The Macroeconomic and Welfare Effects of Targeted Inflation Support Packages*

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Abstract

We evaluate the welfare effects of targeted fiscal responses to inflationary shocks, such as sector-specific subsidies and household-specific transfers. We do so in a multi-sector New Keynesian model nesting a flexible heterogeneous-household block, matched one-for-one to distributional data. We derive a micro-founded welfare loss function that decomposes into macroeconomic and distributional components, as well as into partial- and general-equilibrium effects. Social preferences for distributional fluctuations are disciplined using observed tax and transfer policies over the business cycle. Applying this framework to the UK's 2022 cost-of-living package, which combined an energy price cap with targeted transfers, we obtain three findings. First, the package reduces welfare, at an average cost of about £150 per household, more than three times as large as a partial-equilibrium evaluation would suggest. Second, the dominant channel is an unintended redistribution, with large welfare losses borne by higher-income but heavily indebted households, who are negatively affected both by rising interest rates and by the taxes required to finance the package. Third, the standard central bank practice of "looking through" the mechanical effects of the energy price cap on CPI inflation amplifies these losses: under a rule that does not look through the cap, the same package would slightly raise welfare.

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1 Introduction

When a geopolitical shock sends food and energy prices soaring, governments rarely stand by. Following Russia's 2022 invasion of Ukraine, the direct burden of higher food and energy prices fell most heavily on poorer households, who spend larger budget shares on these necessities (Jaravel, 2019; Levell et al., forthcoming). Nearly every European government responded with a fiscal package: energy subsidies or price caps, combined with special transfers to particular categories of households (Sgaravatti et al., 2021). These interventions were granular by design, targeting specific sectors and specific groups, and intended to shield vulnerable households from the shock. Yet whether such packages actually raise welfare is far from clear, as they affect households heterogeneously, through a variety of micro- and macroeconomic channels.

How should the welfare effects of such detailed packages be evaluated? We argue that a comprehensive quantitative tool needs three properties, which are standard practice in different literatures but are rarely combined. First, it should be matchable directly to micro data, capturing not only the distribution of income and wealth but also households' exposures to specific prices and specific policies. Since at least Deaton and Muellbauer (1980) this has been standard practice in the micro and public finance literatures, where flexible demand systems matched one-for-one to the data make it possible to weigh, say, the distributional benefits of an energy subsidy against the distortions it creates in consumption baskets.

Second, a comprehensive framework should capture not only sectoral, but also macroeconomic channels, which matter directly for welfare (Rotemberg and Woodford, 1997). Sectoral shocks naturally generate macroeconomic effects, for instance by raising the overall level of inflation and creating macroeconomic distortions. Fiscal interventions may either worsen or alleviate these. Moreover, the broader macroeconomic effects of a sectoral shock, together with the monetary policy response, give rise to further distributional consequences. For instance, if an energy shock raises the overall level of inflation and the central bank reacts by raising interest rates, this redistributes resources across households depending on their asset positions (Doepke and Schneider, 2006; Auclert, 2019). Indeed, Pallotti et al. (2024) find that nominal net positions were the key driver of differences in the welfare costs of the recent inflation surge across euro area households. As a result, a fiscal package that targets households purely on the basis of their exposure to energy prices may fail to mitigate the overall redistributive effects of the shock.

Third, any analysis of the redistributive effects of a shock inevitably requires taking a stance on the welfare consequences of redistribution. This is especially important here,

since the fiscal packages we consider include lump-sum transfers designed precisely to address the redistributive effects of sectoral shocks. For a framework to be of practical use to policymakers, it must rest on a social welfare function that adequately reflects the preferences embedded in real-world policy (Piketty and Saez, 2013; Saez and Stantcheva, 2016; Chetty, 2009). In our case specifically, we aim to avoid assumptions that would render certain fiscal interventions welfare-improving by design, even in the absence of a macroeconomic shock.[1]

This paper provides a framework with all three properties, bridging the gap between the micro and macro literatures. We nest a flexible heterogeneous-household block into an otherwise standard multi-sector New Keynesian model, building on Olivi et al. (2026), and embed two targeted fiscal instruments: sector-specific price controls and household-specific transfers. The household block can be matched one-for-one to the empirical distributions of income, wealth and consumption baskets, so that the model captures precisely the incidence of real-world fiscal packages, of sectoral shocks, and of the monetary policy reaction. At the same time, the model retains macro channels from which the applied micro literature typically abstracts, and which we find to be quantitatively important.

To evaluate the welfare effects of fiscal packages, we derive a second-order approximation to the welfare loss function in this environment and present two decompositions of it. The first splits the loss by the distortions it operates through: inflation within and between sectors, the output gap, and redistribution across households. The second splits it into the direct effects of a package and the macro effects it sets in motion, and therefore measures how much of the welfare effect a partial-equilibrium evaluation would miss.

To capture real-world preferences for redistribution, we then adapt the principle of inverse-optimum taxation to second order. Where the standard approach recovers society's valuation of the level of households' consumption from steady-state policy, studying fluctuations also requires pinning down how society values *fluctuations* of the distribution. We recover this from distributional time series, i.e. from the extent to which the observed tax and transfer system has dampened income fluctuations for different groups. If, for instance, taxes and transfers nearly stabilize net income for a particular group of households, we infer that the planner places a high value on avoiding utility fluctuations within that group. Estimating these weights on UK data from the decades preceding the war in Ukraine, we find that for much of the distribution the social welfare function is

[1]For instance, a utilitarian welfare function typically implies that it is optimal to eliminate steady-state inequality entirely through a far-reaching program of lump-sum transfers, even absent any shock. Since we do not observe such policies in practice, the utilitarian welfare function appears poorly suited to capturing the objectives of real-world policymakers.

close to a linear one, i.e. a Pareto-Negishi specification.[2] For the top decile, however, the UK government places significantly lower weights: it dampens income fluctuations for high-income households less than a planner with a linear social welfare function would.

With the framework at hand, we show that a sectoral price subsidy may "backfire": while it mechanically lowers the prices households pay for certain goods, it may lead to higher underlying inflation, push output above its efficient level, and raise the interest rate.[3]

Underlying the macro responses are demand and supply effects created by the price subsidy, which manifest themselves through wedges in the aggregate Euler equation and the New Keynesian Phillips Curve. The demand effect is familiar from the literature: a temporary subsidy boosts aggregate demand through intertemporal substitution, increasing output and inflation and thereby partly undermining the subsidy. The supply effect is, to the best of our knowledge, novel: the subsidy increases households' real wages, which raises labor supply through a substitution effect.[4] This, in turn, reduces firms' marginal costs, increasing output but lowering inflation, and thereby amplifying the effect of the subsidy on prices. Both effects thus tend to push up output but pull inflation in opposite directions. Which one prevails for inflation depends on monetary policy: in a simplified version of the model, underlying inflation rises when the central bank "looks through" the mechanical effects of the subsidy and does not respond strongly to the output gap, two conditions that are likely met in reality. With both the output gap and inflation increasing, the nominal interest rate rises as well.[5]

The macroeconomic channels turn out to be key for the welfare evaluation of fiscal packages, following a negative supply shock that pushes up necessity prices. We illustrate this point by contrasting the overall welfare effects with those obtained in a more narrow, partial-equilibrium evaluation of the policy. In such an evaluation, there is a simple welfare trade-off. On the one hand, the necessity subsidy may mitigate the distributional welfare losses created by the necessity supply shock, as it compensates households who are exposed to necessity prices. On the other hand, it creates a distortion in the consumption basket, as the policy leads to higher consumption of a good that is fundamentally more

[2]When the social preference function G is linear, social welfare takes the form $\int_i \omega_i V_i$, where ω_i is the Pareto-Negishi weight on household i.

[3]The macro effects of transfers depend on the cross-sectional relation between the incidence of the policies and marginal propensities to consume, a point that is well understood in the literature. In our calibrated model, we find these effects to be relatively minor.

[4]Since the subsidy is financed via taxes paid by households, the income effect on labor supply is absent, aside from distributional effects, which tend to be unimportant.

[5]Interestingly, we show that the interest rate also increases if monetary policy does respond strongly to the output gap.

expensive to produce, given the supply shock.

However, the policy's welfare effects may be drastically different in a broader evaluation that accounts for macro channels. In particular, the distributional benefit of the subsidy may turn into a loss once we account for the fact that, in response to the inflationary supply shock, the central bank raises the nominal interest rate, which creates a welfare loss by redistributing income from borrowers to savers. Moreover, the macro effects created by the subsidy make this distributional loss even worse since, as explained above, the subsidy leads to even higher inflation and higher interest rates. Analytically, we show that the subsidy improves the distributional component of welfare if households differ only in their consumption baskets, but worsens it if they differ only in wealth. Since there is a large degree of wealth heterogeneity in the data, we find the latter channel to be particularly important quantitatively. Beyond redistribution, the movements in headline inflation and the output gap created by the subsidy may further affect welfare.[6]

Quantitatively, we use the model to evaluate the UK's response to the 2022 cost-of-living crisis, which combined the Energy Price Guarantee, a cap on consumer energy prices financed by the Treasury, with a sequence of Cost-of-Living Payments, some universal and some targeted to lower-income, elderly, and disabled households. We match the model to the UK Living Costs and Food Survey, representing every household in the survey as a household in the model. In line with our analytical results, the package substantially raises the output gap and the nominal interest rate, while its effect on inflation is positive at first but later turns negative.

We find that the package reduces welfare, at an average cost of about £150 per household, more than three times as large as a partial-equilibrium evaluation would suggest, and driven largely by the price subsidy on electricity and gas.[7] The largest channel is not the textbook distortion of energy consumption but the redistribution channel through interest rates, as described above. In particular, indebted higher-income households help finance the package, receive little of it, and absorb the higher mortgage payments triggered by the monetary tightening that follows the shock.[8] The effect is particularly severe for indebted hand-to-mouth households, who are unable to smooth the impact of the shock, and is further amplified by the demand stimulus from the energy subsidy, which pushes nominal interest rates up even more.

Finally, we show that the standard central-bank practice of "looking through" the

[6] In the quantitative model, we find these effects to create further welfare losses, although the magnitudes are smaller than the losses created by the interest rate channel.

[7] This result is overall similar under a linear social welfare function, and the losses are larger when the package is financed by government debt rather than contemporaneous taxes.

[8] We assume that the taxes financing the package are proportional to steady-state labor income.

mechanical effects of the price cap amplifies these welfare losses. We consider a counterfactual monetary policy rule that responds to the movements in consumer energy inflation generated by the cap, just as it responds to core inflation. The nominal rate then falls as the cap is introduced and rises when it is lifted, which partly offsets the demand wedge and results in a softer interest rate response, with a modest tightening towards the end. This path undoes the adverse redistribution towards indebted households without raising inflation. Under this rule, the same fiscal package would slightly raise welfare rather than strongly reduce it.

Related Literature. This paper relates to three strands of work. First, we connect to microeconomic studies evaluating the welfare effects of energy interventions, see, e.g., Hahn and Metcalfe (2021) and Costa and Gerard (2021). Within this strand of literature, the most closely related paper is Levell et al. (forthcoming) who, like us, evaluate the recent fiscal policy response in the UK. We contribute to this literature by incorporating macroeconomic distortions into the welfare analysis, and show that these are quantitatively important and depend crucially on the monetary policy response. Moreover, we show that even when abstracting from the equilibrium effects triggered by the interventions themselves, accounting for concurrent movements in macroeconomic variables, such as nominal interest rates, can substantially affect welfare calculations.[9] Our welfare decomposition relates to Bergman et al. (2025), who decompose the distributional welfare effects of sectoral shocks into partial- and general-equilibrium components in a static model with flexible preferences. Our framework instead features nominal rigidities, household balance sheets and monetary policy, which drive much of the redistribution.

We also relate to a growing macroeconomic literature studying the aggregate effects of the 2022 energy shock and the role of policy in shaping its transmission, often using HANK frameworks.[10] Auclert et al. (2023) and Bayer et al. (2023) show that energy price subsidies can stimulate aggregate demand and create international spillovers. Andreolli et al. (2024) compare the macroeconomic effects of VAT changes on essential versus non-essential goods in a New Keynesian model with non-homothetic preferences. Relative to this literature, our framework is distinguished by two features: a one-for-one match to

[9]Our exercise further differs from Levell et al. (forthcoming) in that we develop a dynamic rather than a static framework, which lets us capture dynamic distortions, such as those related to consumption smoothing, that turn out to be important. Moreover, they primarily compare policy designs at a fixed budgetary footprint whereas we evaluate the realized package against a no-policy baseline, netting out the taxes that finance it. Finally, they estimate key parameters based on rich transaction-level data, which we rely upon to parametrize our model.

[10]See, e.g., Bachmann et al. (2022); Bayer et al. (2023); Auclert et al. (2023); Kharroubi and Smets (2024); Bobasu et al. (2025); Gnocato (2025); Chan et al. (2024).

micro data, which captures household-level exposures to shocks and policies in as much detail as the micro literature does; and a welfare evaluation based on a micro-founded loss function that decomposes cleanly and whose distributional weights are disciplined empirically.[11]

Our normative analysis connects to the literature on welfare analysis with household heterogeneity, e.g. Dávila and Schaab (2025), and to the inverse-optimum taxation and revealed-preference tradition in public finance (Christiansen and Jansen, 1978; Bourguignon and Spadaro, 2012; Bargain and Orsini, 2006; Saez and Stantcheva, 2016; Hendren, 2020). Like this literature, we assume a social welfare function under which the steady-state distribution is efficient. In our setting, however, fluctuations in the distribution further reduce social welfare. Finally, our paper relates to work emphasizing the importance of non-homothetic demand for optimal tax design (Jaravel and Olivi, 2021).

2 The Model

In this section we introduce fiscal support packages into a multi-sector New Keynesian model with generalized, non-homothetic preferences. The production side is standard, and the household block follows Olivi et al. (2026), which is what allows a one-for-one match to micro data. New here are the two fiscal instruments: a sector-specific price control and a household-specific transfer.

Agents. The economy's production side consists of a finite number of sectors, indexed by $k = 1, 2, \ldots, K$. Within each sector, there is a unit mass continuum of varieties, which are indexed by j. Each of these varieties is produced by a different firm. On the consumption side, there is a unit mass of heterogeneous households, indexed by i. Finally, there is a government which conducts fiscal and monetary policy.

Fiscal support packages. We consider two kinds of fiscal instruments. The first is a sector-specific price control on goods sold to consumers, which takes the form of a time-varying subsidy $\tau_{k,t}$.[12] We assume that the price subsidy does not apply to sales of intermediate goods to other firms. Let $p_{k,t}(j)$ denote the wholesale price of variety j in sector k at time t. We denote the regulated retail price paid by consumers by $q_{k,t}(j) = p_{k,t}(j)(1 - \tau_{k,t})$.

[11] We derive the welfare-relevant New Keynesian Phillips Curve in the spirit of Rubbo (2023).

[12] Under the Energy Price Guarantee (EPG), the UK government capped the per-unit price of electricity and gas sold to consumers, but compensated energy suppliers for the difference between the market price and the regulated price. Therefore, the policy was a time-varying subsidy per unit of electricity and gas.

The second kind of fiscal policy we consider is a lump-sum transfer to households, denoted by $\zeta_t(i)$. Note that the transfer may vary not only over time, but also across households. In this way, the model is able to capture that the UK cost-of-living payments were partly targeted to particular groups: lower-income households, the elderly and the disabled.[13]

Households. We model household preferences and heterogeneity following Olivi et al. (2026). Utility over consumption is given by $u_i(\mathbf{c}_t(i))$, where $\mathbf{c}_t(i) = \{c_{1,t}(i), .., c_{K,t}(i)\}$ is a vector of sectoral consumption bundles and where $u_i(\cdot)$ is twice differentiable, increasing, and concave. This general specification allows for non-homotheticity over sectors as well as household-specific preference shifters. For simplicity, we assume that preferences over varieties within sectors are CES, i.e. $c_{k,t}(i) = \left(\int_0^1 c_{k,t}(i,j)^{\frac{\epsilon_k - 1}{\epsilon_k}} dj\right)^{\frac{\epsilon_k}{\epsilon_k - 1}}$, where ϵ_k is a constant but sector-specific elasticity of substitution between varieties. It follows that the wholesale price index for sector k is given by $P_{k,t} = \left(\int_0^1 p_{k,t}(j)^{1-\epsilon_k} dj\right)^{\frac{1}{1-\epsilon_k}}$, whereas the retail price index is $Q_{k,t} = P_{k,t}(1 - \tau_{k,t})$.

In every period, households decide on consumption ($\mathbf{c}_t(i)$), effective labor supply ($n_t(i)$), and on saving in one-period nominal bonds ($b_{t+1}(i)$). Moreover, households die with a probability $\delta \in [0, 1)$ in every period, and each deceased household is replaced by a new household.

Newborn households vary in a number of dimensions. As mentioned above, their preferences and eligibility for transfers may differ. In addition, they vary in terms of their labor productivity, denoted $\vartheta(i)$, and in terms of their initial nominal wealth positions. Moreover, households own the firms and receive their dividends, but their ownership shares may vary. A household may be either Hand-to-Mouth (HtM) or non-HtM, which we treat as a permanent characteristic and we denote $\varphi(i)$ the share of HtM households within type i. [14] Finally, households discount the future at a rate $\beta \in (0, 1)$.

[13] In the quantitative version of the model, each household i will correspond to a household in a survey, in which we observe many characteristics from which it can be determined whether a household received a certain transfer or not.

[14] Households also differ by their birth date, which we will denote later by a superscript t_0. For now, we suppress this aspect in our notation.

The decision problem of a non-HtM household can be expressed as:

$$\begin{aligned} \max_{\mathbf{c}_t(i),n_t(i),b_{t+1}(i)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1-\delta))^s \left(u_i(\mathbf{c}_{t+s}(i)) - \chi_i \left(\frac{n_{t+s}(i)}{\vartheta(i)} \right) \right), \\ s.t. \qquad & e_t(i) + \frac{b_{t+1}(i)}{R_t} = b_t(i) + W_t n_t(i) + \sum_k \varsigma_k(i) Div_{k,t} + \zeta_t(i). \end{aligned} \tag{1}$$

where $e_t(i) = \sum_{k=1}^{K} e_{k,t}(i) = \sum_{k=1}^{K} Q_{k,t} c_{k,t}(i)$ denotes consumption expenditures, $\frac{n_t(i)}{\vartheta(i)}$ is labor supply in hours, $\chi_i(\cdot)$ is a function which captures disutility from labor supply, R_t is the nominal interest rate, W_t is the nominal wage rate per effective unit of labor, $Div_{k,t}$ denotes firm dividends from sector k, $\varsigma_k(i)$ is the household's ownership share of that sector and $\zeta_t(i)$ is a lump-sum transfer, net of lump-sump taxes. Finally, $\mathbb{E}_t$ is the conditional expectations operator. Hand-to-Mouth households face the same decision problem, but with the additional constraint that $b_{t+1}(i) = b_t(i)$, i.e. these households must consume their current income.

To simplify the expressions, in the remainder of this paper we will assume that the Elasticity of Intertemporal Substitution (EIS) and the Frisch elasticity of labor supply are homogeneous across households and denote them by σ and ψ respectively.[15] For later use, let us define household i's elasticity of substitution between sectors as $\rho_{k,l}(i) \equiv -\left(\partial_{ln\ P_l} ln\ c_k(i) + c_l(i)\partial_e ln\ c_k(i)\right)$. We further assume that dividend shares are proportional to steady-state labor income shares.

