H and product L, and can choose to sell probabilistically as we studied in the basic model, and a *multi-product* competitor, firm T, that sells the two products transparently: a transparent product H which generates consumer valuation v_H with the unit cost c_H, and a transparent product L generating consumer valuation v_L with the unit cost c_L.

Benchmark. As a benchmark, we first analyze the setting when consumers are assumed to be fully rational. The game is as follows: The two firms engage in a simultaneous price competition that is similar in spirit to the Bertrand competition. Following a similar argument as in the basic model, we can show that firm O has no strict incentives to use an opaque selling strategy: If $v_H - c_H \geq v_L - c_L$, then selling product H is optimal; otherwise, selling product L is optimal.

[14] For simplicity, we assume that consumers still have homogeneous valuation v_L for product L. We have confirmed that relaxing this assumption will not qualitatively change our result.

PROPOSITION 4. *Suppose consumers have rational expectations. There exists a unique equilibrium in the duopoly market. In equilibrium, firm O does not use the opaque selling strategy and it sells product m^* that has the highest value of $v_m - c_m$ for $m = H, L$. Both firms set prices at their marginal costs and have equal market shares.*

Proposition 4 shows that each firm virtually competes against each other so that the market share is split evenly between them, i.e., there is indeed intense competition between them.

According to Proposition 4, the equilibrium demand is split equally between firm O and firm T and they both earn zero profit if consumers have rational expectations. We now analyze the case when $v_H - c_H \geq v_L - c_L$ holds (and the main qualitative finding here remains for the other case). We are interested in whether opaque selling can be profitable when consumers have boundedly rational expectations in the sense of anecdotal reasoning. To gain intuition without going into technical complexity, we first analyze the special case of $S(K)$ when $K = 1$ where we have a simple model that is amenable to complete analytical characterization.

PROPOSITION 5. *Suppose consumers use anecdotal reasoning and $K = 1$. (a) If $v_H \geq 2v_L - 2c_L + c_H$, there exists an equilibrium where firm O uses the opaque selling strategy: $\xi_O^* = \frac{1}{2}$, $p_O^* = \frac{1}{2}(v_H + c_H)$, and firm T sells product H at price $p_T^* = v_H$. In this equilibrium, both firms earn strictly positive profits: $\Pi_O^* = \frac{1}{4}(v_H - c_L)$ and $\Pi_T^* = \frac{1}{2}(v_H - c_H)$. (b) If $v_H \in [2c_H - 2c_L + v_L, 2v_L - 2c_L + c_H]$, there exists an equilibrium where firm O uses the opaque selling strategy: $\xi_O^* = \frac{1}{2}$, $p_O^* = v_H - v_L + c_L$, and firm T sells product H at price $p_T^* = 2(v_H - v_L + c_L) - c_H$. In this equilibrium, both firms earn strictly positive profits: $\Pi_O^* = \frac{1}{2}(v_H - v_L) - \frac{1}{4}(c_H - c_L)$ and $\Pi_T^* = v_H - c_H - v_L + c_L$. (c) Otherwise, opaque selling is not adopted, and both firms price at marginal costs and earn zero profits in equilibrium.*

Proposition 5 shows that under reasonable conditions, opaque selling is optimal even in the presence of competition, where the multi-product competitor who sells both products transparently could undercut the high price charged by the opaque selling firm. Surprisingly, opaque selling can create a win-win outcome for the firms when customers are boundedly rational, compared to the equilibrium outcome with rational customers in Proposition 4. Competition certainly puts a pressure for the opaque selling firm to lower its price. However, it may not necessarily diminish the profitability of opaque selling. Firm O can still exploit the fact that some consumers overestimate the probability of offering product H by sometimes offering product L whose marginal cost is low. By doing that, firm O leaves the customers who obtained samples of product L to its competitor firm T, which reduces firm T's incentives to undercut its price. Hence, opaques selling can effectively

soften competition. This finding appears to be consistent with observed practices: an opaque selling firm typically faces competitors who sell transparent goods; this firm may not charge a very high price, yet it still has incentives to offer opaque products because it can offer some products whose marginal costs are lower. This justifies the practice of opaque selling in the presence of competition.

If neither of the conditions in part (a) and (b) of Proposition 5 is satisfied, firm T has an incentive either (i) to undercut firm O's price by selling product H at a lower price to capture all the consumers or (ii) to sell product L to give the consumers who obtained samples of product H more surplus so that they are willing to switch. In those settings, an equilibrium where opaque selling is adopted does not exist.