Firms. Each firm produces a single variety in a particular sector and is therefore monopolistically competitive. In every period, a firm j in sector k decides on the amount of effective labor hired ($n_{k,t}(j)$), as well as on the amount of intermediate inputs, where $\tilde{Y}_{l,k,t}(j)$ denotes input bundles from sector l, which we assume are produced according to the same CES aggregator as the one used by the households. From this it follows that a firm's demand is given by:

$$y_{k,t}(j) = \left(\frac{p_{k,t}(j)}{P_{k,t}} \right)^{-\epsilon_k} (C_{k,t} + \sum_{l=1}^{K} \tilde{Y}_{k,l,t}), \tag{2}$$

where $C_{k,t} = \delta \sum_{t_0=-\infty}^{t} (1-\delta)^{t-t_0} \int_0^1 c_{k,t}^{t_0}(i) di$ is aggregate consumption, where the superscript t_0 denotes the household's birth date. The production function of the firm is given

[15] This can always be achieved, see Olivi et al. (2026).

by

$$y_{k,t}(j) = A_{k,t}F_k(n_{k,t}(j), \tilde{Y}_{1,k,t}(j), \tilde{Y}_{2,k,t}(j), ..., \tilde{Y}_{K,k,t}(j)), \tag{3}$$

Here, $y_{k,t}(j)$ denotes output, $F_k(\cdot)$ is a sector-specific production function with constant returns to scale and $A_{k,t}$ is an exogenous, sector-specific productivity variable, which evolves according to a stationary stochastic process. The firms' total cost of production is given by:

$$TC_{k,t}(j) = W_t n_{k,t}(j) + \sum_{l=1}^{K} P_{l,t}\tilde{Y}_{l,k,t}(j). \tag{4}$$

We introduce nominal rigidities into the model by assuming that a firm can adjust its price only with a probability $1 - \theta_k$ in a given period. The firm's optimal price setting problem is given by:

$$\max_{p_{k,t}(j), n_{k,t+s}(j), \{\tilde{Y}_{l,k,t+s}(j)\}_{l=1}^{K}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta_k^s \left(p_{k,t}(j)y_{k,t+s}(j) - (1-\iota_k)TC_{k,t+s}(j)\right),$$

subject to (2)-(4). Here, $\Lambda_{t,t+s}$ is the firm's stochastic discount factor.[16] In the above equation, ι_k is a time-invariant, sector-specific cost subsidy which may be used by the government to correct markup distortions in the steady state.[17]

Government. Let us now consider the government, which consists of a fiscal authority and a central bank. For now, we assume that fiscal policy follows a balanced budget, an assumption we will later relax in the quantitative analysis. The fiscal authority's budget constraint is given by:

$$\int_0^1 \zeta_t(i)di + \sum_{k=1}^{K}\left(\tau_{k,t}P_{k,t}C_{k,t} + \iota_k \int_0^1 TC_{k,t}(j)dj\right) = 0, \tag{5}$$

Fiscal policy is a time path for $\{\zeta_t(i)\}_{i=0}^1, \tau_{k,t}$, such that the above constraint is satisfied at any time period.[18]

[16] We assume that in the steady state $\Lambda_{t,t+s} = (1-\delta)^s\beta^s$, but we do not need to make further assumptions since we linearize the model around a steady state without inflation.

[17] The subsidy is funded by a lump-sum tax on households, through $\zeta_t(i)$ in 1.

[18] One could decompose the net transfer as $\zeta_t(i) = \zeta_t^{\text{transfer}}(i) - \zeta_t^{\text{tax}}(i)$. In every period, the government adjusts $\zeta_t^{\text{tax}}(i)$ to balance the budget, where we assume that, at the individual level, $\zeta_t^{\text{tax}}(i)$ is proportional to steady-state labor income.

Demographics and Market Clearing. In any period, a fraction δ of all households dies. We assume that each deceased household is replaced by a new household of the same type. A household's type is pinned down by its birth date t_0, labor productivity, $\vartheta(i)$, initial bond holdings, $b_{t_0}^{t_0}(i)$, preferences, u_i and χ_i, transfer eligibility, and HtM status. Bond market clearing implies that the average wealth of households is zero, and hence the same is true for deceased and newborn households, due to i.i.d. death probabilities. Therefore, the wealth given to newborn households can always be financed and the net inheritance from all deceased households is zero. Clearing in the labor market and the bond market requires, respectively:

$$\begin{aligned} N_t \equiv \delta \sum_{t_0=-\infty}^{t} (1-\delta)^{t-t_0} \int_0^1 n_t^{t_0}(i)di = \sum_{k=1}^{K} \int_0^1 n_{k,t}(j)dj, \\ \delta \sum_{t_0=-\infty}^{t} (1-\delta)^{t-t_0} \int_0^1 b_t^{t_0}(i)di = 0. \end{aligned} \tag{6}$$

Goods market clearing requires, for any goods variety:

$$\delta \sum_{t_0=-\infty}^{t} (1-\delta)^{t-t_0} \int_0^1 c_{k,t}^{t_0}(i,j)di + \tilde{y}_{k,t}(j) = y_{k,t}(j), \tag{7}$$

where $\tilde{y}_{k,t}(j) \equiv \left(\frac{p_{k,t}(j)}{P_{k,t}}\right)^{-\epsilon_k} \sum_{l=1}^{K} \tilde{Y}_{k,l,t}$ is firm demand for variety j in sector k. Finally, in every sector:

$$\tilde{Y}_{k,t} = \sum_{l=1}^{K} \int_0^1 \tilde{Y}_{k,l,t}(j)dj. \tag{8}$$

An equilibrium is a law of motion for prices and allocations such that households, firms and the government behave as specified above, and markets clear. It is worth noting that in the deterministic steady state of the model, households keep their bond holdings constant over time.[19] The model is thus consistent with any arbitrary steady-state distribution of wealth, which in the calibration we will take from the data.

3 Macroeconomic Effects of Support Packages

Before evaluating the welfare effects of fiscal packages, we first study their macroeconomic effects. In this section, we show that they create an expansionary effect on both

[19] It can be shown that, in the absence of idiosyncratic income risk and aggregate shocks, the target level of wealth equals current wealth.

aggregate demand and aggregate supply. The equilibrium outcomes are determined by the combination of these effects, which in turn depends on the monetary policy reaction.

To show this, we derive the first-order conditions of agents' optimization problems and linearize the equations around the steady state, and aggregate appropriately. The technical details of this can be found in Appendix A, which also provides the full set of linearized model equations that we use to solve the model quantitatively.

Here, we directly provide the key linearized conditions. To simplify the exposition, we make the following assumptions, which we maintain throughout Sections 3 and 4 and relax in the quantitative analysis (Section 5):

Assumption A. *Assume that there are no input-output linkages ($\tilde{Y}_{k,t} = 0$); that price stickiness and within-sector elasticities of substitution are homogeneous across sectors ($\theta_k = \theta$ and $\epsilon_k = \epsilon$ for all k); and that HtM households are randomly distributed across the population ($\varphi(i) = \varphi$).*

Moreover, monetary policy is assumed to target price stability in the steady state, and the subsidy ι_k is set to eliminate steady-state markups in all sectors. We use a hat to denote variables expressed in log deviations from the steady state, whereas variables without time subscripts denote steady-state values.

Price indices. Before analyzing the macro effects of fiscal policy, let us briefly discuss a number of important price indices that will enter the equations. First, note that consumption baskets are heterogeneous across households, due to non-homothetic preferences and idiosyncratic preference shifters. In what follows, we let $s_k(i) \equiv \frac{e_k(i)}{e(i)}$ denote the steady-state budget share of household i on sector k. The household-level Consumer Price Index (CPI) weighs sectors by these budget shares. In addition, let us define $\partial_e e_k(i) \equiv \frac{\partial e_k(i)}{\partial e(i)}$ as the household's *marginal* budget share. These marginal budget shares capture the allocation of a marginal unit of expenditure over sectors, see Olivi et al. (2026). These marginal shares differ from regular budget shares once preferences are non-homothetic. In particular, at the margin households spend more on luxuries and less on necessities than they do on average.

At the aggregate level, there are four key price indices. The first is the regular Consumer Price Index, which is a post-subsidy price index given by $\hat{Q}_{cpi,t} \equiv \sum \bar{s}_k \hat{Q}_{k,t}$, where $\bar{s}_k = \int_0^1 \frac{e(i)}{E} s_k(i) di$ is the aggregate expenditure weight on sector k. Note that this CPI averages post-subsidy sectoral prices (i.e. $\hat{Q}_{k,t}$) since these are the prices relevant to consumers. Analogously, we define the *Marginal Consumer Price Index* (MCPI) as $\hat{Q}_{mcpi,t} \equiv \sum \overline{\partial_e e_k} \hat{Q}_{k,t}$, where $\overline{\partial_e e_k} = \int_0^1 \frac{e(i)}{E} \partial_e e_k(i) di$ is the aggregated marginal budget share. As discussed in Olivi et al. (2026), the MCPI is a key index associated with aggregate quantities in the model. Intuitively, when households make optimal labor supply and saving decisions,

they consider on which sectors they spend at the margin.

The third important price index is wholesale CPI, i.e. the CPI before consumer subsidies are applied. This price index is given by $\hat{P}_{cpi,t} \equiv \sum \bar{s}_k \hat{P}_{k,t}$. It will turn out that this price index will capture the "pure" welfare cost of inflation, caused by markup dispersion within sectors.[20] Fourth and finally, we define the wholesale MCPI as $\hat{P}_{mcpi,t} \equiv \sum \overline{\partial_e e_k} \hat{P}_{k,t}$.

Aggregate supply effects of a sectoral subsidy. We are now ready to study the aggregate effects of the fiscal interventions. We first show that a sectoral price subsidy has an expansionary effect on aggregate supply. This can be observed from the New Keynesian Phillips Curve (NKPC). Defining wholesale CPI inflation as $\pi_{cpi,t} \equiv \hat{P}_{cpi,t} - \hat{P}_{cpi,t-1}$, the NKPC for this inflation index is given by:

$$\pi_{cpi,t} = \lambda\Big((\sigma^{-1} + \psi^{-1})\tilde{\mathcal{Y}}_t + \mathcal{NH}_t - \underbrace{\mathcal{F}_t}_{\text{fiscal wedge}}\Big) + (1-\delta)\beta\mathbb{E}_t\pi_{cpi,t+1}. \tag{9}$$

Relative to the textbook NKPC, two wedges appear: a non-homotheticity wedge $\mathcal{NH}_t$, which is highlighted in Olivi et al. (2026), and a fiscal wedge $\mathcal{F}_t$, which is new and captures the aggregate supply effect of a price subsidy. We discuss each in turn below. Here, $\lambda = \frac{(1-\theta)(1-\theta/R)}{\theta}$ is a slope coefficient which depends on the degree of price stickiness. The NKPC is in terms of wholesale rather than consumer inflation, since it derives from the price-setting decision of the firms. Like the standard NK model, the NKPC relates inflation to an output gap, which captures the labor supply distortions created by markups. In our case, it is given by $\tilde{\mathcal{Y}}_t = \frac{\psi}{1+\frac{\psi}{\sigma}}\left(\hat{W}_t - \sum_{k=1}^K \overline{\partial_e e_k}\hat{Q}_{k,t} - \sum_{k=1}^K \overline{\partial_e e_k}\hat{A}_{k,t}\right)$. That is, it captures the gap between households' marginal rate of substitution between leisure and consumption (which equals their marginal real wage), and the economy's marginal rate of transformation from labor into goods on the other hand. As we will see later, both $\pi_{cpi,t}$ and $\tilde{\mathcal{Y}}_t$ will capture key macro distortions in the welfare loss function.

Unlike the standard NK model, however, two wedges appear. The first, $\mathcal{NH}_t = (\hat{P}_{mcpi,t} - \hat{P}^*_{mcpi,t}) - (\hat{P}_{cpi,t} - \hat{P}^*_{cpi,t})$ is a non-homotheticity wedge, studied in detail by Olivi et al. (2026). Here, a star denotes variables in a counterfactual equilibrium with flexible prices. Note that under homothetic preferences, we obtain $\mathcal{NH}_t = 0$, i.e. the wedge drops out.

The second, a fiscal wedge $\mathcal{F}_t = -\left(\hat{Q}_{mcpi,t} - \hat{P}_{mcpi,t}\right) = \sum_{k=1}^K \overline{\partial_e e_k}\tau_{k,t}$, is novel and captures the aggregate supply effect of a subsidy. The subsidy pushes the consumer

[20] Implicit in our formulation of the model is the assumption that subsidies do not directly reduce markup dispersion and therefore cannot mechanically reduce the pure cost of inflation.

price index $\hat{Q}_{mcpi,t}$ below the wholesale price index $\hat{P}_{mcpi,t}$. As a result, the wedge $\mathcal{F}_t$ turns positive. From the NKPC we observe that, given inflation expectations, this reduces current inflation and/or increases the output gap. Intuitively, a decline in prices increases households' real wages,[21] akin to a reduction in labor income taxes. This induces them to increase their labor supply via a substitution effect.[22] The increase in labor supply in turn exerts downward pressure on nominal wages, which lowers firms' marginal costs, which in turn reduces inflation and/or increases labor demand.

Note that a movement in the fiscal wedge on its own creates a trade-off for monetary policy, which becomes unable to jointly stabilize inflation and the output gap, i.e. the "divine coincidence" breaks down. However, if the $\mathcal{NH}$ wedge moves as well, then fiscal policy may be used strategically to offset this wedge. Olivi et al. (2026) show that following a negative supply shock to a necessity sector, such as energy, the $\mathcal{NH}$ wedge increases. A subsidy increases $\mathcal{F}_t$ and may then offset this effect, restoring the divine coincidence,[23] allowing the central bank to jointly stabilize the output gap and inflation. Thus, a price subsidy may potentially help to reduce macroeconomic distortions by facilitating monetary policy. Finally, note that the lump-sum transfers do not matter for the NKPC.

Aggregate demand effects of a sectoral subsidy. Next, let us turn to the aggregate demand effects of the subsidy. These effects can be understood from the aggregate Euler Equation (EE) for the output gap, which can be expressed as:

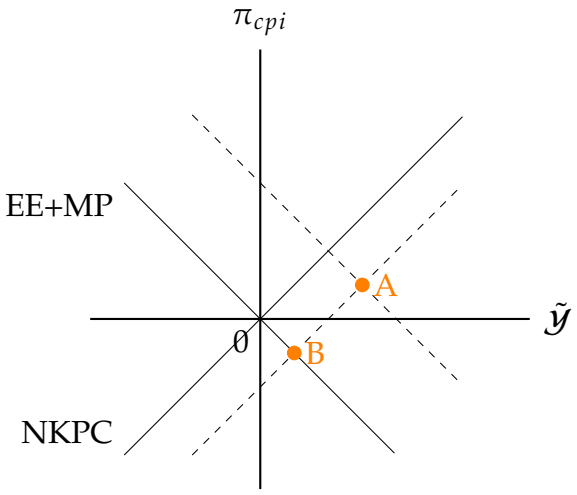
$$\tilde{\mathcal{Y}}_t = -\sigma\Big(\hat{R}_t - \mathbb{E}_t\pi_{mcpi,t+1} + \underbrace{\mathbb{E}_t\mathcal{F}_{t+1} - \mathcal{F}_t}_{\text{fiscal wedge}} - \hat{r}^*_t\Big) + \mathbb{E}_t\tilde{\mathcal{Y}}_{t+1},$$

where $\hat{r}^*_t = \frac{1}{\sigma+\psi}\sum_{k=1}^{K}\left(\psi\overline{\partial_e e_k} + \bar{s}_k\right)(\mathbb{E}_t\hat{A}_{k,t+1} - \hat{A}_{k,t})$ is the natural rate of interest, i.e. the real interest rate obtained under flexible prices. This equation is again very similar to its counterpart in the textbook NK model, with two main differences. First, the nominal interest rate is deflated by an inflation index weighing sectors by marginal rather than regular budget shares (see also Olivi et al. (2026)). The second main difference is that it the expected change in the fiscal wedge $\mathbb{E}_t\mathcal{F}_{t+1} - \mathcal{F}_t$, which captures the direct effect of a price

[21] As explained in Olivi et al. (2026), household i's real wage is $\hat{W}_t - \sum_{k=1}^{K}\partial_e e_k(i)\hat{Q}_{k,t}$, so the wage distortion created by a subsidy on good k is proportional to the average marginal budget share of k. Subsidies on necessities (i.e. lower $\overline{\partial_e e_k}$) generate relatively fewer wage distortions.

[22] Note that households pay for the subsidy via lump-sum taxes, which eliminates much of income effect on labour supply, except for a distributional effect.

[23] We show in section 4.3 that this is exactly what a planner would do at the optimum. The planner uses a flat VAT cut to undo the $\mathcal{NH}$ wedge via the $\mathcal{F}$ wedge and uses relative subsidies to stabilize relative prices.



Notes: solid lines illustrate the New Keynesian Phillips Curve and Euler Equation + Monetary Policy response, before the subsidy. Dashed lines are the curves after the subsidy. Point A illustrates the equilibrium when the nominal interest rate is kept unchanged, whereas point B illustrates a case in which monetary policy tightens to fully neutralize the effect of the fiscal wedge in the Euler equation, so that only the supply effect in the NKPC remains.

Figure 1: Demand and supply effect of a sectoral subsidy on wholesale inflation (π_{cpi}) and the output gap ($\tilde{y}$).

subsidy on aggregate demand. Intuitively, a subsidy mechanically causes a temporary decline in prices paid by consumers, and hence increases their inflation expectations. The latter lowers the real interest rate and stimulates aggregate demand today. Unlike the labor supply effect above, the demand effect has been emphasized in the New-Keynesian literature on the macro effects of temporary price subsidies (see e.g. Auclert et al. (2023); Bayer et al. (2023)). Note further that the demand effect may be undone by an adjustment of the nominal interest rate $\hat{R}_t$.

Finally, note that the fiscal transfers again do not enter the aggregate Euler equation. This result breaks down in the full quantitative model, if there is a correlation between the fiscal transfer and HtM status across households.

Equilibrium effects and the role of monetary policy. From the above discussion it follows that sectoral subsidies have both a positive aggregate supply effect and a positive aggregate demand effect. We are now ready to study the aggregate outcomes in equilibrium (abstracting for the moment from any sectoral productivity shocks). These outcomes are illustrated in Figure 1, which for simplicity takes expectations of the output gap ($\mathbb{E}_t\tilde{\mathcal{Y}}_{t+1}$) and wholesale inflation ($\mathbb{E}_t\pi_{cpi,t+1}$) as given, which would be the case if the intervention is purely transitory. The solid lines plot the aggregate supply curve (i.e. the

NKPC) and the aggregate demand curve, i.e. the EE curve combined with the monetary policy (MP) response. Solid lines represent these curves before the subsidy, whereas the dashed lines represent the two curves after a downward price subsidy, keeping the nominal interest rate constant.

The figure illustrates that, following the subsidy, the EE+MP curve shifts upward due to the demand effect, whereas the NKPC shifts downward, because of the supply effect. In the absence of an interest rate response, the new equilibrium is at point A, with the output gap and wholesale inflation both increasing. Thus, the demand effect dominates the inflation response in this example.

Point B illustrates a case in which the central bank raises the nominal interest rate, and does so sufficiently to undo the demand shift completely, so the EE+MP curve remains at the solid line. The NKPC still shifts down, as the change in the nominal interest rate does not alter the supply effect of the subsidy. At point B, the output gap still increases, but inflation now falls.

Let us now study equilibrium outcomes more formally. To this end, let us assume that monetary policy takes the following form:

$$\hat{R}_t = \hat{r}_t^* + \phi_{\tilde{\boldsymbol{y}}} \tilde{\boldsymbol{y}}_t + \phi_\pi \pi_{cpi,t}. \tag{10}$$

where we assume that the coefficients $\phi_{\tilde{\boldsymbol{y}}}, \phi_\pi \geq 0$ are such that the equilibrium is locally determinate.[24] We assume this rule throughout Sections 3 and 4 (Section 5 presents quantitative analysis under a more realistic rule with interest rate smoothing). Under the above rule, monetary policy moves one-for-one with the natural rate, $\hat{r}_t^*$ and, with some coefficients, to the output gap and wholesale CPI inflation. Note that with this rule, the central bank "looks through" the mechanical effects of a subsidy, as typically stated by central banks in practice.

Let us now consider the effect of a purely transitory subsidy, i.e. $\mathcal{F}_t > 0$ and $\mathcal{F}_{t+1} = \mathcal{F}_{t+2} = .. = 0$. We can derive the following result:

Proposition 1 (macro effects of a sectoral price subsidy). *Under Assumption A and the policy rule (10), following a purely transitory sectoral price subsidy, the output gap $\tilde{\boldsymbol{y}}_t$ and the nominal interest rate R_t both increase. Wholesale CPI inflation $\pi_{cpi,t}$ decreases if and only if $\psi \phi_{\tilde{\boldsymbol{y}}} > 1$.*

This result has a number of immediate implications. First, the response of the output gap to the subsidy is always positive, that is, $\tilde{\boldsymbol{y}}_t \geq 0$. Intuitively, this reflects the fact that both

[24] It can be shown that determinacy requires $\left(1 - \frac{1}{R}\right) \sigma \phi_{\tilde{\boldsymbol{y}}} + \lambda \left(1 + \frac{\sigma}{\psi}\right) \phi_\pi \geq \lambda \left(1 + \frac{\sigma}{\psi}\right)$

demand and supply effects push the output gap upward. Second, the nominal interest rate always increases in response to the subsidy. Indeed, the demand effect dominates the supply effect at $R_0 = 0$ so both inflation and the output gap are positive (as depicted in point A). Any stricter rule therefore gives a positive interest rate.[25]

Third, the inflation response is negative if and only if $\phi_{\tilde{y}} > \frac{1}{\psi}$: that is monetary policy must be sufficiently responsive to the output gap to lower inflation, independently of its sensitivity to inflation. Indeed, monetary policy needs to at least partially undo the demand effect of the price subsidy (shifting the Euler equation towards point B) to lower the inflation response. This could be done by responding directly to the fiscal wedge in the Euler equation[26]. However, if monetary policy looks through the subsidy, the output gap is the right variable to respond to, because it is the only variable the subsidy moves unambiguously upward, through both the demand and supply channels. In other words, the output gap is the observable indicator of the demand wedge that the rule otherwise ignores. In addition, the strength of the response depends on the Frisch elasticity. When labor supply is elastic (high ψ), most of the subsidy shows up as extra hours and output, so a mild response to the output gap suffices. When labor supply is inelastic, little of the boost is supply, so the central bank must correct most of the response of the output gap. By contrast, a rule that only responds to inflation would impose an interest rate of 0 at 0 inflation. Therefore, mechanically, an inflation rule cannot undo the demand wedge at 0 inflation: the demand effect of the subsidy always dominates and inflation is always positive.

If we allow the rule to directly respond to the price subsidy (no looking-through), for example, by directly correcting the demand wedge through

$$\hat{R}_t = r_t^* + \phi_y \tilde{\mathcal{Y}}_t + \phi_\pi \pi_{cpi,t} + \phi_\tau \left(\mathcal{F}_t - \mathbb{E}_t \mathcal{F}_{t+1}\right),$$

then monetary policy can lower inflation as long as $\left(1 + \frac{\psi}{\sigma}\right)\phi_\tau + \psi\phi_y > 1$, that is even without a strong response to the output gap.

In the Appendix, we generalize Proposition 1 to study persistent subsidies. We show that, inflation decreases on impact if and only if $\psi\phi_{\tilde{y}} > 1$, and that inflation is negative at all dates for $\psi\phi_{\tilde{y}} > 1$.

[25] To see this more formally suppose the interest rate turns negative. This implies the output gap is higher than under $R_0 = 0$ so inflation is also higher, since the coefficient on output gap and inflation are non negative that contradicts a negative rate.

[26] Which is optimal as shown in section 4.3

4 Welfare

4.1 Welfare loss function

Having studied the macro effects of the fiscal interventions, our next goal is to conduct a welfare evaluation of such policies, implemented following a negative productivity shock to a necessity sector. Let social welfare take the following form, at some initial date $t = 0$:

$$\mathcal{W} = (1-\delta)\int G(V^{-}(i), i)di + \delta\mathbb{E}_0 \sum_{t_0=0}^{\infty} \beta^{t_0} \int G(V^{t_0}(i), i)di, \tag{11}$$

where $G(\cdot, i)$ is a differentiable function which captures the social planner's preferences over utility levels of different households, which may condition on the household type i. In the above expression, the first term on the right-hand side corresponds to households who are already alive at $t = 0$.[27] The lifetime value of these households is given by $V^{-}(i) = \mathbb{E}_0 \sum_{s=0}^{\infty}(\beta(1-\delta))^s(u_i(\mathbf{c}_s^{-}(i)) - \chi(n_s^{-}(i)/\vartheta(i)))$. The second term corresponds to households born after time $t = 0$, where the lifetime welfare of household i born at t_0 is given by $V^{t_0}(i) = \mathbb{E}_{t_0} \sum_{s=0}^{\infty}(\beta(1-\delta))^s(u_i(\mathbf{c}_{t_0+s}^{t_0}(i)) - \chi(n_{t_0+s}^{t_0}(i)/\vartheta(i)))$.

Our goal is to derive a second-order approximation to $\mathcal{W}$, as a function of the policy instruments and equilibrium variables. To this end, we must make assumptions on the first- and second-order derivatives of the social preference function G. Our first assumption follows the literature on inverse optimum taxation and implies that the steady-state distribution is efficient, see also McKay and Wolf (2023). This means that, absent any aggregate shocks, there is no motive for policy to manipulate distributional outcomes.[28] This assumption is implemented via the following condition on the first-order derivative of G:

$$G'\left(V^{t_0}(i), i\right)\partial_e v_i\left(e_{t_0}^{t_0}(i), \mathbf{P}_{t_0}\right) = 1,$$

where $v_i\left(e(i), \mathbf{P}\right) = \max_{\mathbf{c}(i)} u_i(\mathbf{c}(i))$ s.t. $\sum_{k=1}^{K}\int_0^1 P_k(j)c_k(i,j)dj \leq e(i)$ is the indirect utility function of consumption.

While the steady-state distribution is treated as efficient, *fluctuations* in the distribution following aggregate shocks still create welfare losses. In order to pin these down, we must make an assumption on the second-order derivative $G''\left(V^{t_0}(i), i\right)$, which captures

[27] We assume the economy starts from the steady state, so households of a given type i born before $t = 0$ are identical.

[28] This is important in our case since, as in the data, the fiscal authority in charge of the relief program can use lump sum transfers, for example based on steady state income. If the fiscal authority values lower income households more in steady state, then those transfers should be used even without aggregate shocks.

social preferences regarding distributional fluctuations. A simple assumption would be to set $G''\left(V^{t_0}(i), i\right) = 0$. In this case, the social preference function is linear (conditional on type i), which implies that the planner cares equally about utility fluctuations of all households.[29] Alternatively, one could recover $G''\left(V^{t_0}(i), i\right)$ from the data. This is the approach we will follow in the quantitative section of this paper. For now, we assume $G''\left(V^{t_0}(i), i\right) = 0$.

In the Appendix, we derive the loss function for the full model (including input-output-linkages, non-randomly distributed HtM households, and heterogeneity in price stickiness across sectors). Since the welfare loss function is a rather large equation with some relatively unimportant components, we further simplify the model along the following dimensions:[30]

Assumption B. *Assume no OLG structure ($\delta = 0$); a linear social preference function ($G''\left(V^{t_0}(i), i\right) = 0$); and no heterogeneity in marginal budget shares ($\partial_e e_k(i) = \overline{\partial_e e_k}$).*

We now derive the simplified social welfare loss function:

Proposition 2 (welfare loss function). *Under Assumptions A and B, the social welfare loss function is given by $\mathcal{L} = \mathcal{L}^{\pi,within} + \mathcal{L}^{\pi,between} + \mathcal{L}^{\tilde{\mathcal{Y}}} + \mathcal{L}^{redist}$, where:*

$$\mathcal{L}^{\pi,\text{within}} = \sum_{t\geq 0}\beta^t \frac{\epsilon}{\lambda}\pi^2_{cpi,t}$$

$$\mathcal{L}^{\pi,\text{between}} = \sum_{t\geq 0}\beta^t \sum_k \bar{s}_k \sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t} + \hat{A}_{k,t}\right)\left(\hat{Q}_{l,t} + \hat{A}_{l,t}\right)$$

$$\mathcal{L}^{\tilde{\mathcal{Y}}} = \left(\psi^{-1} + \sigma^{-1}\right)\left(\underbrace{\sum_{t\geq 0}\beta^t \tilde{\mathcal{Y}}_t^2}_{\text{standard}} - \underbrace{\eta\left(\varphi\sum_{t\geq 0}\beta^t\tilde{\mathcal{Y}}_t^2 + (1-\varphi)(1-\beta)\left(\sum_{t\geq 0}\beta^t\tilde{\mathcal{Y}}_t\right)^2\right)}_{\text{heterogeneous wealth effects}}\right)$$

$$\mathcal{L}^{\text{redist}} = \underbrace{(1-\beta)(1-\varphi)\int g(i)\left(\sum_{t\geq 0}\beta^t \mathcal{I}_t(i)\right)^2 di}_{\text{non-HtM}} + \underbrace{\varphi\sum_{t\geq 0}\beta^t\int g(i)\left(\mathcal{I}_t(i)\right)^2 di}_{\text{HtM}}$$

where

[29] We then obtain the linear form $\int G(V(i), i)di \propto \int G'\left(V^{t_0}(i)\right)V(i)di$, where $G'\left(V^{t_0}(i)\right)$ is the Pareto-Negishi weight.

[30] The loss function in the general case is presented in Appendix C and derived in Appendix B

$$\mathcal{I}_t(i) = \zeta_t(i) - \sum_k \left(\frac{e(i)}{E} \left(s_k(i) - \bar{s}_k \right) \right) \hat{Q}_{k,t} + \frac{b(i)}{E} \left(\frac{\hat{R}_t}{R} - \left(1 - \frac{1}{R} \right) \hat{Q}_{cpi,t} - \left(1 - \frac{1}{R} \right) \hat{\mathcal{Y}}_t^* \right),$$

$$g(i) = \frac{E}{\sigma e(i) + Wn(i)\psi},$$

$$\hat{Q}_{k,t} = \hat{P}_{k,t} - \tau_{k,t}$$

and where $\hat{\mathcal{Y}}_t^* = \sum_{k=1}^K \frac{\psi \overline{\partial_e e_k} + \bar{s}_k}{1 + \psi/\sigma} \hat{A}_{k,t}$ is the natural level of aggregate demand, i.e. the level that would prevail in a version of the economy with flexible prices. Moreover, $\bar{\rho}_{k,l} = \int_0^1 \frac{c_k(i)}{C_k} \rho_{k,l}(i) di$ is the expenditure-weighted elasticity of substitution between sectors.

The first component, $\mathcal{L}^{\pi,\text{within}}$, is the inflation distortion in the textbook New Keynesian model. In the presence of aggregate shocks, nominal rigidities create price dispersion across varieties, which lowers consumer welfare. Note that this term depends on wholesale inflation, $\pi_{cpi,t}$, which excludes the mechanical effects of a price control, as such controls do not affect within-sector price dispersion.

The second term, $\mathcal{L}^{\pi,\text{between}}$, captures relative sectoral price distortions. If relative sectoral prices faced by consumers respond one-for-one to movements in relative sectoral productivity levels, then this term is zero. A price control directly affects this component. It captures a common criticism of energy price controls, namely that they induce overconsumption of energy. The magnitude of this distortion is governed by the elasticity of substitution between sectors, $\overline{\rho}_{k,l}$.

The third welfare component, $\mathcal{L}^{\tilde{\mathcal{Y}}}$, captures welfare loss arising from the output gap. That is, it captures labor market inefficiencies induced by the presence of markups, which distort real wages. Within this component, the second term captures heterogeneous wealth effects on labor supply. Here, $\eta \equiv \left(1 - \frac{1}{R}\right) \int \frac{\frac{b(i)}{E}}{1 + \frac{Wn(i)\psi}{\sigma e(i)}} di \geq 0$ captures the wedge between individual and representative-agent Hicksian labor supply elasticities. In our quantitative model, this second term is relatively unimportant.

The fourth component, $\mathcal{L}^{\text{redist}}$ captures welfare losses arising from distributional fluctuations. Within this term, $\mathcal{I}_t(i)$ is a distributional variable that is zero on average and captures changes in household i's real budget, relative to other households, due to variations in equilibrium prices and transfers. Moreover, $g(i)$ denotes the weight of household i in the welfare loss function. Note that under $G''\left(V^{t_0}(i), i\right) = 0$, this weight is declining in steady-state expenditures and income; that is, poorer households receive a higher weight. Intuitively, a given income shock reduces expenditure by proportionally more for poorer

households:[31] the same income fluctuations generate larger utility fluctuations and hence welfare losses for the poor.

Next, note that $\mathcal{L}^{\text{redist}}$ is the sum of two components. The first arises from non-HtM households, which comprise a share $1-\varphi$ of the population, while the second arises from HtM households. The first term is multiplied by $1-\beta$ and therefore tends to be relatively small, since the discount factor is close to one. Intuitively, non-HtM households are able to smooth any flow income gains or losses over their entire lifetime, thereby mitigating the associated welfare loss. By contrast, HtM households must absorb any current income changes through current consumption, generating a relatively large utility loss.

Let us now unpack the distributional variable $\mathcal{I}_t(i)$, which captures the relative effects of equilibrium prices and quantities on households' budgets. It contains several components. First, $\zeta_t(i)$ captures the direct income effect of a government transfer on household i's budget. The second component captures redistribution arising from relative sectoral prices. For example, a household that consumes a relatively large amount of energy suffers, relative to other households, from an increase in energy prices. Indeed, a key goal of the cost-of-living policies implemented in practice is precisely to compensate such households.

The remainder of $\mathcal{I}_t(i)$ is related to wealth redistribution (note that the term in large brackets is multiplied by $b(i)$). In particular, an increase in the nominal interest rate $\hat{R}_t$ directly redistributes wealth from indebted households to savers. Given that the variation in wealth in the data is much greater than the variation in consumption baskets, it may therefore be the case that the distributional effects of the monetary policy response to an energy price shock dominate the distributional effects arising from changes in energy prices and consumption basket heterogeneity. Similarly, fluctuations in the consumer price level $\hat{Q}_{cpi,t}$ generate wealth redistribution, although this effect is relatively small, as it is multiplied by the term $(1-1/R)$.[32] Finally, fluctuations in the natural level of output $\hat{\mathcal{Y}}_t^*$ generate a distributional effect related to efficiency considerations. In particular, households differ in the extent to which they allocate additional income to consumption versus leisure. During periods of low aggregate productivity, goods are relatively costly to produce, and it is therefore efficient to redistribute toward households that consume relatively more leisure at the margin, i.e. indebted households.

[31] The semi-elasticity of expenditure with respect to income I is $\partial_I ln(e(i)) = (e(i) + Wn(i)\psi/\sigma)^{-1}$ which declines with labor income and expenditure.

[32] This is because valuation effects only matter to the extent that households receive flow income from an asset or change their asset positions. In the steady state, asset levels are constant. Therefore, valuation effects operate only through changes in real interest income or payments on the asset.

Nonlinear social preference function. Under a more general social preference function that departs from linearity, the individual weight in the loss function is given by:

$$g(i) = \frac{E}{\sigma e(i) + Wn(i)\psi} - \frac{R}{R-1}\frac{G''\left(V^{t_0}(i), i\right) v'\left(e(i)\right) E}{G'\left(V^{t_0}(i), i\right)}$$

The first term is the same as before, but now a second term is added, which captures preferences of the planner over *fluctuations* in the distribution. In Section 5.5 we will present a method to recover these more general welfare weights from the data, based on the observed properties of tax/transfer systems over the business cycle.

4.2 Welfare effects of fiscal packages following a sectoral shock

Our ultimate goal is to evaluate and understand the welfare effects of a fiscal package following a sectoral shock. To understand the welfare effects, we present two decompositions. To this end, let us adopt sequence-space notation and write the welfare loss function as a function $\mathcal{L}(\mathbf{P}(\mathbf{A}, \mathbf{Z}), \mathbf{X}(\mathbf{A}, \mathbf{Z}); \mathbf{A}, \mathbf{Z})$, where $\mathbf{A}$ is a matrix containing the time paths of sectoral productivities ($\hat{A}_{k,t}$), while $\mathbf{Z}$ contains the time paths of the fiscal policy variables ($\zeta_t(i), \tau_{k,t}$). Moreover $\mathbf{P}(\mathbf{A}, \mathbf{Z})$ contains sectoral producer prices ($\hat{P}_{k,t}$), while $\mathbf{X}(\mathbf{A}, \mathbf{Z})$ is a matrix with the time paths of other macro variables entering the loss function ($\hat{R}_t, \tilde{\mathcal{Y}}_t$). Both $\mathbf{P}$ and $\mathbf{X}$ are written here explicitly as functions of the shocks ($\mathbf{A}$) and fiscal policies ($\mathbf{Z}$), since they determine the paths of the equilibrium objects.[33]

4.2.1 Decomposition 1: the roles of distortions

The first decomposition follows directly from the welfare loss function and decomposes the effect of the fiscal package into the contributions of the various distortions. Let $\Delta\mathcal{L}_{\text{total}} \equiv \mathcal{L}(\mathbf{P}(\mathbf{A}, \mathbf{Z}), \mathbf{X}(\mathbf{A}, \mathbf{Z}); \mathbf{A}, \mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A}, \mathbf{0}), \mathbf{X}(\mathbf{A}, \mathbf{0}); \mathbf{A}, \mathbf{0})$ denote the difference in the welfare loss in the baseline version of the model and a counterfactual without the fiscal package ($\mathbf{Z} = 0$), but with the same shock $\mathbf{A}$. Thus, $\Delta\mathcal{L}_{\text{total}}$ captures the negative of the welfare effect of the fiscal package, given the shock. From Proposition 2 it immediately follows that we can decompose it as:

$$\Delta\mathcal{L}_{\text{total}} = \Delta\mathcal{L}^{\pi,\text{within}} + \Delta\mathcal{L}^{\pi,\text{between}} + \Delta\mathcal{L}^{\tilde{\mathcal{Y}}} + \Delta\mathcal{L}^{\text{redist}}.$$

[33] Here, we treat the nominal interest rate also as an equilibrium variable since we assume the same monetary policy rule across counterfactuals. At the end of Section 5.5, we consider a different counterfactual for the monetary policy response.

With this decomposition, we can evaluate which distortions the fiscal package alleviates and which ones it worsens, following a shock.