The impact of consumer bounded rationality. Compared with the competition case with fully rational customers in Proposition 4, firm O earns strictly positive profit by opaque selling. Interestingly, opaque selling also strictly benefits the transparent firm. With boundedly rational consumers and opaque selling, the equilibrium prices are higher. The interesting finding is that, opaque selling can *soften* price competition, which creates a win-win outcome for the two competing firms. This finding is complementary to both Shapiro and Shi (2008) who show that opaque selling enables sellers to price discriminate between consumers with different price sensitivities, and Fay (2008b) who finds that with sufficient brand loyalty, opaque sales help reduce price rivalry. In our model, in contrast, all consumers are homogeneous and hence have the same price sensitivity and there is no difference in brand loyalty. Interestingly, we show that, in the absence of consumer heterogeneity for price discrimination, opaque selling may still soften competition because of consumer bounded rationality. Intuitively, opaque selling provides an effective market segmentation mechanism that allows both firms to make positive profits in equilibrium, due to the heterogeneity in consumers' anecdotes/samples despite of consumer homogeneity.

The fact that firm O leaves the customers who obtained samples of product L to its competitor firm T, which reduces firm T's incentives to undercut its price, has interesting managerial implications. Given that consumers are heavily influenced by anecdotes/samples, would firm O have an incentive to give the consumers who got product H an incentive to spread the word (e.g., post on discussion boards, social media, etc.) and discourage those who obtained product L from doing so? If that is the case, then that would undermine the integrity/credibility of the information source. Our analysis shows that, interestingly, firm O actually does not have an incentive to do so, since doing that would *intensify* price competition with firm T and thus makes both firms worse off. The reason can be seen by noticing that, to ensure that in equilibrium, firm T is satisfied with demand quantity $1-\xi$ (i.e., consumers who obtained samples of product L) and does not cut

price to just below firm O's price p_O a little to attract the ξ number of consumers (who obtained samples of product H) to buy product H, we require $p_T - c_H \leq (1-\xi)(v_H - c_H)$, where $p_T = p_O - \epsilon$ for sufficiently small $\epsilon > 0$. If firm O artificially increases the fraction of consumers who obtained product H from ξ to $\widetilde{\xi} > \xi$, then firm T will take this fact into account since its expected profit $(1-\widetilde{\xi})(v_H - c_H)$ (from selling to only those $1-\widetilde{\xi}$ consumers who obtained samples of product L) decreases. This means that firm T has a stronger incentive to cut its price to capture all the consumers from firm O, which results in a fierce price competition. Hence, the fact that opaque selling can soften price competition *helps* the credibility of anecdotes/samples.

Finally, we would like to point out that, the intuition gained from the $K=1$ case extends to the settings when $K = 2, 3, ...$, following the same argument. In that case, the firm O would solve a constrained optimization problem, where the opaque selling price has to satisfy $p_O \leq [1-\gamma_K(\xi)](v_H - c_H) + c_H$, where $\gamma_K(\xi) = 1 - \Sigma_{n=0}^{\lfloor \frac{K(p_O - v_L)}{(v_H - v_L)} - \epsilon \rfloor} B(n; K, \xi)$ is the fraction of customers that purchase from firm O in equilibrium, and firm O's expected profit function is $\Pi_K^O(\xi, p_O) = \left[1 - \Sigma_{n=0}^{\lfloor \frac{K(p_O - v_L)}{(v_H - v_L)} - \epsilon \rfloor} B(n; K, \xi)\right] [p_O - \xi c_H - (1-\xi)c_L]$. Using a similar algorithm as in Proposition 2, we can show that the firm's constraint optimization problem with two variables can be equivalently reduced to the following simpler problem: For each $j = 1, 2, ..., K-1$, the firm chooses ξ_j that maximizes its expected profit function