4.2.2 Decomposition 2: partial-equilibrium versus macro channels

A different, complementary way to understand the welfare effect of a fiscal package is to decompose into the contributions of direct, partial equilibrium effects and macro channels:

$$\begin{aligned}\Delta\mathcal{L}_{\text{total}} = & \underbrace{\mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{0})}_{\text{Partial Equilibrium } (\Delta\mathcal{L}_{\text{PE}})} \\ & + \underbrace{\begin{aligned}&(\mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{Z})) \\ &\quad - (\mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{0}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{0}))\end{aligned}}_{\text{Macro Exposures } (\Delta\mathcal{L}_{\text{ME}})} \\ & + \underbrace{\mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{0}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{0}),\mathbf{X}(\mathbf{A},\mathbf{0});\mathbf{A},\mathbf{0})}_{\text{General Equilibrium } (\Delta\mathcal{L}_{\text{GE}})}\end{aligned}$$

This decomposition is helpful in understanding to what extent welfare effects are driven by the immediate, partial equilibrium effects of a fiscal support package, which are often at the forefront of policy discussions, and to what extent by macro channels. Note further that we can also implement the decompositions jointly, i.e. decompose each distortion term into PE, ME and GE effects. For instance, we can decompose the effect of a fiscal package on the within-sector inflation distortion as $\Delta\mathcal{L}^{\pi,\text{within}} = \Delta\mathcal{L}^{\pi,\text{within}}_{\text{PE}} + \Delta\mathcal{L}^{\pi,\text{within}}_{\text{ME}} + \Delta\mathcal{L}^{\pi,\text{within}}_{\text{GE}}$. Let us now discuss the components in Decomposition 2 (continuing to make Assumptions *A* and *B*):

Partial Equilibrium (PE) effect. The Partial Equilibrium effect (or "direct effect") captures the effect of the policy in a case in which only prices and productivities change following the shock. This can be thought of as the welfare effect obtained in a micro-oriented welfare calculation which ignores any macro effect beyond the shock and the price changes. Note that $\Delta\mathcal{L}_{PE} = \Delta\mathcal{L}^{\pi,\text{between}}_{PE} + \Delta\mathcal{L}^{\text{redist}}_{PE}$, because the fiscal policy variables in $\mathbf{Z}$ only enter through those terms. That is, the PE effect purely captures the trade-off between the consumption basket distortion and the distributional effects of a fiscal package. Indeed, these two effects are typically central in policy discussions.

Macro exposures. The PE effect abstracts from the fact that, following a sectoral shock, not only sectoral prices move, but other macro variables as well. This in turn creates redis-

tributional effects, beyond those arising from sectoral price movements. $\Delta\mathcal{L}_{\text{ME}}$ captures these effects. Intuitively, this adjustment captures the extent to which households' individual exposures to the macro effects co-vary with the incidence of the fiscal policy. For instance, following a negative supply shock, the central bank may increase the nominal interest rate, which benefits savers but hurts borrowers. If it is the case that borrowers also stand to gain from the fiscal package, then this raises the welfare effect of the package beyond the direct effect. If not, welfare is reduced.

Note that $\Delta\mathcal{L}_{\text{ME}} = \Delta\mathcal{L}_{\text{ME}}^{redist}$, i.e. the macro exposure adjustment enters purely through the redistribution term. Indeed, this is the only term in which the fiscal policy variables in $\mathbf{Z}$ interact with the macro variables in $\mathbf{X}$. Note further that, within $\mathbf{X}$, only the nominal interest rate matters for the macro exposures, since the output gap $\tilde{\mathcal{Y}}_t$ does not enter in the redistribution term.

General Equilibrium (GE) effects of the fiscal package. The final term of the equation captures the fact that the fiscal package itself creates welfare effects, which potentially affects all components of the welfare loss function. For instance, the demand effects of a sectoral subsidy may increase inflation, leading to a direct welfare loss and additional redistribution between borrowers and savers. On the other hand, the subsidy may also dampen the decline in the output gap, which may improve welfare.

Overall welfare effects. The welfare implications of direct effects, macro exposures, and equilibrium effects are all ambiguous. Hence, the overall welfare effect is generally ambiguous as well. Yet, based on the positive results in the previous section we can obtain some insight into under which circumstances a subsidy improves/reduces social welfare:

Proposition 3 (welfare effect of a sectoral price subsidy). *Consider a purely transitory, negative productivity shock to a necessity sector k. Under Assumptions A and B, the policy rule (10) and $(R-1)\left(\sigma+\psi\right) \leq 1$,*[34] *a small transitory subsidy to the necessity sector has the following effects on the components of the welfare loss function:*

- $\Delta\mathcal{L}_{\text{total}}^{\pi,\text{between}} > 0$,
- $\Delta\mathcal{L}_{\text{total}}^{\pi,\text{within}} > 0$ if and only if $\psi\phi_{\tilde{\mathcal{Y}}} < 1$,
- $\Delta\mathcal{L}_{\text{total}}^{\tilde{\mathcal{Y}}} < 0$,
- $\Delta\mathcal{L}_{\text{total}}^{\text{redist}} < 0$ if $b(i) = b = 0$ for every i (only consumption basket heterogeneity),

[34]This condition is very likely to be satisfied in practice, since $R-1$ is close to zero. In our calibrated model we have $(R-1)\left(\sigma+\psi\right) = 0.0272$.

- $\Delta\mathcal{L}^{\text{redist}}_{\text{total}} > 0$ if $s_k(i) = \bar{s}_k$ for every k, i (only wealth heterogeneity).

The welfare losses from the fiscal package are independent from the size of the subsidy, its gains turn into losses if the subsidy is large enough.

The proposition implies that the welfare effect of a sectoral price subsidy following a negative necessity shock is generally ambiguous. On the one hand, it increases welfare losses from consumption basket distortions (the between-sector component), as it worsens the misalignment of relative sectoral prices with relative productivity levels. That is, the subsidy worsens the distortion in consumption baskets. This effect is entirely captured by the PE channel, i.e. $\Delta\mathcal{L}^{\pi,\text{between}}_{\text{total}} = \Delta\mathcal{L}^{\pi,\text{between}}_{\text{PE}} > 0$.[35]

On the other hand, absent any wealth heterogeneity, the subsidy may improve the distributional welfare components, as it directly benefits households with high exposures to the subsidized sector. In particular, without heterogeneity in nominal wealth ($Var(b_i) = 0$) it holds that $\Delta\mathcal{L}^{\text{redist}}_{\text{total}} = \Delta\mathcal{L}^{\text{redist}}_{\text{PE}} < 0$, i.e. the policy reduces the redistribution losses from the shock, and this effect enters entirely via the PE channel.[36] However, once wealth heterogeneity is taken into account, PE, ME and GE channels all matter for the redistribution term.

Proposition 3 characterizes the welfare impact of the policy through the macro components related to the output gap and CPI inflation. These effects matter purely through the GE channel, i.e. $\Delta\mathcal{L}^{\pi,\text{within}}_{\text{total}} = \Delta\mathcal{L}^{\pi,\text{within}}_{\text{GE}} \lesseqgtr 0$ and $\Delta\mathcal{L}^{\tilde{y}}_{\text{total}} = \Delta\mathcal{L}^{\tilde{y}}_{\text{GE}} < 0$.[37] Indeed, a negative shock in a necessity sector – absent any fiscal stimulus – increases CPI inflation and reduces the output gap through the $\mathcal{NH}$ wedge (see Olivi et al. (2026)). As a price subsidy always increases the output gap (see Proposition 1), the subsidy reduces the welfare loss from the output gap component. However, the output gap gain is only local, for a large enough subsidy, the increase in the output gap overshoots and the contribution becomes negative. Proposition 3 also states that the effect of the subsidy on the within-sector inflation component depends on the monetary policy response. Unless monetary policy reacts sufficiently aggressively to the output gap, the subsidy also increases within-sector inflation, which also reduces welfare. As explained in Proposition 1, without a strong response to the output gap, the subsidy stimulates aggregate demand and thereby fuels

[35] With homogeneous price stickiness across sectors, neither fiscal nor monetary policy affects relative producer prices $\hat{P}_{k,t} - \hat{P}_{cpi,t}$, so the distortion in consumption baskets is fully captured by the direct impact of the subsidies on consumer prices. Quantitatively, the PE effect explains most of the between sector component of the welfare loss with heterogeneous price stickiness.

[36] This again follows from the homogeneous price stickiness assumption since $\mathcal{L}^{\text{redist}}$ only depends on relative prices without wealth heterogeneity.

[37] Note that the fiscal policy variables do not enter directly into these components.

inflation.

Finally, Proposition 3 makes clear that, in the presence of wealth heterogeneity, a sectoral subsidy may worsen the redistribution component of the welfare loss function. Indeed, in the special case without consumption basket heterogeneity ($Var(s_i) = 0$), we obtain $\Delta\mathcal{L}^{\text{redist}}_{\text{total}} > 0$. The overall effect is a combination of PE, ME and GE channels. The PE channel, which enters through the CPI $\hat{Q}_{cpi,t}$, creates a revaluation of nominal positions, while the natural level $\hat{\mathcal{Y}}^*_t$ redistributes between households based on their relative dependence on labor versus financial income. These effects tend to be relatively small, however.[38]

More important is the ME channel, which incorporates the redistributive effects of the nominal interest rate. Following a negative productivity shock, the nominal rate increases which redistributes between borrowers and savers. In addition, the direct impact of the subsidy is to lower the price level thereby increasing the real wealth of savers. The subsidy therefore amplifies redistribution towards savers and thereby creates a welfare loss through the macro exposures effects: $\Delta\mathcal{L}^{\text{redist}}_{\text{ME}} > 0$. Finally, GE forces tend to exacerbate these losses. As shown in Proposition 1, in response to the subsidy, the nominal rate increases (which redistributes towards savers) and usually dominates the increase in the price level (which redistributes towards debtors). This is the case if the central bank responds sufficiently strongly to either the output gap or inflation, $\Delta\mathcal{L}^{\text{redist}}_{\text{GE}} > 0$.[39]

Although the overall welfare impact of the subsidy is ambiguous, under monetary rules responding weakly to the output gap (as is the case in the UK) if the variance of wealth in the population and the subsidy itself are large enough, the overall impact of the price subsidy will reduce welfare.

4.3 Optimal Policy

While the main purpose of this paper is to evaluate the welfare effects of existing policies, it is instructive to understand what the optimal policy would look like in this environment. We can derive the following result:

Proposition 4 (optimal monetary/fiscal policy). *Under Assumptions A and B, the joint-optimal monetary-fiscal policy consists of a path of transfers, sector-level price controls, and nominal interest rates such that* $\tilde{\mathcal{Y}}_t = \pi_{cpi,t} = \mathcal{I}_t(i) = 0$ *and* $\hat{Q}_{k,t} = -\hat{A}_{k,t} - \tau_t$ *for all* $t \geq 0$. *This is*

[38]Note that in the redistribution component of the loss function, both variables are multiplied by $\left(1 - \frac{1}{R}\right)$, which is a small number.

[39]A sufficient condition for $\Delta\mathcal{L}^{\text{redist}}_{\text{GE}} > 0$ is $\phi_\pi + \left(\lambda^{-1} + R\right)\psi\phi_{\tilde{\mathcal{Y}}} > R$.

achieved by setting:

$$\begin{aligned}
\tau_{k,t} &= \hat{P}_{k,t} + \hat{A}_{k,t} + \tau_t, \\
\tau_t &= \mathcal{NH}_t - \left(\hat{P}_{mcpi,t} - \hat{P}^*_{mcpi,t}\right), \\
\hat{R}_t &= \hat{r}^*_t + \mathbb{E}_t \pi_{mcpi,t+1} - \left(\mathbb{E}_t \mathcal{F}_{t+1} - \mathcal{F}_t\right), \\
\zeta_t(i) &= \sum_k \left(\frac{e(i)}{E}\left(s_k(i) - \bar{s}_k\right)\right) \hat{Q}_{k,t} - \frac{b(i)}{E}\left(\frac{\hat{R}_t}{R} - \left(1 - \frac{1}{R}\right)\hat{Q}_{cpi,t} - \left(1 - \frac{1}{R}\right)\hat{\mathcal{Y}}^*_t\right).
\end{aligned}$$

As mentioned in section 2, the optimal policy uses an economy-wide price control (τ_t) to offset the $\mathcal{NH}$ wedge in the NKPC, and differentiates the price control across sectors to correct relative prices ($\tau_{k,t}$). With the NKPC wedge being offset, monetary policy then exploits the "divine coincidence" to jointly stabilize CPI inflation and the output gap. Finally, optimal transfers between households correct any redistributional effects.

Note that under the optimal policy, the central bank does not merely move the nominal rate to let the real rate mimic its natural counterpart, as in the standard NK model. Indeed, it actively tightens in response to a temporary downward price control, in order to eliminate the aggregate demand effect discussed in the previous section. Thus, the optimal policy does not "look through" the price control.

Arguably, the joint optimal policy is not very realistic as it requires not only coordination between fiscal and monetary policy, but also complex fiscal transfers. For example, the optimal transfer redistributes based simultaneously on heterogeneity in wealth and consumption baskets. Moreover, the implementation of time-varying price controls or VAT taxes/subsidies in all sectors seems unrealistic. The question is therefore what are the welfare effects of realistic policies. We evaluate this quantitatively in the next section.

5 Evaluating the 2022 UK fiscal package

We now use the model to quantitatively evaluate the UK's cost-of-living policies in response to the large increase in Electricity and Gas prices following the invasion of Ukraine in 2022. We evaluate these policies in the full model.

5.1 Calibration

The model contains eight broad COICOP sectors: *Food*, *Clothing*, *Electricity & Gas*, *Furniture*, *Transport*, *Recreation*, *Restaurants & Hotels*, and *Miscellaneous*. The time period is one

quarter. We discipline the steady-state distributions of the model using micro data from the UK Living Costs and Food (LCF) Survey, which contains detailed information on around 6000 households. The model is sufficiently flexible for us to include each household in the data as a household in the model (where we use survey population weights for aggregation).

Steady-state distributions and preferences. The calibration of the steady-state distributions follows Olivi et al. (2026). That is, we impute nominal wealth at the individual level ($b(i)$) from interest payments on mortgages and credit cards, and from interest income. We then add a constant to ensure that the aggregate net wealth position is zero, as the model is for a closed economy. Next, we take total expenditures on the 8 sectors ($e(i)$) from the data, and back out labor income as a residual of the individual budget constraint, which we then use to calibrate the labor productivity parameter $\vartheta(i)$. The left panel of Figure 2 plots the joint distributions of steady-state income and nominal wealth.

To parametrize preferences, we estimate a Non-Homothetic CES demand system, following Comin et al. (2021), allowing for idiosyncratic preference shifters, in order to exactly match consumption baskets in the LCF. We set the elasticity of substitution between sectors to 0.3, following recent evidence by Levell et al. (forthcoming). We then back out preference shifters such that the model exactly matches the distribution of consumption baskets in the LCF, at the individual level. For illustration, the right panel of Figure 2 plots the average expenditure share on energy by income decile, as well as the range between the 10th and 90th percentile of this expenditure share, within each income decile. Clearly, poorer households devote a large share of their spending to energy. At the same time, there is a large amount of variation within each decile, suggesting that income-based transfers may not be perfectly targeted (Levell et al. (forthcoming)).

For firms, we assume the same elasticity of substitution, which is in the range of empirical estimates reported by Bachmann et al. (2022). The elasticity of intertemporal substitution and the Frisch elasticity of labor supply are both set to be common across households and equal to one. The subjective discount factor is set to $\beta = 0.995$, and the death probability is set to $\delta = 0.0083$.

The fraction of hand-to-mouth households is calibrated within each types so that the model exactly reproduces regression evidence in Albuquerque and Green (2022) who elicit individual MPCs in a survey and regress them on individual characteristics, which we also observe in the LCF. The calibration of input-output linkages is based on ONS data on sectoral input-output tables and price rigidities are set following previous empirical evidence on sectoral price adjustment frequencies in the UK, as in Olivi et al. (2026).

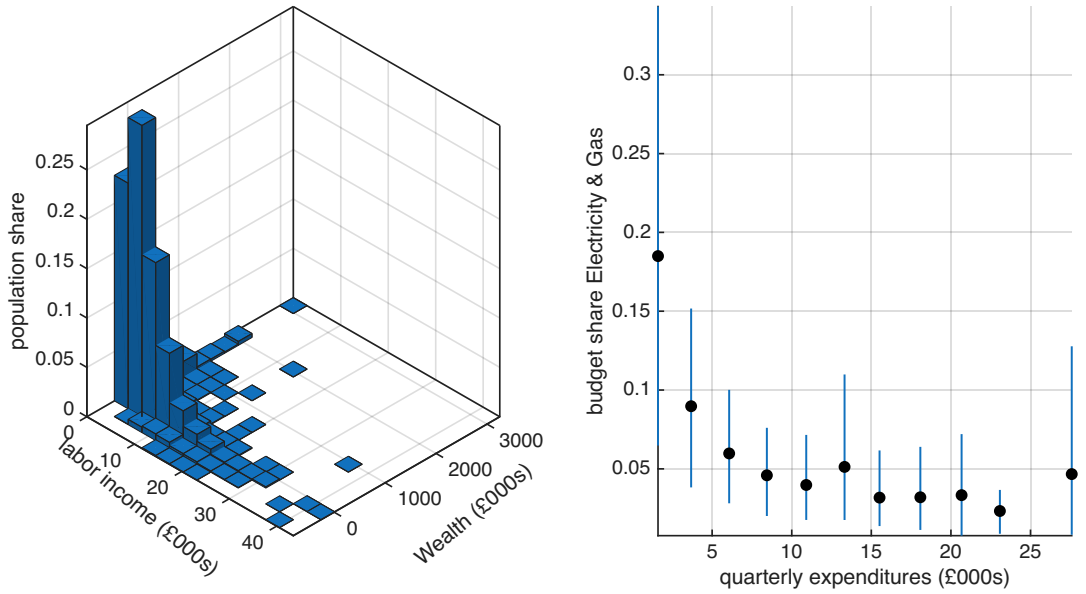


Figure 2: Steady-state distribution.

Notes: Left panel: histogram of the steady-state distribution of income and wealth. Right panel: steady-state budget shares on Electricity & Gas, by deciles of total expenditure. Vertical bars denote the 10th-90th percentile ranges within each total expenditure decile.

The Ukraine shock. We feed productivity shocks to the all sectors the model. We back out these shocks from the sector-level NKPCs expressed in terms of marginal costs, and using data on the wage and sectoral prices. We assume that the time path of the shock is announced at the time of the invasion, i.e. in the first quarter of 2022.

Energy Price Guarantee. Next, we introduce the two fiscal policies, again assuming that their time paths are announced at the time of the shock. The first policy captures the UK's Energy Price Guarantee (EPG), which capped the energy bill for a typical household at £2,500 for three quarters starting in October 2022. This cap corresponded to about a 95 percent increase in energy prices relative to the Ofgem cap in January 2022, just before the shock hit. The price cap is implemented as a price subsidy on energy τ_{energy} such that the increase in the price of Electricity and Gas faced by consumers is limited to 95 percent, in line with the policy.[40]

[40]The EPG was financed by the UK government, which compensated energy companies for the difference between the market price and the capped price. In reality, the EPG was introduced on top of a price cap set by the UK energy regulator Ofgem. This Ofgem cap, however, is designed to reflect market prices and costs of production, and therefore is not a subsidy. We can therefore think of the Ofgem cap as the market price.

Cost-of-Living payments. The second policy we introduce are the cost-of-living payments paid over the period 2022Q3-2024Q1. There were a number of transfers, paid out at different points in time and to different groups of households. They are summarized in Table 1. First, there were transfers paid out to households who were eligible for certain pre-existing means-tested welfare programs. Roughly speaking, these transfers were paid to households in the bottom thirty percent of the income distribution. In the LCF, we observe whether households are currently receiving certain benefits, from which we can determine whether they are eligible for these payments. Similarly, cost-of-living payments were made to disabled people as well as to those at or above the state pension age, and the payment doubled if the pensioner was aged 80 or above. Since every household type in the model corresponds to a household in the LCF, we can calibrate directly which households received a given transfer. Finally, there were two payments made to all households.

Quarter	**Means-tested (£)**	**Disability (£)**	**Pensioner (£), no 80+**	**Pensioner (£), 80+**	**All households (£)**
2022Q1	0	0	0	0	0
2022Q2	0	0	0	0	0
2022Q3	326	150	0	0	0
2022Q4	324	0	150	300	200
2023Q1	0	0	0	0	200
2023Q2	301	150	0	0	0
2023Q3	0	0	0	0	0
2023Q4	300	0	150	300	0
2024Q1	299	0	0	0	0

Table 1: *Notes:* Cost of Living Payments and Energy Bill Support by quarter.

Monetary policy. Our assumed monetary policy rule follows Nakamura et al. (2025), who estimate the following specification for the United Kingdom:

$$\hat{R}_t = \phi_\pi \pi^{1y}_{core,t} + \phi_{\tilde{y}} \tilde{\boldsymbol{y}}_t + \phi_R \hat{R}_{t-1} + \varepsilon_t,$$

where $\pi^{1y}_{core,t}$ is annual core inflation. We calibrate the coefficients using the estimates reported in Nakamura et al. (2025) for the UK (their Table B.2), converted into a rule for the quarterly interest rate. Accordingly, we set $\phi_\pi = 0.17/4 = 0.0425$, $\phi_{\tilde{y}} = 0.06/4 = 0.015$ and $\phi_R = 0.84$. To account for the fact that monetary policy likely did not strictly follow this rule in the post-2022 period, we additionally include a sequence of policy shocks, ε_t. We assume that these shocks are announced in 2022Q1 and calibrate their values so that

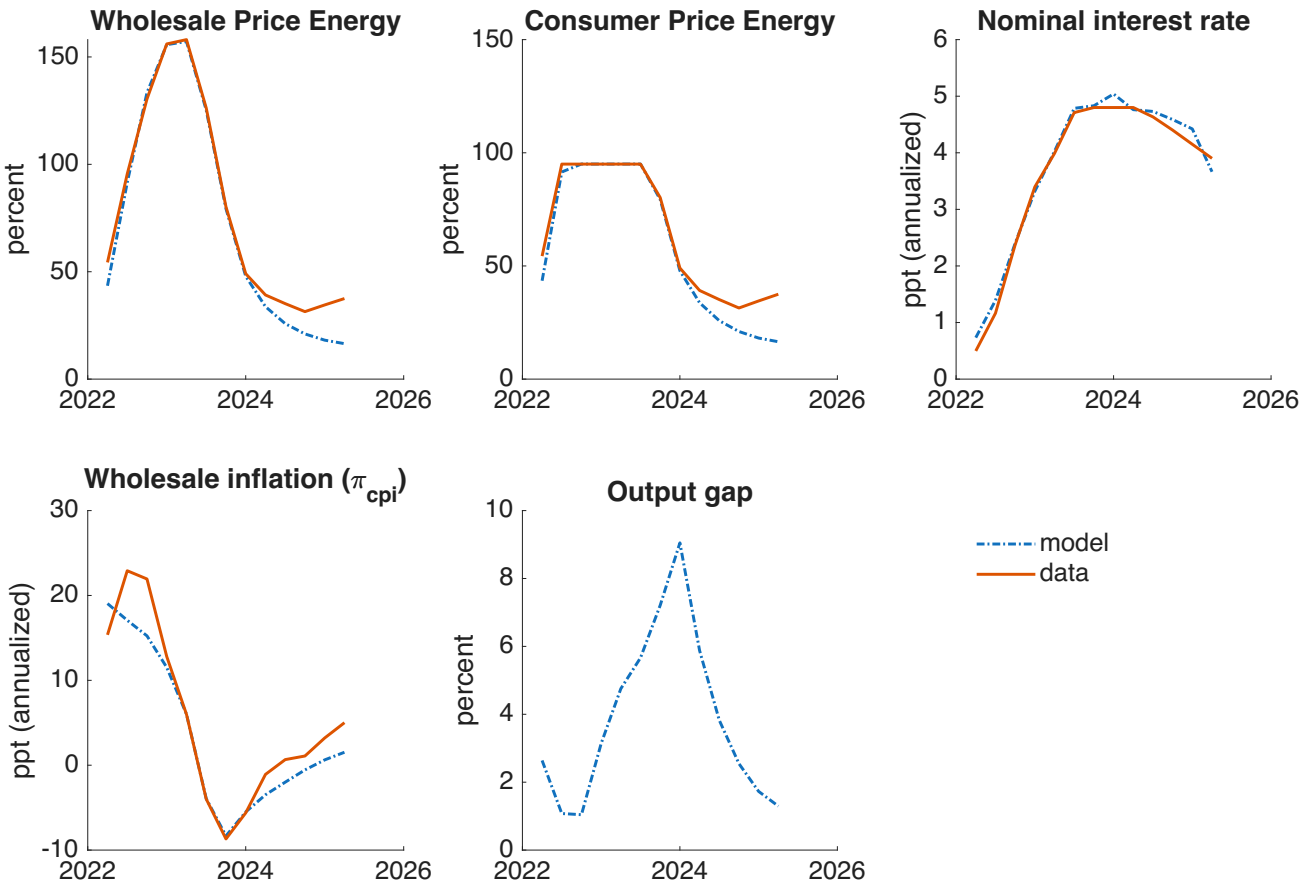


Figure 3: Baseline simulation and counterfactuals.

Notes: See main text.

the model reasonably replicates the observed paths of the nominal interest rate ($\hat{R}_t$) and wholesale inflation ($\pi_{cpi,t}$) over the sample period.

Model fit Figure 3 plots the fit of the model. Following the shock, there is a large increase in the price of Electricity & Gas. The energy price guarantee significantly limits the price increase for consumers. Contemporaneously, there is a strong increase in wholesale CPI inflation (driven largely by the energy price), as well as in the nominal interest rate. The fit of the model along these dimensions appears reasonable. Finally, the figure also plots the output gap (not directly observable), which remains positive throughout, dipping briefly in 2022 before peaking in 2024.

Social welfare weights. Finally, we parametrize the social welfare weight $g(i)$ in the quadratic loss function, for each household. The estimation of these weights, an independent contribution of this paper, is based on cyclical fluctuations in income and taxes/transfers over the decades before the Ukraine shock. Intuitively, the planner will adjust transfers to household i following shocks more strongly the higher is $g(i)$, allowing us to infer weights from observed cyclical movements.

The full procedure is described in detail in the Appendix, but the main idea is as follows. Suppose that both labor productivity $\vartheta_t(i)$ and the net lump-sum transfer vary by household over time. Let ω denote some bin of the distribution, and let the net transfer to this bin be given by $\tau_t(\omega)$. In the Appendix, we show that the model implies the following relation between transfers and productivity revisions:

$$\sum_s \beta^s \hat{\vartheta}_{t+s}(\omega) = \gamma(\omega) \sum_s \beta^s \left(\hat{\vartheta}_{t+s}(\omega) + \hat{\tau}_{t+s}(\omega)\right) + b(\omega)\xi_t + b_R(\omega) \sum_s \beta^s \left(\hat{R}_{t+s} - \pi^Q_{CPI,t+s+1}\right) + b_Q(\omega)\hat{Q}_{CPI,t},$$

where ξ_t is an unobserved factor with heterogeneous loadings given by $b(\omega)$, and the other macro controls capture the redistribution that happens through wealth. The object of interest is $\gamma(\omega)$, which captures how large (average) welfare weights are relative to the linear benchmark $G'' = 0$.[41] In the extreme case, if income fluctuations were completely stabilized for some group, then $\gamma(\omega)$ approaches infinity.

We use the LCF household-level quarterly data from 2002-2019 to construct labor income deciles, and estimate a joint VAR of the decile-level and four macro variables: real GDP, wages, prices and interest rates. We then use the quarterly revisions to estimate the equation above, which recovers average weights for each decile. Lastly, we recover the underlying welfare function by expanding the curvature term $\chi(V)$ evaluated at the household's lifetime utility V in a polynomial and fitting it to the estimated decile moments.[42]

The results are shown in Figure 4. The left panel shows the point estimates for each bin with its 95% confidence interval. For the bottom nine deciles the estimates are close to the linear benchmark. The exception is the top income decile of the distribution, where the estimates are significantly below the linear case. The right panel plots each household's empirical weight against its labor income. It highlights that while there is a strong negative

[41] The exact expression for the coefficient is given by $\gamma(\omega) = \frac{\int g(i)\frac{Wn(i)}{WN}\pi(i|\omega)di}{\int g^{\text{linear}}(i)\frac{Wn(i)}{WN}\pi(i|\omega)di}$ which collapses to one in the case of a linear social welfare function.

[42] When $\sigma = 1$, the exact expression is $V(e, Wn) = \frac{1}{1-R^{-1}}\left(\ln(e) - \frac{Wn}{(1+\psi^{-1})e}\right)$.

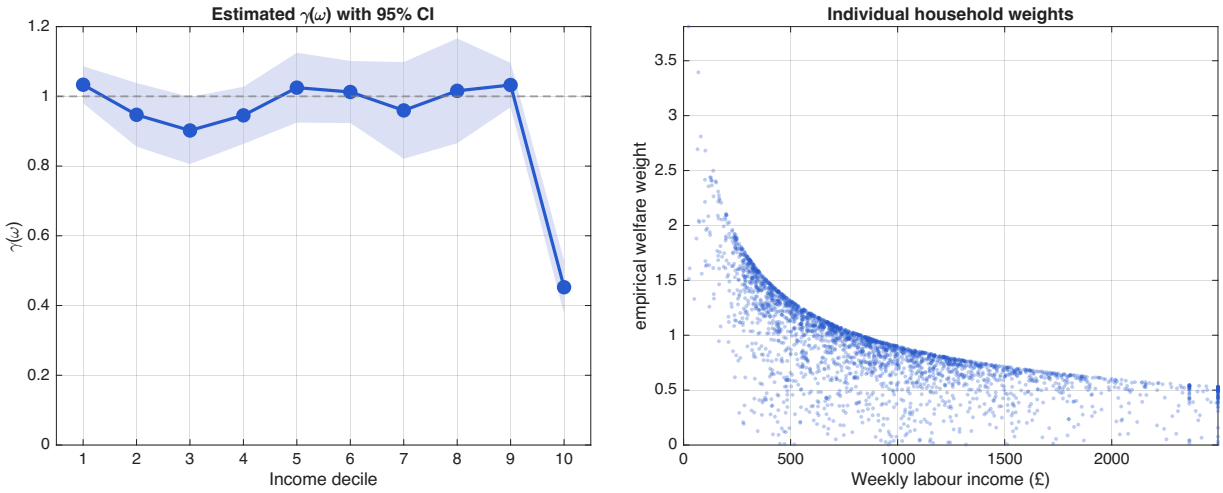


Figure 4: Estimated welfare weights.

Notes: The left panel reports the estimated $\hat{\gamma}(\omega)$, i.e. welfare weight relative to the the linear benchmark ($\gamma = 1$), for each decile of the residualised labour income distribution, with 95% confidence intervals. The right panel plots the fitted welfare weight for each household in the 2019 cross-section against its weekly labor income. The projection returns a negative weight for about 6% of households, which are not shown here.

gradient with respect to income, a significant degree of heterogeneity still remains.

5.2 Macro effects of the UK support package

Having parametrized the model, we first study the macro effects of the UK fiscal package, shown in Figure 5. It turns out that the fiscal transfers (yellow line) have relatively limited macroeconomic effects, and hence the overall effect is mostly driven by the Energy Price Guarantee (EPG), which drastically limited the increase in the price of energy paid by consumers. The figure also shows that the EPG had a substantial positive effect on the output gap and the nominal interest rate, in line with our analytical results. The effect on inflation is positive (after a small initial dip), but is followed by a negative effect.

To better understand the macro implication of the EPG, Figure 6 plots its demand and supply effects of the fiscal wedge. These are obtained by shutting down, in turn, the fiscal wedge in the NKPC and Euler equation for the output gap. As expected, the supply and demand effects on inflation work in opposite directions, whereas for the output gap the two forces tend to reinforce each other. Importantly, both effects are quantitatively large, underscoring the significant macroeconomic implications of the fiscal intervention. Given the monetary policy rule, the demand effects outweigh the supply effects in terms of the inflation response.

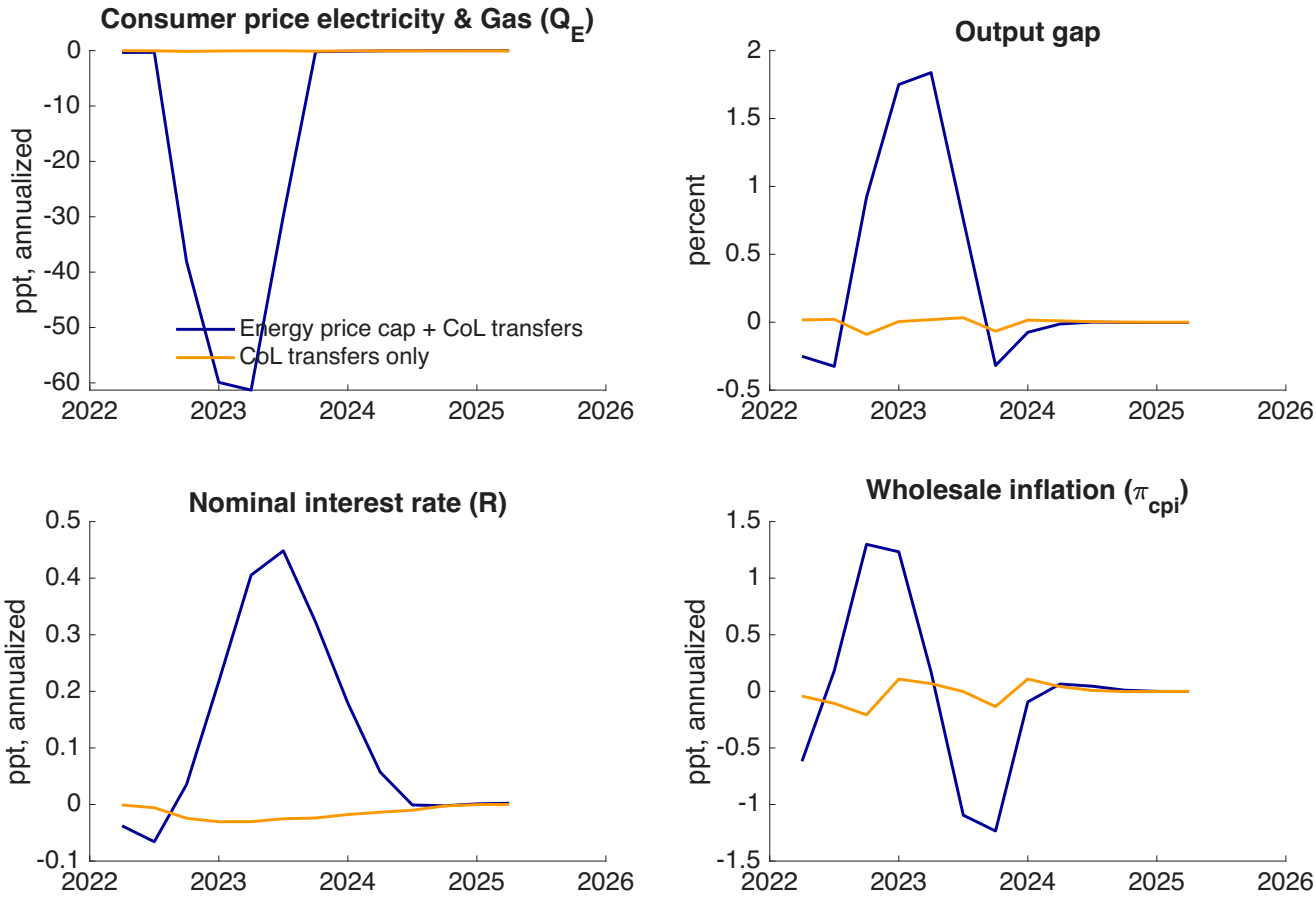


Figure 5: Macro effects of the UK support package

Notes: difference between a simulation of the baseline model and a simulation without the full fiscal package (blue line), and without only the CoL payments (yellow line).

5.3 Distributional effects

Having studied the macro effects, let us now consider the distributional effects of the Ukraine shock. The blue solid line in Figure 7 plots the effects of these on the budgets of savers and borrowers with different levels of labor income. Here, the solid blue line captures the direct distributional effects of the Ukraine shock via prices and productivity, whereas the dashed blue line captures the distributional effects of the change in the nominal interest rate. The red lines capture the distributional effects of the fiscal package, net of the lump-sum taxes required to finance the package.

Interestingly, the group that faced the most negative budgetary hit from the shock consists of borrowers with high labor income, plotted in the upper right panel. These

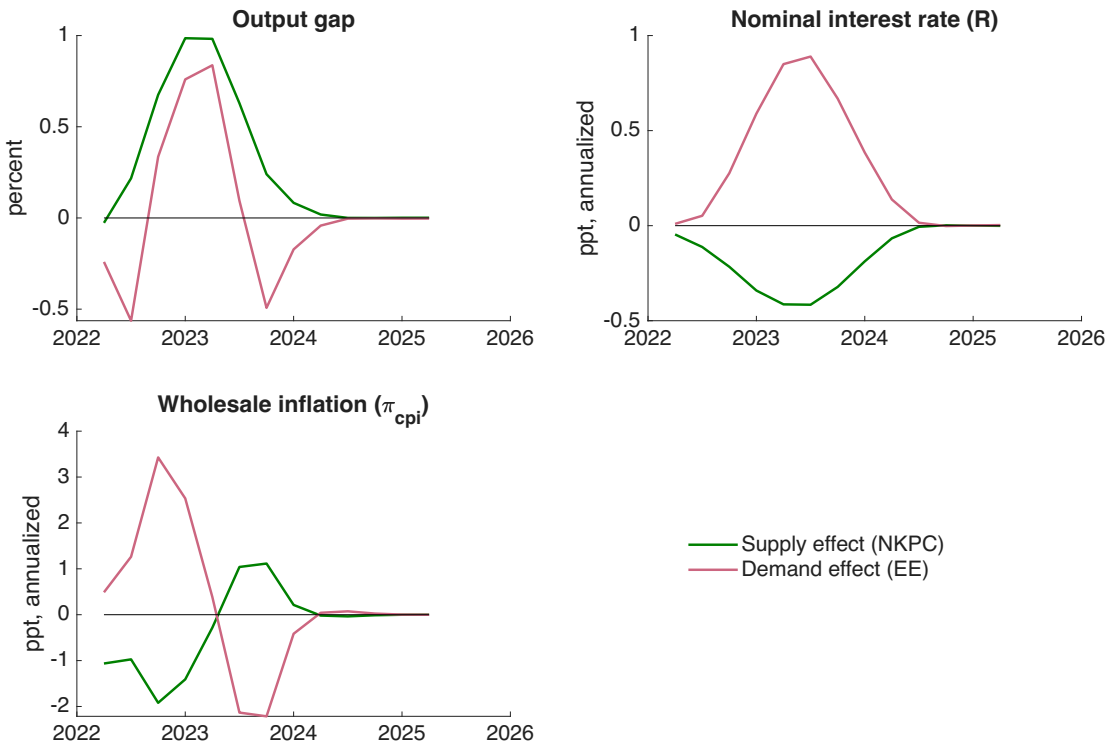


Figure 6: Response to the support package: demand and supply effects.