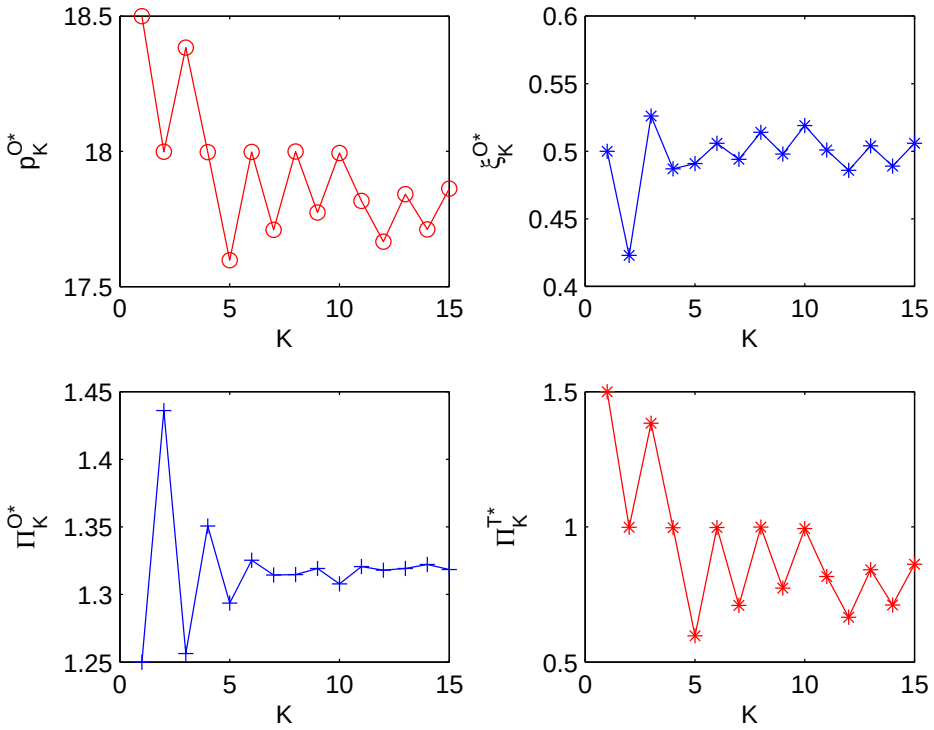
$$\Pi_K^O(\xi, p_O(j)) = \left[1 - \Sigma_{n=0}^{j-1} B(n; K, \xi)\right] \left[\Sigma_{n=0}^{j-1} B(n; K, \xi)(v_H - c_H) + (1-\xi)(c_H - c_L)\right],$$

subject to $\xi \in [0, 1]$ and

$$p_O(j) = \Sigma_{n=0}^{j-1} B(n; K, \xi)(v_H - c_H) + c_H \in \left(v_L + \frac{j-1}{K}(v_H - v_L), v_L + \frac{j}{K}(v_H - v_L)\right].$$

Then, the firm chooses j^* that corresponds to the maximum profit among the $K-1$ number of candidate profits. If $\xi_{j^*} \in (0,1)$, $v_H \geq 2(v_L - c_L) + c_H$ and $p_O(j^*) > v_L$, then there exits an opaque-selling equilibrium where opaque selling brings positive profits to both firms. To investigate whether such an equilibrium indeed exists, we conduct a numerical study. Figure 2 shows the opaque-selling equilibrium outcomes and firms' profits, for a set of numerical examples where we use the parameters $v_H = 20, c_H = 17, v_L = 16, c_L = 15$ and $K = 1, 2, ..., 15$. We can see that the qualitative finding from Proposition 5 still holds. It is interesting that, although for a general $K > 1$, the equilibrium product assignment strategy ξ_{j^*} can be larger or smaller than 0.5, it appears that, it is close to and fluctuates around 0.5, as shown in the upper-right panel of Figure 2. This result in our competition setting is complementary to the prediction of symmetric product assignment offered in Fay and Xie (2008) in a monopoly setting.

Figure 2 **Opaque-selling equilibrium outcomes in a competitive market for different** K **(**$v_H = 20, c_H = 17, v_L = 16, c_L = 15, K = 1, 2, ..., 15$**)**



5. Discussion and Conclusion

The probabilistic/opaque selling strategy, in which a seller guarantees one of several fully specified products, but hides the identity of the product that the consumer will actually obtain until after the purchase is completed, has been used by many firms in practice and received a lot of attention in the recent marketing, economics, and operations management literature. Under what conditions, and why, is opaque selling attractive to firms? The extant literature has offered several convincing and insightful explanations: price discriminate heterogeneous consumers with different preferences, reduce mismatches between uncertain demand and capacity, and soften price competition. In a market where all these considerations are absent or non-salient, the existing literature would suggest practitioners not to use opaque selling. In this paper, we provided a behavioral explanation: to *exploit* or take advantage of consumer bounded rationality in the sense of anecdotal reasoning. This research is vital because we found that opaque selling may still be recommended in such a market. We built a simple model where the firm is a monopoly, consumers are homogeneous, and there is no demand uncertainty or capacity constraint. This model allowed us to isolate the impact of consumer bounded rationality on the adoption of the opaque selling strategy. Our modeling framework can