Notes: Supply and demand effects are computed as the difference between a simulation from the baseline model and a counterfactual version in which the fiscal wedge $\mathcal{F}$ is suppressed from, respectively, the NKPC and the Euler equation.

households suffered from a decline in labor productivity and from the increase in interest rate. By contrast, savers with low labor income (a group that includes pensioners), plotted in the bottom left panel, gained from the shock, as they did not rely much on wage income but did gain from the increase in interest on their savings.

Next, let us consider the budgetary effects of the EPG and transfers, as well as the taxes needed to finance these measures. As expected, low income households gain from both fiscal measures, whereas high income households lose on net, considering that the package is financed via increased taxes. Thus, for high income borrowers the fiscal package comes as a *double whammy*: not only are they hit relatively hard by the decline in wages and increase in interest rates resulting from the Ukraine shock, they also end up losing on net from the fiscal package.

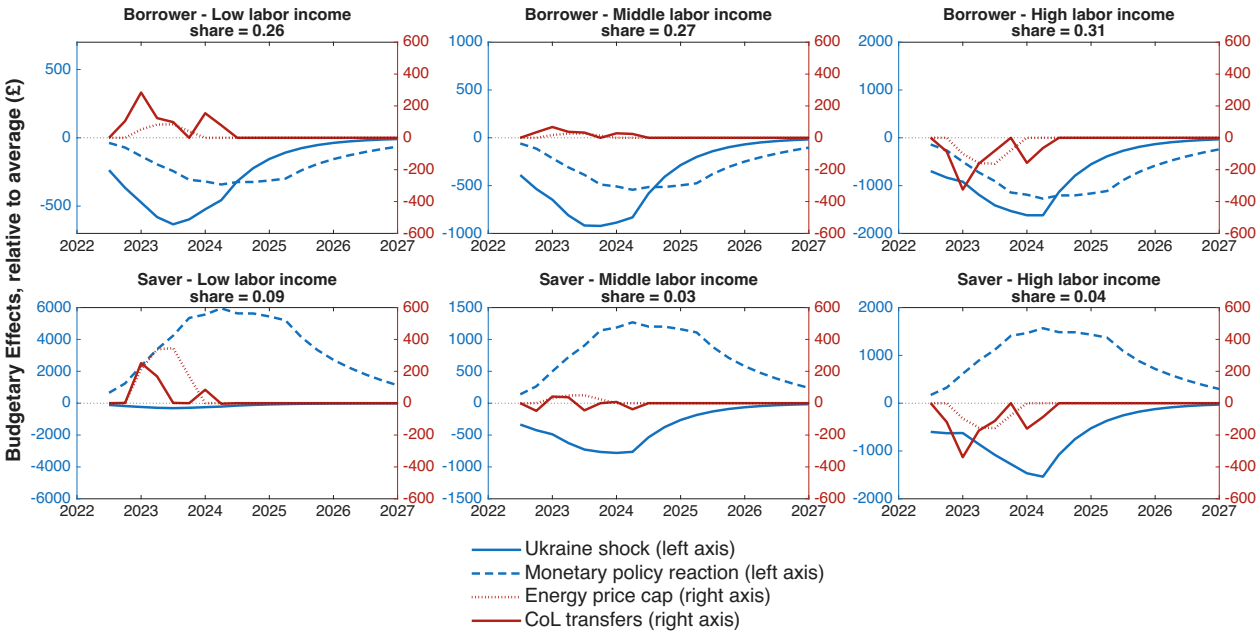


Figure 7: Distributional effects of the shock and support package.

Notes: Average budgetary effects by group, in £ per quarter. Groups are defined by nominal wealth position and labor income terciles, with their population share given at the top of each panel. The effects of the two fiscal measures are shown net of the lump-sum taxes.

5.4 Welfare evaluation

We now evaluate the welfare effects of the policies by comparing welfare losses in a baseline simulation (i.e. the Ukraine shock plus the fiscal response) with those in a counterfactual simulation without the fiscal measures. In order to understand the effects well, we apply the decomposition of Section 4.2. That is, we first consider only the partial equilibrium effects, then we add the effects of macro exposures, and finally we compute the overall welfare effect of the fiscal package. More formally, recall from Section 4.2 that:

$$\begin{aligned}
\Delta\mathcal{L}_{\text{PE}} &= \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{0};\mathbf{A},\mathbf{0}) \\
\Delta\mathcal{L}_{\text{PE}} + \Delta\mathcal{L}_{\text{ME}} &= \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{0}) \\
\Delta\mathcal{L}_{\text{PE}} + \Delta\mathcal{L}_{\text{ME}} + \Delta\mathcal{L}_{\text{GE}} &= \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{0}),\mathbf{X}(\mathbf{A},\mathbf{0});\mathbf{A},\mathbf{0})
\end{aligned}$$

where $\mathbf{A}$, $\mathbf{P}$, $\mathbf{X}$ and $\mathbf{Z}$ are defined as in Section 4.2, except that here we add the nominal wage $\hat{W}_t$ to $\mathbf{X}$ since in the general case with input-output linkages it enters the welfare loss function separately from the output gap. Note also that each of these components we can further decompose into the contributions of inflation within and between sectors, the

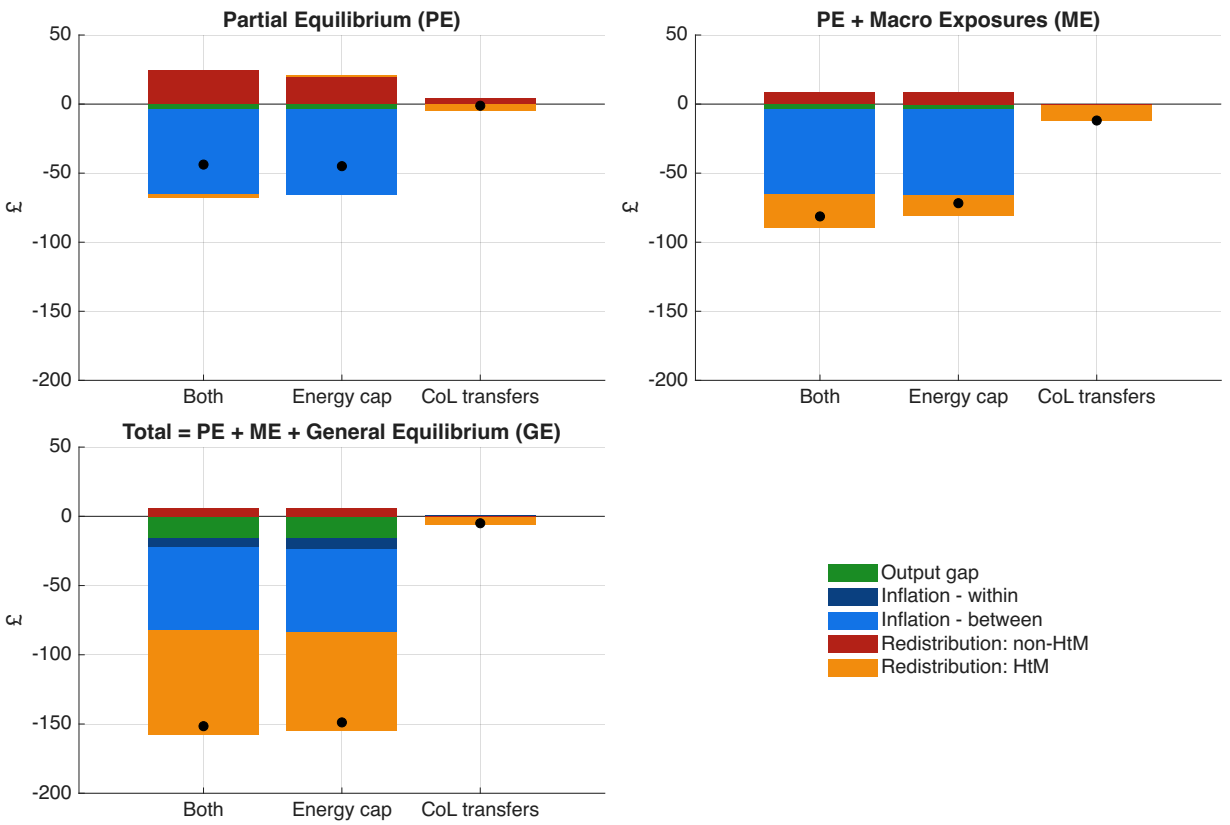


Figure 8: Welfare effects of the support package.

Notes: See main text.

output gap, and redistribution. The results are shown in Figure 8.

Partial Equilibrium effects. This welfare calculation, most closely in spirit to a typical evaluation in the applied micro literature, is shown in the top left panel of Figure 8. It considers only the effects of changes in relative prices and sectoral productivity, shutting down fluctuations in the output gap and the interest rate, i.e. it sets $\mathbf{X} = 0$, in the loss evaluation both with and without the fiscal intervention.

For the energy price cap (i.e. the EPG) the main welfare effect is driven by the between-sector inflation component, which captures the distortion of the consumption basket created by the subsidy, which encourages energy consumption. Unsurprisingly, this creates a negative welfare effect. In addition, there are significant redistribution effects which are on net positive as the price cap benefits exactly those who are most affected by the increase in energy prices. However, the welfare effect turns out to be dominated by the consumption basket distortion, so that the overall effect is negative. Finally, there is a very small output gap term, which arises from heterogeneity in marginal budget shares,

which affects labor supply responses.

The panel further shows that the PE welfare effect of the Cost-of-Living transfers is negative, but only slightly so. An important reason for this moderate magnitude of the redistribution term is that the transfers were based on characteristics such as age or income, and while these correlated with exposure to energy prices, the correlation in the data is relatively weak, since there is a large amount of idiosyncratic variation in consumption baskets in the data. Thus, the transfers are only moderately successful in compensating households most affected by price changes. In addition, since the transfers were paid out in particular quarters, they induced consumption volatility among HtM households, reducing welfare.

Macro exposures. The top right panel of Figure 8 adds the Macro Exposures to the welfare calculation. That is, the panel shows $\Delta\mathcal{L}_{\text{PE}} + \Delta\mathcal{L}_{\text{ME}} = \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{0})$. Importantly, the calculation now includes the redistributive effects of the nominal interest rate (but not the fact that the interest rate reacted endogenously to the fiscal policy). Interestingly, the redistribution now becomes negative on net, driven by the HtM households. This is due to the perverse redistributive effects discussed in Section 5.3: higher-income, indebted households suffered from the Ukraine shock and the interest rate hike this triggered, and the fiscal package exacerbated this loss as these households benefited relatively little from the package, while still paying for it through taxes. This reduces their welfare in particular to the extent that these indebted households are also HtM.

General Equilibrium effects. Finally, we add the General Equilibrium (GE) effects. We then obtain the total welfare effect of the fiscal intervention, plotted in the bottom left panel of Figure 8. This panel shows $\Delta\mathcal{L}_{\text{PE}} + \Delta\mathcal{L}_{\text{ME}} + \Delta\mathcal{L}_{\text{GE}} = \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{Z}),\mathbf{X}(\mathbf{A},\mathbf{Z});\mathbf{A},\mathbf{Z}) - \mathcal{L}(\mathbf{P}(\mathbf{A},\mathbf{0}),\mathbf{X}(\mathbf{A},\mathbf{0});\mathbf{A},\mathbf{0})$. Now, the welfare effects of the price cap are even more negative, driven by the redistribution/smoothing term. As studied above, the package has a stimulative effect on the output gap and inflation, to which the central bank reacts endogenously through its policy rule by increasing even further the nominal interest rate (see Figure 5). This in turn creates even larger welfare losses for the most indebted households, who were already most negatively affected.

Interestingly, welfare loss created by the Cost-of-Living transfers alone is nearly zero, which is in line with their limited macro effects in our model.

Finally, we note that the overall welfare loss of the fiscal package is about £150 per household, roughly 3.3 times as large as in the Partial Equilibrium, which underscores the

importance of accounting for macro channels.

5.4.1 Robustness

In the Appendix, we consider two robustness exercises. In the first, we compute welfare under the assumption of a linear social welfare function ($G'' = 0$). Results are quantitatively somewhat different in this version, but overall very similar to our baseline.

In a second robustness exercise, we allow for the fiscal policy to be financed by government debt. During the 2022 Cost-of-Living crisis, the UK had relatively limited fiscal space. Accordingly, our baseline model assumes that the fiscal packages are financed through contemporaneous taxes. We now extend the model to allow the policies to be financed via an increase in government debt, repaid at some later date.

We find that the welfare effect of the fiscal package becomes even *more* negative than in the baseline without government debt, and the welfare loss from the smoothing component is also larger. The mechanism runs as follows. Delaying the tax increase stimulates aggregate demand in the wake of the shock, which raises the output gap and inflation and, through the monetary policy rule, the nominal interest rate. The higher nominal rate amplifies the smoothing losses associated with the interest rate increase, and these dominate the overall welfare effects.

5.5 Looking through the energy price subsidy?

In practice, central banks often state that they "look through" the purely mechanical effects of fiscal measures on inflation, and our baseline rule does the same by responding only to core inflation. Our analytical results suggest this practice may be costly. A central bank that responds to the mechanical inflation induced by the cap offsets the demand wedge, while leaving intact the deflationary pressure of the supply wedge in the NKPC, and can potentially lessen the distortions of the fiscal package.

To study this, we consider an alternative rule under which the central bank does not look through the energy price cap:

$$\hat{R}_t = \phi_\pi \, \pi^{1y}_{core,t} + \phi_{\tilde{\mathcal{Y}}} \, \tilde{\boldsymbol{\mathcal{Y}}}_t + \phi_R \, \hat{R}_{t-1} + \phi_\pi \bar{s}_E \left(\pi^{Q,1y}_{E,t} - \pi^{1y}_{E,t} \right) + \varepsilon_t,$$

where $\pi^{Q,1y}_{E,t}$ and $\pi^{1y}_{E,t}$ denote, respectively, the one-year inflation rates of the consumer and producer prices in *Electricity & Gas*, and $\bar{s}_E$ is the CPI weight on energy. In the absence of a price subsidy, $\pi^{Q,1y}_{E,t} = \pi^{1y}_{E,t}$, and the rule reduces to the baseline specification. With the cap, by contrast, monetary policy responds to the policy-induced wedge in energy

inflation in the same way as it responds to core inflation, i.e. with coefficient ϕ_π, and can therefore lessen the impact of the demand wedge.

Figure 9 compares simulations under the baseline and the alternative rule. The additional term in the alternative rule is negative when the cap is introduced in 2022, as consumer energy prices fall relative to producer prices, and positive when the cap is lifted and consumer prices catch up. Since consumer and producer prices end up at the same level once the cap is removed, this term sums to zero over the episode: in that sense, not looking through does not tighten or loosen policy overall, but retimes the mechanical interest rate responses. However, through the endogeneous responses (smoothing, wholesale inflation and the output gap), the nominal rate is sharply lower when the cap is introduced and slightly higher towards the end. Despite this large initial loosening, neither the output gap nor wholesale inflation increases, as the supply wedge keeps exerting downward pressure on inflation. Not looking through therefore allows the central bank to set a lower path of interest rates early on without creating additional macroeconomic distortions. Figure 10 shows that this has a large welfare payoff. Under the alternative rule, the welfare effect of the fiscal package turns from strongly negative to slightly positive. The improvement runs through the redistribution component: the lower interest rate path in 2022–2023 limits the rise in debt servicing costs for indebted households, precisely when they are hit hardest by the shock, thereby reversing the redistribution generated by the package.

As discussed above, the effects of fiscal interventions depend on how monetary policy responds to them. In practice, central banks often state that they "look through" the purely mechanical effects that fiscal measures may have on inflation. In the baseline model, the monetary policy rule indeed looks through the mechanical effects of the price subsidy, as it responds only to core inflation, which excludes the *Electricity & Gas* sector.

In this subsection, we consider an alternative policy reaction under which the central bank does not look through the energy price cap. Specifically, we augment the policy rule as follows:

$$\hat{R}_t = \phi_\pi \, \pi^{1y}_{core,t} + \phi_{\tilde{\boldsymbol{y}}} \, \tilde{\boldsymbol{y}}_t + \phi_R \, \hat{R}_{t-1} + \phi_\pi \bar{s}_E \left(\pi^{Q,1y}_{E,t} - \pi^{1y}_{E,t} \right) + \varepsilon_t,$$

where $\pi^{Q,1y}_{E,t}$ and $\pi^{1y}_{E,t}$ denote, respectively, the one-year inflation rates of the consumer and producer prices in *Electricity & Gas*, and $\bar{s}_E$ is the CPI weight on energy. In the absence of a price subsidy, $\pi^{Q,1y}_{E,t} = \pi^{1y}_{E,t}$, and the rule reduces to the baseline specification. With

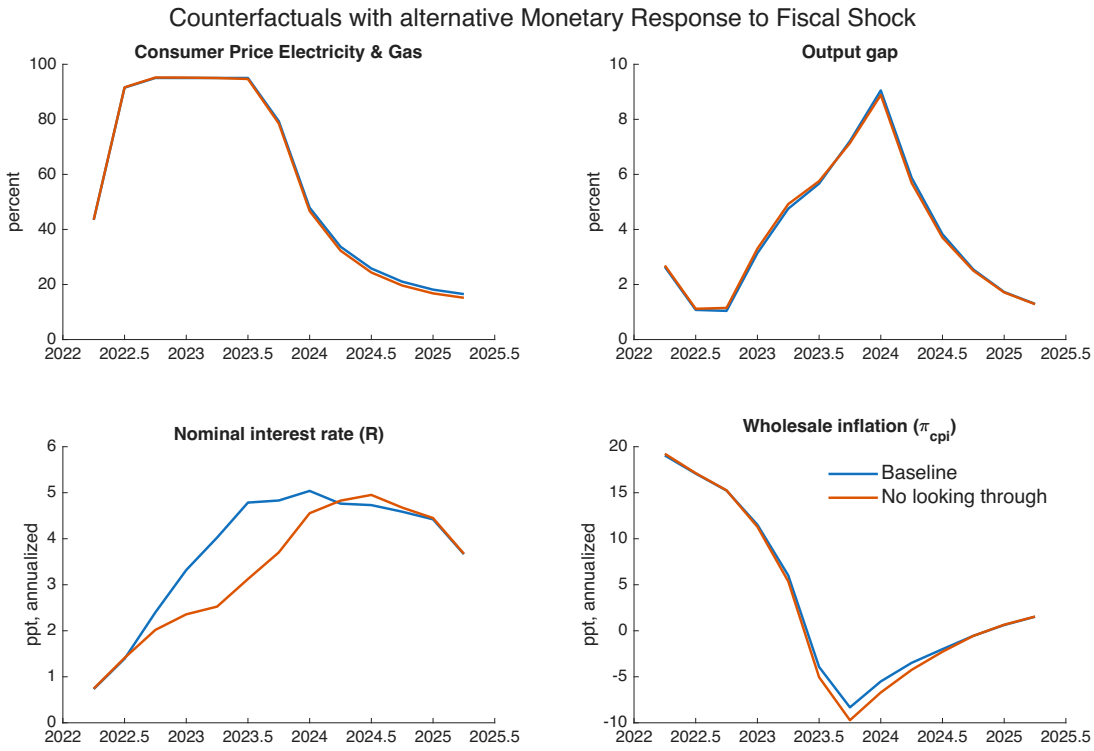


Figure 9: Macroeconomic effects of monetary policy "looking through" the price subsidy.