include the rational-expectations model as a special case. We found that, while it is never optimal to adopt the opaque selling strategy when consumers have rational expectations, it can be optimal under certain conditions when consumers are boundedly rational. We showed that opaque selling may soften price competition and increase the industry profits due to consumer bounded rationality. Our study complements the existing literature in several aspects (e.g., asymmetric costs, nonequal product offering, etc.), underscores the importance of considering consumer bounded rationality when deciding whether or not to adopt the opaque selling strategy, and shows that this strategy might be even more attractive than previously thought.

Our study shows that the firm may profit from systematic misunderstanding of its product offering strategy by consumers using probabilistic selling, in the absence of all other advantages proposed in the literature. Such a misunderstanding comes from anecdotal reasoning, a type of consumer bounded rationality. Marketers are actually familiar with adopting a certain marketing strategy to exploit consumer bounded rationality. For example, nine-ending pricing has been frequently used to take advantage of consumer cognitive biases – anchoring or misunderstanding the price information (Anderson and Simester (2003), Thomas and Morwitz (2005) and references therein). Interestingly, we show that probabilistic selling can share a similar mechanism of making a profit as nine-ending pricing. Notably, in contrast to nine-ending pricing, probabilistic selling can actually strictly improve some consumers' surplus if those consumers are lucky to obtain the favorable product (at a lower price than when it were sold transparently).

To isolate and focus on the impact of consumer bounded rationality, we have purposely chosen a simple model. Future research is needed to incorporate other rich factors into the model to develop modern decision support systems. For example, we have assumed that consumers are risk neutral throughout the paper, which is consistent with the literature. It will be of interest to consider consumer risk attitude. While risk aversion would reduce a consumer's willingness-to-pay of the opaque product, probabilistic selling may provide a device to discriminate consumers according to variation of their disposition toward risk (Fay and Xie (2008)). It would also be of interest to investigate how the intangibility of the opaque product may affect perceived risk (Laroche et al. (2005), Fay (2008b)).

For parsimony of the model, we have assumed that consumers' anecdotes/samples are independent from each other. One can extend our model to a setting where samples are correlated. The simplest way would be to assume that a group of N customers share the same samples, in which case the customers in the same group obtain perfectly correlated samples. All our results would remain in that setting because it is simply a re-scaling of the market size by N. For other

more sophisticated correlation structures, as long as the firm's tradeoff between profit margin and demand quantity exists due to consumer overestimation and underestimation as we discussed in the basic model, our results will continue to hold. Finally, we also assumed that each consumer has the same number K of samples. It would be of interest to consider the setting where consumers may have different numbers of samples. While we lose analytical tractability in that setting, we believe that our main qualitative results will likely remain given that introducing that heterogeneity does not appear to fundamentally change the firm's tradeoff in its decision-making. Moreover, in that setting, the firm might be able to use opaque selling to discriminate consumers based on the heterogeneity of their sample sizes. We hope that this study stimulates more future research in this area.

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Appendix.

This Appendix has two parts: Appendix A includes all the proofs of the results in the main paper; Appendix B provides additional simple sufficient conditions for the optimality of opaque selling based on a price that is strictly lower than v_H.

Appendix A: Proofs

In this part, we provide the detailed proofs of our results in the main paper.

Proof of Proposition 1. Suppose the firm uses opaque selling with probability ξ to offer product H. A rational consumer purchases the opaque good if her expected utility is greater than the price:

$$u_R \equiv \xi v_H + (1-\xi)v_L \geq p.$$

Hence, the optimal price for the firm is

$$p_R^*(\xi) = \xi v_H + (1-\xi)v_L$$

Then the firm expected profit is

$$\Pi_R(\xi, p_R^*(\xi)) = \xi(v_H - c_H) + (1-\xi)(v_L - c_L) \leq \max\{v_H - c_H, v_L - c_L\},$$

where the inequality is strict for any $\xi \in (0,1)$ if $v_H - c_H \neq v_L - c_L$. Hence, the firm is always weakly better off by selling one of the products only. □