Notes: See main text.

the cap, by contrast, monetary policy responds to the policy-induced wedge in inflation in the same manner as it responds to core inflation, i.e. with coefficient ϕ_π.

Figure 9 compares a simulation under the baseline rule with one under the alternative rule ("no looking through"). Under the latter, the nominal interest rate rises by less in 2022–2023 and somewhat more in 2024. Figure 10 reports the welfare effect under the "no looking through" policy, compared to the baseline. Under the alternative monetary policy, the welfare effect of the fiscal package is positive. Intuitively, when the central bank does not look through the mechanical effect, the path of the nominal interest rate is smoother and less front-loaded, limiting the rise in debt servicing costs for indebted households and thereby attenuating the adverse redistributive consequences of the shock.

6 Conclusions

In this paper, we evaluate the welfare and macroeconomic effects of household support packages implemented during the 2022 cost-of-living crisis, with a particular focus on

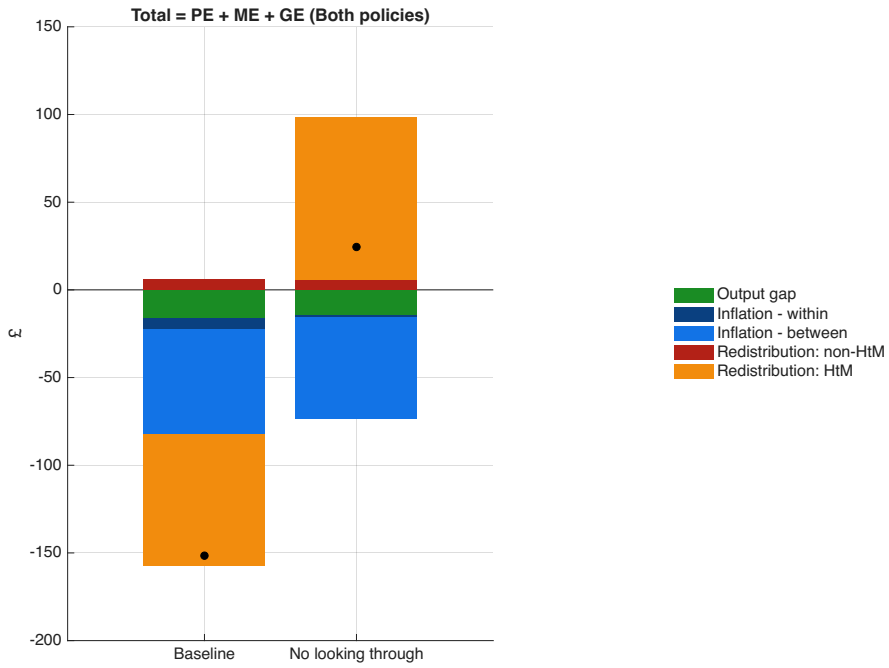


Figure 10: Welfare effect of the fiscal intervention: dependence on monetary policy.

Notes: The left bar shows the welfare effect of the fiscal policy in the baseline model. The right bar shows the same effect, but under an alternative monetary policy rule without "looking through", as described in the main text.

the United Kingdom. Using a multi-sector New Keynesian model disciplined by detailed micro data, we show that sectoral price subsidies (or price caps) and targeted transfers have substantial aggregate and distributional consequences that extend well beyond their direct effects on household budgets. While price caps can redistribute toward energy-exposed households, they also generate sizable macroeconomic adjustments through both demand and supply channels, especially when monetary policy responds endogenously. Quantitatively, accounting for these macro equilibrium effects significantly amplifies the welfare losses relative to partial-equilibrium assessments.

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A Model derivations

A.1 Households

This section lays out the household side of the model, building closely on Olivi et al. (2026). Households have unit mass, are uniformly distributed, and are indexed by i. Preferences are additively separable over time, with a discount factor β and a death rate δ common to all households, while intratemporal preferences over consumption and labor are allowed to differ by type i. Throughout we impose that dividend shares equal steady-state labor income shares ($\varsigma_k(i) = Wn(i)/WN$). Within each type i, a fraction $\varphi(i)$ of households is Hand-to-Mouth (HtM), with the remaining $1-\varphi(i)$ unconstrained.

A.1.1 Unconstrained Households

Unconstrained households can save and borrow through a risk-free bond b_t, subject to the natural borrowing limit (equivalently, a no-Ponzi condition).

1. Time separability lets us write the **intertemporal problem** of unconstrained households as:

$$V_t(i) = \sup_{\{\mathcal{I}_{t+s}, b_{t+s+1}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1-\delta))^s \mathcal{V}_i(\mathcal{I}_{t+s}, W_{t+s}, \mathbf{Q}_{t+s}) \tag{12}$$

$$s.t. \mathcal{I}_{t+s} = b_{t+s} + \sum_k \varsigma_k(i) Div_{k,t+s} - \frac{b_{t+s+1}}{R_{t+s}} + \zeta_{t+s}(i), \quad \lim_{s\to\infty} b_{t+s} / \prod_{u=0}^{s-1} R_{t+u} = 0 \; a.s$$

$$b_t(i) = b(i)\left(1 + \sum_k \bar{s}_k \frac{Q_{k,t} - Q_k^*}{Q_k^*}\right).$$

Here $\varsigma_k(i)$ is i's constant share of dividends ($Div_{k,t}$) paid by firms in sector k, and $\zeta_t(i)$ is the lump-sum transfer (or tax) received by i. Household i's initial real wealth is given by $b_t(i) = b(i)\left(1 + \sum_k \bar{s}_k \frac{Q_{k,t}-Q_k^*}{Q_k^*}\right)$, with Q_k^* the (initial) steady-state consumer price (so $Q_k^* = P_k^*$) and $\bar{s}_k$ the steady-state expenditure share of sector k; this formulation pins real wealth (measured in time-t CPI) of a household i born at t to its initial value. We assume $\int_0^1 b(i)\, di = 0$. $\mathcal{V}_i(\mathcal{I}, W, \mathbf{Q})$, defined below, is the intratemporal indirect utility function and depends on intratemporal income $\mathcal{I}$, the wage W, and the vector of sectoral consumer prices $\mathbf{Q} = \{Q_1, ..., Q_K\}$.

2. The **intratemporal problem** pins down the optimal allocation of labor n and consumption expenditure e, and in turn defines $\mathcal{V}_i(\mathcal{I}, W, \mathbf{Q})$:

$$\mathcal{V}_i(\mathcal{I}, W, \mathbf{Q}) = \sup_{e,n} v_i(e, \mathbf{Q}) - \chi_i\left(\frac{n}{\vartheta(i)}\right) \quad s.t. \quad e - Wn = \mathcal{I}. \tag{13}$$

$\vartheta(i)$ is (constant) labor productivity, χ_i is increasing, C^2, and strictly convex, and $v_i(e, \mathbf{Q})$, defined next, is the indirect utility from consumption.

3. Last, the indirect consumption utility $v_i(e, \mathbf{Q})$ comes from the **consumption problem**, which allocates expenditure e optimally across sectors. Within a sector k, households share a common CES aggregator over varieties: $c_k = \left(\int_0^1 c_k(j)^{\frac{\epsilon_k - 1}{\epsilon_k}} dj\right)^{\frac{\epsilon_k}{\epsilon_k - 1}}$. This delivers variety demand $c_k(j) = \left((1-\tau_k) p_k(j)/Q_k\right)^{-\epsilon_k} c_k = \left(p_k(j)/P_k\right)^{-\epsilon_k} c_k$, with sectoral price index

$$Q_k = (1-\tau_k) P_k = (1-\tau_k)\left(\int_0^1 p_k(j)^{1-\epsilon_k} dj\right)^{\frac{1}{1-\epsilon_k}}.$$

The consumption problem is then:

$$v_i(e, \mathbf{Q}) = \max_{\{c_k\}_{k=1}^K} u_i(c_1, c_2, \ldots, c_K), \quad s.t. \quad \sum_{k=1}^K Q_k c_k = e, \; Q_k = (1-\tau_k)\left(\int_0^1 p_k(j)^{1-\epsilon_k} dj\right)^{\frac{1}{1-\epsilon_k}} \tag{14}$$

u_i is increasing, C^2, and strictly concave.

First-order conditions and log-linearization Here we characterize how optimal consumption and labor supply respond to price changes (variety prices, sectoral subsidies, the wage, and the interest rate) in the neighborhood of a steady state in which variety prices are constant and equal within sectors and the real interest rate satisfies $R_t^{-1} = (1-\delta)\beta$. To keep notation light, time subscripts are dropped for the consumption and intratemporal problems throughout this appendix unless noted otherwise, and hats denote log deviations.

Consumption. Problem 14yields a C^2 Marshallian demand function $c_{k,i}(e, \boldsymbol{Q})$ and expenditure function

$$e_{k,i}(e, \boldsymbol{Q}) = Q_k c_{k,i}(e, \boldsymbol{Q}).$$

Sectoral consumption then responds to price and expenditure changes as:

$$\hat{c}_{k,i}(e,\boldsymbol{Q}) = \partial_e e_k(i)\left(\hat{e} - \sum_{l=1}^{K} s_l(i)\hat{Q}_l\right)\frac{1}{s_k(i)} + \sum_{l=1}^{K}\rho_{k,l}(i)\,\hat{Q}_l,$$

where

$$\rho_{k,l}(i) = \left(Q_k\partial_{\ln Q_l}c_{k,i}(e,\boldsymbol{Q}) + e_{l,i}(e,\boldsymbol{Q})\,\partial_e e_{k,i}(e,\boldsymbol{Q})\right)/e_{k,i}(e,\boldsymbol{Q})$$

denotes the compensated price elasticity of c_k, while $s_l(i) = e_{l,i}(e,\boldsymbol{Q})/e$ and $\partial_e e_k(i) = \partial_e e_{k,i}(e,\boldsymbol{Q})$ denote, respectively, sector l's expenditure share and marginal expenditure.

Applying Lemma 1 of Olivi et al. (2026) gives the derivatives of the marginal value of income $\partial_e v_i(e,\boldsymbol{Q})$, which we use below to pin down the optimal labor supply and consumption expenditure choices in the intratemporal problem:

$$Q_k\partial_{Q_k}\{\partial_e v_i(e,\boldsymbol{Q})\} = -\partial_e v_i(e,\boldsymbol{Q})\left(\partial_e e_k(i) - \frac{1}{\sigma(i)}s_k(i)\right) \tag{15}$$

with $\sigma(i) \equiv -\partial_e v_i(e,\boldsymbol{Q})/(e\partial_{ee}v_i(e,\boldsymbol{Q}))$. u_i is renormalized so that, at initial prices, $\sigma(i)$ is the same across households, equal to σ, without altering the demand functions $c_{k,i}(e,\boldsymbol{Q})$.

Intratemporal Problem. Turning to the optimal choice of labor supply, the first-order condition of the intratemporal problem 13governing optimal labor supply and expenditure reads:

$$W\partial_e v_i(e,\boldsymbol{Q}) = \frac{1}{\vartheta(i)}\chi_i'\left(\frac{n}{\vartheta(i)}\right).$$

Combined with the intratemporal budget constraint, this pins down labor supply $n(i) = n_i(\mathcal{I},W,\boldsymbol{Q})$ and expenditure $e(i) = e_i(\mathcal{I},W,\boldsymbol{Q})$. Log-linearizing around the optimal allocation gives (see Lemma 2 of Olivi et al. (2026)):

$$\hat{n}(i) = \psi(i)\left\{\hat{W} - \sum_{k=1}^{K}\partial_e e_k(i)\hat{Q}_k\right\} - \frac{\psi(i)}{\sigma(i)}\left(\hat{e}(i) - \sum_{k=1}^{K}s_k(i)\hat{Q}_k\right), \tag{16}$$

with $\psi(i) \equiv \chi_i'\left(\frac{n(i)}{\vartheta(i)}\right)/\left(\frac{n(i)}{\vartheta(i)}\chi_i''\left(\frac{n(i)}{\vartheta(i)}\right)\right)$ the Frisch elasticity of labor supply. Going forward we take $\chi_i\left(\frac{n}{\vartheta(i)}\right) = \kappa_i\left(\frac{n}{\vartheta(i)}\right)^{1+\frac{1}{\psi}}$ with $\kappa_i > 0$, so that ψ_i is common across households and equal to ψ. The envelope condition applied to the intratemporal problem further delivers:

$$\partial_{\mathcal{I}}\mathcal{V}_i(\mathcal{I},W,\mathbf{Q}) = \partial_e v_i(e(i),\boldsymbol{Q}). \tag{17}$$

Intertemporal Problem. Reintroducing time subscripts, the optimal expenditure path is obtained by log-linearizing the first-order conditions of the intertemporal problem 12, given by:

$$\frac{\partial\mathcal{V}_i(\mathcal{I}_{t+s},W_{t+s},\mathbf{Q}_{t+s})}{\partial\mathcal{I}_{t+s}} = \beta(1-\delta)R_{t+s}\mathbb{E}_{t+s}\frac{\partial\mathcal{V}_i(\mathcal{I}_{t+s+1},W_{t+s+1},\mathbf{Q}_{t+s+1})}{\partial\mathcal{I}_{t+s+1}},$$
$$\Rightarrow \partial_e v_{i,t+s} = \beta(1-\delta)R_{t+s}\mathbb{E}_{t+s}\partial_e v_{i,t+s+1},$$

where the second line follows from equation (17), and $\partial_e v_{i,t} = \partial_e v_i(e(\mathcal{I}_t,W_t,\mathbf{Q}_t),\boldsymbol{Q}_t)$ is shorthand notation. Lemma 3 of Olivi et al. (2026) then implies:

$$\left(\hat{e}_t(i) - \sum_k s_k(i)\hat{Q}_{k,t}\right) = \mathbb{E}_t\left(\hat{e}_{t+1}(i) - \sum_k s_k(i)\hat{Q}_{k,t+1}\right) - \mathbb{E}_t\sigma\left(\hat{R}_t - \sum_k\partial_e e_k(i)\pi^{Q}_{k,t+1}\right),$$

with $\pi^{Q}_{k,t} = \hat{Q}_{k,t} - \hat{Q}_{k,t-1}$.

A.1.2 Hand-to-Mouth households

HtM households spend all of their current income and never adjust their bond holdings. Recall a household of type i born in t_0 starts with bond holdings $b_{t_0}(i) = b(i)\left(1 + \sum_{k=1}^{K}\bar{s}_k\frac{Q_{k,t_0}-Q_k^*}{Q_k^*}\right)$. Their problem can then be rewritten as:

$$\mathcal{V}_i(\mathcal{I}_t,W_t,\mathbf{Q_t}) = \sup_{e,n} v_i(e,\mathbf{Q}_t) - \chi_i\left(\frac{n}{\vartheta(i)}\right) \quad s.t. \quad e - W_t n = \mathcal{I}_t,\ \mathcal{I}_t = \left(1 - \frac{1}{R_t}\right)b_{t_0}(i) + \sum_k\varsigma_k(i)Div_{k,t} + \zeta_t(i)$$

Combining the demand function with equations (15), (16), and (17) yields:

$$\begin{aligned}
\hat{c}_{k,t}(i) &= \partial_e e_k(i)\left(\hat{e}_t(i) - \sum_{l=1}^{K} s_l(i)\hat{Q}_{l,t}\right)\frac{1}{s_k(i)} \\
&\quad + \sum_{l=1}^{K}\rho_{k,l}(i)\hat{Q}_{l,t}, \\
\hat{n}_t(i) &= \psi\left\{\hat{W}_t - \sum_{k=1}^{K}\partial_e e_k(i)\hat{Q}_{k,t}\right\} - \frac{\psi}{\sigma}\left(\hat{e}_t(i) - \sum_{k=1}^{K} s_k(i)\hat{Q}_{k,t}\right), \\
\hat{e}_t(i) - \sum_{k=1}^{K} s_k(i)\hat{Q}_{k,t} &= \frac{1}{e(i) + \frac{\psi}{\sigma}Wn(i)}\left(\hat{R}_t\frac{b(i)}{R} + Wn(i)\left(\hat{W}_t - \sum_{k=1}^{K}\bar{s}_k\hat{Q}_{k,t}\right) + \sum_k \varsigma_k(i)dDiv_{k,t} + d\zeta_t(i)\right. \\
&\quad + Wn(i)\psi\left(\hat{W}_t - \sum_{k=1}^{K}\partial_e e_k(i)\hat{Q}_{k,t}\right) - \sum_{k=1}^{K}\left(e(i)\left(s_k(i) - \bar{s}_k\right)\right)\hat{Q}_{k,t} \\
&\quad \left. + \left(1 - \frac{1}{R}\right)b(i)\sum_{k=1}^{K}\bar{s}_k\left(\hat{Q}_{k,t_0} - \hat{Q}_{k,t}\right)\right).
\end{aligned}$$

A.2 Firms

This section derives the sectoral New Keynesian Phillips Curves. Within each sector, firms are ex-ante identical, operate a constant-returns-to-scale technology, and produce varieties of good k using labor together with a bundle of sector-l goods assembled by a representative intermediary. We proceed in three steps: first, the firm's pricing equation away from steady state is written as a function of the change in unit marginal cost; second, the firm's intratemporal problem delivers the resulting changes in demand for intermediate inputs and labor; third, combining these with market clearing in goods and labor produces the sectoral NKPCs in terms of sectoral prices and the output gap.

A.2.1 Intermediate inputs producers

Consider first the competitive intermediaries that produce intermediate inputs. They bundle differentiated varieties into $\tilde{Y}_k$ using the same CES aggregator as households, then sell the bundle to firms at price P_k^I:

$$\sup_{\{\tilde{y}_k(j)\}_{0\le j\le 1}} P_k^I\tilde{Y}_k - \int_0^1 p_k(j)\tilde{y}_k(j)dj \quad s.t. \tilde{Y}_k = \left(\int_0^1 \tilde{y}_k(j)^{\frac{\epsilon_k-1}{\epsilon_k}}dj\right)^{\frac{\epsilon_k}{\epsilon_k-1}}.$$

This gives $P_k^I = \left(\int_0^1 p_k^{1-\epsilon_k}(j)dj\right)^{\frac{1}{1-\epsilon_k}} = P_k$ and $\tilde{y}_k(j) = \left(p_k(j)/P_k\right)^{-\epsilon_k}\tilde{Y}_k$.