Proof of Lemma 1. Simplifying inequality (2) and rearranging, we obtain the result. □

Proof of Proposition 2. (i) First, observing (4), it is clear that it is optimal for the firm to charge p such that $\frac{K(p-v_L)}{(v_H-v_L)}$ is an integer, for any ξ. The reason comes from the floor function in (4). The first term $1 - \Sigma_{n=0}^{\lfloor \frac{K(p-v_L)}{(v_H-v_L)} - \epsilon \rfloor} B(n; K, \xi)$ does not change with respect to p in each range where $\frac{K(p-v_L)}{(v_H-v_L)}$ is not an integer, while the second term $p - \xi c_H - (1-\xi)c_L$ strictly increases in p. Hence, the firm will rationally charge p such that $\frac{K(p-v_L)}{(v_H-v_L)}$ is an integer for any ξ. The optimal product offering strategy can be found by solving a one-variable optimization problem: $\xi_K^* \in \arg\max_\xi \Pi_K(\xi, p^*(K))$.

(ii) First, notice that the cumulative distribution function of the binomial distribution is a polynomial function that can be expressed in terms of the regularized incomplete beta function

$$P\left(\sum_{j=1}^{K} I_{\{i,j\}} \leq s\right) = \Sigma_{n=0}^{\lfloor s \rfloor} B(n; K, \xi) = (K-s)\binom{K}{s}\int_0^{1-\xi} t^{K-s-1}(1-t)^s dt.$$

Let $s = \frac{K(p-v_L)}{(v_H-v_L)} - \epsilon$. Then

$$\Pi_K(\xi, p) = \gamma_K(\xi, p)\left[p - \xi c_H - (1-\xi)c_L\right] = \left[1 - \Sigma_{n=0}^{\lfloor \frac{K(p-v_L)}{(v_H-v_L)} - \epsilon \rfloor} B(n; K, \xi)\right]\left[p - \xi c_H - (1-\xi)c_L\right],$$

$$=(1-(K-s)\binom{K}{s}\int_0^{1-\xi} t^{K-s-1}(1-t)^s dt)\left[p-\xi c_H-(1-\xi)c_L\right].$$

We can verify that $(1-(K-s)\binom{K}{s}\int_0^{1-\xi} t^{K-s-1}(1-t)^s dt)$ is an increasing and concave function of ξ if $\xi \le \frac{s}{K-1}$. Then the first-order condition is

$$\begin{aligned}\frac{\partial \Pi_K(\xi,p)}{\partial \xi} =&(K-s)\binom{K}{s}(1-\xi)^{K-s-1}(\xi)^s\left[p-\xi c_H-(1-\xi)c_L\right]\\ &+\left[1-(K-s)\binom{K}{s}\int_0^{1-\xi} t^{K-s-1}(1-t)^s dt\right](-(c_H-c_L))=0.\end{aligned}$$

for any fixed p and $s=j-1$. Combining with part (i), we have the algorithm. □

Proof of Proposition 3. We only need to find an opaque selling strategy that strictly dominates transparent selling. We discuss two cases. Suppose $v_H-c_H \ge v_L-c_L$, then the optimal transparent selling strategy is to sell product H only, which yields firm profit $\Pi_K(1,v_H)=v_H-c_H$.

Let $p^*=v_H$, then $\gamma_K(\xi,v_H)=\xi^K$. Hence,

$$\Pi_K(\xi,v_H)=\gamma_K(\xi,v_H)\left[p-\xi c_H-(1-\xi)c_L\right]=-(c_H-c_L)\xi^{K+1}+(v_H-c_L)\xi^K.$$

Taking the first-order derivative, we obtain

$$\frac{\partial \Pi_K(\xi,v_H)}{\partial \xi}=-(K+1)(c_H-c_L)\xi^K+K(v_H-c_L)\xi^{K-1}.$$

If at the point $\xi=1$, the profit function is strictly decreasing, we can conclude that there exists some $\xi<1$ that yields a strictly higher profit for the firm. That condition is simply

$$\frac{\partial \Pi_K(\xi,v_H)}{\partial \xi}|_{\xi=1}=-(K+1)(c_H-c_L)+K(v_H-c_L)<0,$$

which is equivalent to $v_H<\frac{(K+1)c_H-c_L}{K}$.

Suppose $v_H-c_H<v_L-c_L$, then the optimal transparent selling strategy is to sell product L only with profit v_L-c_L. Let $p^*=v_H$, the first-order condition yields

$$\widehat{\xi}=\frac{K(v_H-c_L)}{(K+1)(c_H-c_L)},$$

which is strictly less than 1 if $v_H<\frac{(K+1)c_H-c_L}{K}$. The second-order condition $\frac{\partial^2 \Pi_K}{\partial \xi^2}|_{\xi=\widehat{\xi}}=-K(v_H-c_L)\widehat{\xi}^{K-2}<0$. If the corresponding profit $\widehat{\Pi}_K=\frac{K^K(v_H-c_L)^{K+1}}{(K+1)^{K+1}(c_H-c_L)^K}>v_L-c_L$, then transparent selling is strictly dominated. □

Proof of Lemma 2. To analyze the optimal strategy, we introduce the Hoeffding's inequality. Let $N=\sum_{j=1}^K I_{\{i,j\}}$, i.e., it is the number of samples in which product H is observed/realized for a consumer (say, consumer i). Then we have

$$P(N\le(\xi-\epsilon)K)\le\exp(-2\epsilon^2K),$$

and

$$P(N \geq (\xi+\epsilon)K) \leq \exp(-2\epsilon^2 K),$$

based on Hoeffding's inequality.

We now show the lemma by contradiction. Suppose that $\frac{p_K^*-v_L}{v_H-v_L} - \xi = \epsilon > 0$. But $1 - \sum_{n=0}^{\lfloor \frac{K(p_K^*-v_L)}{v_H-v_L} \rfloor} B(n,K,\xi) = P(N > \lfloor \frac{K(p_K^*-v_L)}{v_H-v_L} \rfloor) = P(N > \lfloor (\xi+\epsilon)K \rfloor) \leq \exp(-2\epsilon^2 K)$, which is sufficiently small if K is large. As a result, this cannot be true under the optimal policy. □

Proof of Proposition 4. The argument is the same as that for proving the Nash equilibrium for the Bertrand competition. First, if each firm sets its price equal to its marginal cost (unit cost), neither firm will earn any profits. However, if one firm sets price equal to marginal cost, then if the other firm raises its price above unit cost, then it will earn nothing, since all consumers will buy from the firm still setting the competitive price. No other price is an equilibrium. If both firms set the same price above their unit costs and share the market, then each firm has an incentive to undercut the other by an arbitrarily small amount and capture the whole market and almost double its profits. So there can be no equilibrium with both firms setting the same price strictly above their marginal costs. Hence the only equilibrium occurs when both firms set price equal to their unit costs. Consumers are split equally between the two firms since they have the same consumer utility. □

Proof of Proposition 5. (a) To ensure that in equilibrium, firm T is satisfied with demand $1-\xi$ (i.e., consumers who obtained samples of product L) and does not cut price to just below p_O a little to attract the other ξ consumers to buy product H, we require

$$p_T - c_H \leq (1-\xi)(v_H - c_H),$$

where $p_T = p_O - \epsilon$ for sufficiently small $\epsilon > 0$. We also require that $p_O > v_L$ so that the $1-\xi$ consumers indeed purchase product H from firm T given that firm T charges v_H to sell product H. Notice that under the condition that $v_L - c_L \leq v_H - c_H$, i.e., product H has a higher profit margin, firm T prefers to sell product H (rather than product L) to the $1-\xi$ customers who obtained samples of product L. Firm O's profit is then

$$\Pi_O = \xi[p_O - \xi c_H - (1-\xi)c_L] < -(v_H - c_L)\xi^2 + (v_H - c_L + \epsilon)\xi,$$

for $\xi \in [0,1]$, which is maximized at $\xi^* = \frac{1}{2}$, and $p_O^* = \frac{1}{2}(v_H + c_H)$. Notice that, when $\xi = 0$ or 1, $\Pi_O = 0$ since that would result in intense price competition between the two firms.

The optimal profits are

$$\Pi_O^* = \frac{1}{4}(v_H - c_L),$$

and

$$\Pi_T^* = \frac{1}{2}(v_H - c_H).$$

Given that firm T has product L, it also has the option to attract the ξ customers who obtained samples of product H by selling them product L at price p_{TL}^* if $v_H - p_O^* < v_L - p_{TL}^*$, which requires that $\frac{1}{2}(v_H - c_H) < v_L - p_{TL}^*$. However, we require $p_{TL}^* \geq c_L$. Hence, we obtain the condition $c_L \leq p_{TL}^* < v_L - \frac{1}{2}(v_H - c_H)$ under which firm T has an incentive to deviate. If $v_H - c_H \geq 2(v_L - c_L)$, this condition cannot hold. Hence, the condition $v_H \geq 2(v_L - c_L) + c_H$ ensures that firm T does not have an incentive to attract the ξ customers from firm O by selling product L to them. We now check for the condition: We require $p_O^* = \frac{1}{2}(v_H + c_H) > v_L$ so that the $1 - \xi^* = 1/2$ consumers indeed purchase from firm T at price v_H. Hence, we have the condition $v_H > 2v_L - c_H$.

Therefore, we conclude that, if $v_H \geq \max\{2(v_L - c_L) + c_H, 2v_L - c_H\}$, then the stated outcome is indeed a Nash equilibrium where no firm has strict incentives to deviate given its competitor's strategy. Notice that $2(v_L - c_L) + c_H = 2v_L - c_H + 2(c_H - c_L) > 2v_L - c_H$, we have the equivalent condition $v_H \geq 2v_L - 2c_L + c_H$. Under this condition, we have checked that neither of the two firms has an incentive to deviate given its competitor's strategy.

(b) Suppose in equilibrium, firm T sells product H at a price p_{TH}^* strictly lower than v_H. Following the same argument as in part (a), firm T is satisfied with demand $1 - \xi$ (i.e., consumers who obtained samples of product L) and does not cut price to just below p_O a little to attract the ξ consumers who obtained samples of product H to buy product H, we require

$$p_T - c_H \leq (1 - \xi)(p_{TH}^* - c_H),$$

where $p_T = p_O - \epsilon$ for sufficiently small $\epsilon > 0$. We also require that $v_H - p_{TH}^* \geq v_L - p_O$ so that the $1 - \xi$ consumers who obtained samples of product L indeed purchase product H from firm T given that firm T charges p_{TH}^* to sell product H. Notice that we also need the condition that $p_{TL} - c_L \leq p_{TH}^* - c_H$, so that firm T prefers to sell product H (rather than product L) to the $1 - \xi$ customers who obtained samples of product L, where $p_{TL} \leq p_O$ is the price that firm T would charge for product L in order to attract them from firm O. This condition simplifies to $p_{TH}^* \geq 3c_H - 2c_L$. Firm O's profit is then

$$\Pi_O = \xi[p_O - \xi c_H - (1 - \xi)c_L] < -(p_{TH}^* - c_L)\xi^2 + (p_{TH}^* - c_L + \epsilon)\xi,$$

for $\xi \in [0,1]$, which is maximized at $\xi^* = \frac{1}{2}$, and $p_O^* = \frac{1}{2}(p_{TH}^* + c_H)$. Notice that, when $\xi = 0$ or 1, $\Pi_O = 0$ since that would result in intense price competition between the two firms.

The optimal profits are

$$\Pi_O^* = \frac{1}{4}(p_{TH}^* - c_L),$$

and

$$\Pi_T^* = \frac{1}{2}(p_{TH}^* - c_H).$$

The conditions above are equivalent to $p_{TH}^* \in [3c_H - 2c_L, 2(v_H - v_L) + c_H]$. Notice that $3c_H - 2c_L \leq 2(v_H - v_L) + c_H$ is equivalent to $v_H \geq v_L - c_L + c_H$.

Given that firm T has product L, it also has the option to attract the ξ customers who obtained samples of product H by selling them product L at price p_{TL}^* if $v_H - p_O^* < v_L - p_{TL}^*$, which requires that $v_H - \frac{1}{2}(p_{TH}^* + c_H) < v_L - p_{TL}^*$. However, we require $p_{TL}^* \geq c_L$. Hence, we obtain the condition $c_L \leq p_{TL}^* < v_L - v_H + \frac{1}{2}(p_{TH}^* + c_H)$ under which firm T has an incentive to deviate. If $p_{TH}^* \leq 2(v_H - v_L + c_L) - c_H$, this condition cannot hold. Hence, the condition $p_{TH}^* \leq 2(v_H - v_L + c_L) - c_H$ ensures that firm T does not have an incentive to attract the ξ customers from firm O by selling product L to them.

Combining the conditions together, we have $p_{TH}^* \leq \min\{2(v_H - v_L + c_L) - c_H, 2(v_H - v_L) + c_H, v_H\}$. Notice that $2(v_H - v_L + c_L) - c_H < 2(v_H - v_L) + c_H$. Given the assumption that $v_H \leq 2(v_L - c_L) + c_H$, we know $2(v_H - v_L + c_L) - c_H \leq v_H$. Hence, $p_{TH}^* = 2(v_H - v_L + c_L) - c_H$. Recall that we require $p_{TH}^* = 2(v_H - v_L + c_L) - c_H \geq 3c_H - 2c_L$, which is equivalent to $v_H \geq 2c_H - 2c_L + v_L$. Note that $2c_H - 2c_L + v_L > v_L - c_L + c_H$ since $c_H > c_L$.

The optimal profits are

$$\Pi_O^* = \frac{1}{2}(v_H - v_L) - \frac{1}{4}(c_H - c_L) > 0$$

and

$$\Pi_T^* = v_H - c_H - v_L + c_L > 0,$$

given that $v_H \geq v_L - 2c_L + 2c_H$.

Therefore, we conclude that, if $v_H \in [2c_H - 2c_L + v_L, 2v_L - 2c_L + c_H]$, then the stated outcome is indeed an equilibrium where no firm has strict incentives to deviate given its competitor's strategy.

(c) If neither of the conditions in part (a) and (b) is satisfied, then opaque selling is not adopted in equilibrium, because of the arguments shown above. In this case, the two firms price at their marginal costs in equilibrium as characterized in Proposition 4. □

Appendix B: Additional Sufficient Conditions

In this section, we provide some additional simple sufficient conditions for the optimality of opaque selling for a price that is lower than v_H, for a given $K = 3, 4, 5, ...$ The main intention is to show that it is possible to provide relatively simple sufficient conditions for the opaque selling to be optimal, by using a price that is not the highest price v_H.

Depending on which product is more profitable, we discuss two cases:

First, suppose $v_H - c_H \geq v_L - c_L$. Consider the opaque selling strategy such that $p = v_L + \frac{K-1}{K}(v_H - v_L)$ and $\xi = \frac{K-1}{K}\frac{v_L - c_L}{c_H - c_L} \in (0,1)$. In this case $\gamma_K(\xi, p) = \xi^K + K(1-\xi)\xi^{K-1}$. The firm profit is

$$\Pi_K(\xi, p) = \left[\xi^K + K(1-\xi)\xi^{K-1}\right]\left[\frac{1}{K}(v_L - c_L) + \frac{K-1}{K}(v_H - v_L)\right].$$

But

$$\xi^K + K(1-\xi)\xi^{K-1} = \left[\frac{(K-1)(v_L - c_L)}{K(c_H - c_L)}\right]^K + K\left[1 - \frac{(K-1)(v_L - c_L)}{K(c_H - c_L)}\right]\left[\frac{(K-1)(v_L - c_L)}{K(c_H - c_L)}\right]^{K-1}$$

To ensure the firm profit is more than $v_H - c_H$, it is sufficient to have

$$\frac{K-1}{K}(v_H - v_L)\left[\frac{(K-1)(v_L - c_L)}{K(c_H - c_L)}\right]^K \geq v_H - c_H.$$

Second, suppose $v_H - c_H < v_L - c_L$. Consider the opaque selling strategy such that $p = v_L + \frac{K-1}{K}(v_H - v_L)$ and $\xi = \frac{K-2}{K}\frac{v_H - v_L}{c_H - c_L} \in (0,1)$ for $K = 3, 4, 5, ...$ In this case $\gamma_K(\xi, p) = \xi^K + K(1-\xi)\xi^{K-1}$. The firm profit is

$$\Pi_K(\xi, p) = \left[\xi^K + K(1-\xi)\xi^{K-1}\right]\left[v_L - c_L + \frac{1}{K}(v_H - v_L)\right].$$

To ensure the firm profit is more than $v_L - c_L$, it is sufficient to have

$$\left[v_L - c_L + \frac{1}{K}(v_H - v_L)\right]\left[\frac{(K-2)(v_H - v_L)}{K(c_H - c_L)}\right]^K \geq v_L - c_L,$$

or equivalently

$$\left[1 + \frac{v_H - v_L}{K(v_L - c_L)}\right]\left[\frac{(K-2)(v_H - v_L)}{K(c_H - c_L)}\right]^K \geq 1,$$

for $K = 3, 4, 5, ...$ Using a similar logic, one can certainly provide many other sufficient conditions.