A.2.2 Goods varieties firms: price setting

Turning to the firms producing individual varieties, let $Y_k = \tilde{Y}_k + C_k$ denote total demand for sector k, so that demand for variety j is $y_k(j) = \left(p_k(j)/P_k\right)^{-\epsilon_k}Y_k$. Exploiting constant returns to scale in sector-k production, the present value of firm profits can be expressed in terms of the reset price as:

$$\max_{p_{k,t}(j^*)} \mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta_k^s\left\{\left(p_{k,t}(j^*) - (1-\iota_k)MC_k\left(W_{t+s}, \boldsymbol{P}_{t+s}\right)\right)\left(\frac{p_{k,t}(j^*)}{P_{k,t+s}}\right)^{-\epsilon_k}Y_{k,t+s} - T_{k,t+s}\right\},$$

where MC_k denotes marginal cost, specified below. The first-order condition yields the standard CES pricing condition:

$$\mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta_k^s\left(p_{k,t}(j^*) - \frac{\epsilon_k}{\epsilon_k-1}(1-\iota_k)MC_k\left(W_{t+s}, \boldsymbol{P}_{t+s}\right)\right)\left(\frac{p_{k,t}(j^*)}{P_{k,t+s}}\right)^{-\epsilon_k}Y_{k,t+s} = 0.$$

The steady state satisfies $p_{k,ss}(j^*) = P_{k,ss} = \frac{\epsilon_k}{\epsilon_k-1}(1-\iota_k)MC_k\left(W_{ss}, \boldsymbol{P}_{ss}\right)$. A first-order approximation of the optimality condition around this steady state, using the expressions above, gives:

$$\hat{p}_{k,t}\left(j^*\right) = \left(1-\tilde{\beta}\theta_k\right)\mathbb{E}_t\sum_{s=0}^{\infty}\left(\tilde{\beta}\theta_k\right)^s\widehat{MC}_{k,t+s},$$

using that $\Lambda_{t,t+s} = \tilde{\beta}^s$ in steady state, with $\tilde{\beta} = (1-\delta)\beta = 1/R$, and that the subsidy satisfies $\frac{\epsilon_k}{\epsilon_k-1}(1-\iota_k) = 1$. Since all reset prices at t coincide, $\hat{P}_{k,t} = (1-\theta_k)\hat{P}_{k,t}^* + \theta_k\hat{P}_{k,t-1}$; denoting $\hat{mc}_{k,t} \equiv \hat{MC}_{k,t} - \hat{P}_{k,t}$ then gives:

$$\pi_{k,t} = \lambda_k\hat{mc}_{k,t} + \tilde{\beta}\mathbb{E}_t\pi_{k,t+1}, \tag{18}$$

with $\lambda_k = \left(1-\tilde{\beta}\theta_k\right)\left(1-\theta_k\right)/\theta_k$.

A.2.3 Goods varieties firms: intermediate input choice

The firm's cost-minimization problem is: $\min W n_k(j) + \sum_l P_l \tilde{Y}_{l,k}(j) \quad s.t. \quad A_k F_k\left(n_k(j), \tilde{Y}_{1,k}(j), \tilde{Y}_{2,k}(j), \ldots, \tilde{Y}_{K,k}(j)\right) \geq y_k(j)$. Because F_k exhibits constant returns to scale, input demand takes the form $\tilde{Y}_{l,k}(j) = \mathcal{Y}_{l,k}(\boldsymbol{P}, W)\frac{y_k(j)}{A_k}$, $n_k(j) = \mathcal{N}_k(\boldsymbol{P}, W)\frac{y_k(j)}{A_k}$, where the unit demand for input l by firms in k, $\mathcal{Y}_{l,k}(\boldsymbol{P}, W)$, is common across firms in k. This lets us write the change in marginal cost as:

$$\hat{MC}_k = \left(\Omega_{N,k}\hat{W} + \sum_l \Omega_{k,l}\hat{P}_l\right) - \hat{A}_k. \tag{19}$$

Here Ω is the matrix of intermediate input shares ($\Omega_{k,l} = \frac{P_l \mathcal{Y}_{l,k}(\boldsymbol{P},W)}{W\mathcal{N}_k(\boldsymbol{P},W)+\sum_{l=1}^K P_l \mathcal{Y}_{l,k}(\boldsymbol{P},W)} = \frac{P_l \tilde{Y}_{l,k}}{P_k Y_k}$ under equal steady-state variety prices) and Ω_N is the length-K column vector of labor shares ($\Omega_{N,k} = 1 - \sum_{l=1}^K \Omega_{k,l}$). Again exploiting CRS in F_k, the change in aggregate demand for input bundle l around the steady state with equal variety prices is:

$$\frac{d\tilde{Y}_l}{\tilde{Y}_l} = \sum_k \boldsymbol{Q}_{l,k}\left(\hat{Y}_k - \hat{A}_k\right) + \tilde{\mathcal{T}}_{l,W}\hat{W} + \sum_k \tilde{\mathcal{T}}_{l,k}\hat{P}_k.$$

where $\tilde{\mathcal{T}}$ is the matrix of aggregate input price elasticities, $\tilde{\mathcal{T}}_{l,k} = \sum_m \frac{\tilde{Y}_{l,m}}{\tilde{Y}_l}\frac{\partial \mathcal{Y}_{l,m}}{\partial P_k}\frac{P_k}{\mathcal{Y}_{l,m}}$, $\tilde{\mathcal{T}}_{l,W} = \sum_m \frac{\tilde{Y}_{l,m}}{\tilde{Y}_l}\frac{\partial \mathcal{Y}_{l,m}}{\partial W}\frac{W}{\mathcal{Y}_{l,m}}$ is the vector of wage elasticities, and $\boldsymbol{Q}_{l,k} = \frac{\tilde{Y}_{l,k}}{\tilde{Y}_l} = \frac{Y_k}{A_k \tilde{Y}_l}\mathcal{Y}_{l,k}$. Because intermediary input producers use a CRS technology, market clearing for variety k requires $\hat{Y}_k = s_k^C \hat{C}_k + (1-s_k^C)\widehat{\tilde{Y}}_k$. Letting $\mathcal{D}[s^c]$ and $\mathcal{D}[PY]$ denote the diagonal matrices of consumption-demand shares and sectoral revenue ($s_k^C = \frac{E_k}{P_k Y_k}$ and $P_k Y_k$), and $\hat{\boldsymbol{Y}}, \hat{\boldsymbol{C}}, \hat{\boldsymbol{A}}, \hat{\boldsymbol{P}}$ the column vectors of sectoral output, consumption, TFP shocks, and prices, we get:

$$\left(Id - \mathcal{D}[PY]^{-1}\Omega^T \mathcal{D}[PY]\right)\hat{\boldsymbol{Y}} = \mathcal{D}[s^c]\hat{\boldsymbol{C}} - \mathcal{D}[PY]^{-1}\Omega^T \mathcal{D}[PY]\hat{\boldsymbol{A}} + \mathcal{T}_W \hat{W} + \mathcal{T}\hat{\boldsymbol{P}}.$$

using $\left[\left(Id - \mathcal{D}[s^c]\right)\boldsymbol{Q}\right]_{k,l} = \frac{P_k \tilde{Y}_k}{P_k Y_k}\frac{\tilde{Y}_{k,l}}{\tilde{Y}_k} = \frac{P_l Y_l}{P_k Y_k}\Omega_{l,k}$. It follows that $\mathcal{T}_W = \left(Id - \mathcal{D}[s^c]\right)\tilde{\mathcal{T}}_W$ and, analogously, $\mathcal{T} = \left(Id - \mathcal{D}[s^c]\right)\tilde{\mathcal{T}}$.

A.2.4 Labour Demand Response

A firm j in sector k likewise demands labor according to $n_k(j) = \mathcal{N}_k(\boldsymbol{P}, W)\frac{y_k(j)}{A_k}$. Differentiating and aggregating, the percentage change in aggregate labor demand can be written:

$$\hat{N} = \boldsymbol{s^N}\left((\mathbf{Y} - \mathbf{A}) + \tilde{\mathcal{T}}_W^N \hat{W} + \tilde{\mathcal{T}}^N \mathbf{P}\right),$$

with $\boldsymbol{s^N} = \left[\frac{W\int n_1(j)dj}{WN}, \ldots, \frac{W\int n_K(j)dj}{WN}\right]$, $\tilde{\mathcal{T}}_W^N = \left[\partial_{\ln(W)}\ln(\mathcal{N}_1), \ldots, \partial_{\ln(W)}\ln(\mathcal{N}_K)\right]^T$, and $\tilde{\mathcal{T}}_{k,l}^N = \partial_{\ln(P_l)}\ln(\mathcal{N}_k)$. Substituting the expression for $\mathbf{Y}$ and using the cost-minimization identity

$$W\, d\mathcal{N}_k + \sum_l P_l\, d\mathcal{Y}_{l,k} = 0$$

for changes in input prices at fixed unit output gives

$$\begin{aligned}\hat{N} &= \boldsymbol{s^N}\left(Id - \mathcal{D}[PY]^{-1}\Omega^T \mathcal{D}[PY]\right)^{-1}\left(\mathcal{D}[s^c]\hat{C} - \hat{A}\right)\\ &= \sum_{k=1}^K \bar{s}_k \hat{C}_k - \sum_{k=1}^K \frac{P_k Y_k}{E}\hat{A}_k.\end{aligned}$$

The second line follows from $[WN_1, \ldots, WN_K]\left(Id - \mathcal{D}[PY]^{-1}\Omega^T \mathcal{D}[PY]\right)^{-1} = [PY_1, \ldots, PY_K]$.

A.2.5 Labor Market Clearing

Throughout the aggregation, $E_k = P_k C_k = \int e_k(i)\,di$, $E = \sum_k E_k = \int e(i)\,di$, and $N = \int n(i)\,di$ denote steady-state quantities. The markup-offsetting production subsidy and its labor-income-proportional financing imply $E = WN$ and $e(i) - Wn(i) = (1-1/R)b(i)$. We use

$$\bar{s}_k = \frac{E_k}{E}, \qquad \overline{\partial_e e}_k = \int \frac{e(i)}{E}\partial_e e_k(i)\,di, \qquad \bar{\rho}_{k,l} = \int \frac{e_k(i)}{E_k}\rho_{k,l}(i)\,di,$$

and

$$\varphi^E = \int \varphi(i)\frac{e(i)}{E}\,di, \qquad \varphi^N = \int \varphi(i)\frac{Wn(i)}{WN}\,di.$$

In aggregate expressions without a u or HtM superscript, $\hat{e}_t^{t_0}(i)$ averages the two household groups with weights $1-\varphi(i)$ and $\varphi(i)$.

Reintroducing time subscripts, define the operator $\mathbb{E}_{\delta,t} X_{t_0} \equiv (1-\delta)^{t+1} X_- + \delta \sum_{t_0=0}^{t} (1-\delta)^{t-t_0} X_{t_0}$, which averages the variable X_{t_0} across generations. Aggregating households' labor supply decisions gives:

$$\hat{N}_t^s = \psi \left(\hat{W}_t - \sum_{k=1}^{K} \int \frac{Wn(i)}{WN} \partial_e e_k(i) di \hat{Q}_{k,t} \right) - \frac{\psi}{\sigma} \int \frac{Wn(i)}{WN} \mathbb{E}_{\delta,t} \left(\hat{e}_t^{t_0}(i) - \sum_{k=1}^{K} s_k(i) \hat{Q}_{k,t} \right) di.$$

Recall labor demand is given by:

$$\hat{N}_t = \sum_{k=1}^{K} \bar{s}_k \hat{C}_{k,t} - \sum_{k=1}^{K} \frac{P_k Y_k}{E} \hat{A}_{k,t},$$

$$\hat{C}_{k,t} = \int \frac{e(i)}{E_k} \partial_e e_k(i) \mathbb{E}_{\delta,t} \left(\hat{e}_t^{t_0}(i) - \sum_{l=1}^{K} s_l(i) \hat{Q}_{l,t} \right) di + \sum_{l=1}^{K} \bar{\rho}_{k,l} \hat{Q}_{l,t},$$

where the second line aggregates households' consumption decisions. Given $\sum_{k=1}^{K} \bar{s}_k \bar{\rho}_{k,l} = 0$, labor market clearing then yields

$$\hat{\mathcal{Y}}_t = \frac{\sigma}{\sigma+\psi} \int \frac{e(i)}{E} \left(1 + \frac{\psi W n(i)}{e(i)\sigma} \right) \mathbb{E}_{\delta,t} \left(\hat{e}_t^{t_0}(i) - \sum_{k=1}^{K} s_k(i) \hat{Q}_{k,t} \right) di + \frac{\sigma\psi}{\sigma+\psi} \sum_{k=1}^{K} \int \frac{Wn(i) - e(i)}{E} \partial_e e_k(i) di \hat{Q}_{k,t}. \tag{20}$$

where the following definitions were used:

$$\mathbf{A}_t \equiv (Id - \Omega)^{-1} \mathbf{A}_t,$$

$$\hat{\mathcal{Y}}_t \equiv \frac{\sigma}{\sigma+\psi} \left(\psi \hat{W}_t - \psi \sum_{k=1}^{K} \overline{\partial_e e}_k \hat{Q}_{k,t} + \sum_{k=1}^{K} \bar{s}_k \tilde{A}_{k,t} \right).$$

Defining the natural level of aggregate demand as the level that would prevail absent markup distortions and consumption subsidies gives our formula for the output gap:

$$\hat{\mathcal{Y}}^*{}_t \equiv \frac{\sigma}{\sigma+\psi} \sum_{k=1}^{K} \left(\psi \overline{\partial_e e}_k + \bar{s}_k \right) \tilde{A}_{k,t},$$

$$\tilde{\mathcal{Y}}_t = \frac{\sigma\psi}{\sigma+\psi} \left(\hat{W}_t - \sum_{k=1}^{K} \overline{\partial_e e}_k \left(\hat{Q}_{k,t} + \tilde{A}_{k,t} \right) \right).$$

As in Olivi et al. (2026), this corresponds to the welfare-relevant notion of the output gap.

A.3 Sectoral NKPC

Combining equations (19) and (18) with the output gap definition gives:

$$\begin{aligned} \pi_{k,t} = \lambda_k \Bigg\{ & \Omega_{N,k} \left(\frac{1}{\sigma} + \frac{1}{\psi} \right) \tilde{\mathcal{Y}}_t + \Omega_{N,k} \sum_{l=1}^{K} \overline{\partial_e e}_l \left(\hat{Q}_{l,t} - \hat{P}_{l,t} \right) \\ & + \Omega_{N,k} \sum_{l=1}^{K} \overline{\partial_e e}_l \left(\hat{P}_{l,t} + \tilde{A}_{l,t} - \left(\hat{P}_{k,t} + \tilde{A}_{k,t} \right) \right) + \sum_{l=1}^{K} \Omega_{k,l} \left(\hat{P}_{l,t} + \tilde{A}_{l,t} - \left(\hat{P}_{k,t} + \tilde{A}_{k,t} \right) \right) \Bigg\} \\ & + \tilde{\beta} \mathbb{E}_t \pi_{k,t+1}, \end{aligned}$$

Finally, define

$$\mathcal{NH}_t \equiv \sum_{l=1}^{K} (\overline{\partial_e e}_l - \bar{s}_l)(\hat{P}_{l,t} + \tilde{A}_{l,t}),$$

$$\bar{\mathcal{P}}_{k,t} \equiv (\hat{P}_{k,t} - \sum_{l=1}^{K} \bar{s}_l \hat{P}_{l,t}) + (\tilde{A}_{k,t} - \sum_{l=1}^{K} \bar{s}_l \tilde{A}_{l,t}),$$

$$\mathcal{P}_{k,t} \equiv \Omega_{N,k} \bar{\mathcal{P}}_{k,t} - \sum_{l=1}^{K} \Omega_{k,l} \left(\bar{\mathcal{P}}_{l,t} - \bar{\mathcal{P}}_{k,t} \right),$$

$$\mathcal{F}_t \equiv \sum_{l=1}^{K} \overline{\partial_e e}_l \left(\hat{Q}_{l,t} - \hat{P}_{l,t} \right).$$

Setting $\Omega_{N,k} = 1$ and $\mathcal{P}_{k,t} = \bar{\mathcal{P}}_{k,t}$ recovers the equations of the main text absent the Input-Output structure:

$$\pi_{k,t} = \lambda_k \left(\left(\frac{1}{\sigma} + \frac{1}{\psi} \right) \tilde{\boldsymbol{y}}_t + \mathcal{F}_t + \mathcal{NH}_t - \mathcal{P}_{k,t} \right) + \tilde{\beta} \mathbb{E}_t \pi_{k,t+1}.$$

A.4 Government

A household's debt obligations are cancelled upon death, so government assets evolve as:

$$\mathbb{E}_t B_{t+1} = R_t \left((1-\delta) B_t + T_t \right),$$

with $T_t = -\int \zeta_t(i)\, di - \sum_{k=1}^{K} \left(\tau_{k,t} P_{k,t} C_{k,t} + \iota_k \int TC_{k,t}(j)\, dj \right)$. Linearizing around a steady state with zero government assets and zero government surplus, and letting B_t, T_t, ζ_t denote the linearized versions of the corresponding variables for notational simplicity, gives:

$$T_t = (1-\delta) \left(\beta \mathbb{E}_t B_{t+1} - B_t \right)$$

$$T_t = -\int \zeta_t(i)\, di - \sum_{k=1}^{K} \left(\tau_{k,t} P_k C_k + \iota_k \int dTC_{k,t}(j)\, dj \right).$$

The consumption subsidy, firm cost subsidy, and government surplus are assumed to be funded by a tax proportional to steady-state labor income, so that:

$$\zeta_t(i) = \tilde{\zeta}_t(i) + Wn(i) \sum \bar{s}_k \left(\hat{Q}_{k,t} - \hat{P}_{k,t} \right) - \frac{Wn(i)}{WN} \sum_{k=1}^{K} \iota_k \int dTC_{k,t}(j)\, dj.$$

Here $dTC_{k,t}(j)$ is the change in the firm's total cost. Integrating the transfer decomposition and using $E = WN$ yields

$$\int \tilde{\zeta}_t(i)\, di = -T_t.$$

A.5 Evolution of the Output Gap

We turn now to the evolution of the output gap. Equation (20) gives:

$$\begin{aligned} \tilde{\boldsymbol{y}}_t = & \frac{\sigma}{\sigma+\psi} \mathcal{E}_t - \hat{\boldsymbol{y}}_t^* \\ & + \frac{\sigma\psi}{\sigma+\psi} \sum_{k=1}^{K} \int \frac{Wn(i) - e(i)}{E} \partial_e e_k(i) di \hat{Q}_{k,t} \end{aligned}$$

Let $\mathcal{E}_t = \int \frac{e(i)}{E} \left(1 + \frac{\psi W n(i)}{e(i)\sigma} \right) \mathbb{E}_{\delta,t} \left(\hat{e}_t^{t_0}(i) - \sum_{k=1}^{K} s_k(i) \hat{Q}_{k,t} \right) di$ denote the consumption index associated with the output gap, and $\mathcal{E}_t^u = \int (1 - \varphi(i)) \frac{e(i)}{E} \left(1 + \frac{\psi W n(i)}{e(i)\sigma} \right) \mathbb{E}_{\delta,t} \left(\hat{e}_t^{t_0,u}(i) - \sum_{k=1}^{K} s_k(i) \hat{Q}_{k,t} \right) di$, with $\mathcal{E}_t^{HtM} = \mathcal{E}_t - \mathcal{E}_t^u$. Aggregating the unconstrained households' Euler equations gives:

$$\mathbb{E}_t \mathcal{E}_{t+1}^u = (1-\delta) \mathcal{E}_t^u + (1-\delta) \mathbb{E}_t \int (1 - \varphi(i)) \left(\sigma \frac{e(i)}{E} + \psi \frac{Wn(i)}{WN} \right) \left(\hat{R}_t - \sum_{k=1}^{K} \partial_e e_k(i) \pi_{k,t+1}^Q \right) di + \delta \mathbb{E}_t \tilde{\mathcal{E}}_{t+1}^{u,0}$$

with $\tilde{\mathcal{E}}_{t+1}^{u,0}$ the consumption index of households born at $t+1$. Applying the lifetime budget constraint of households born at t, their labor supply decisions, and the aggregate net-dividend identity

$$dDiv_t - \sum_k \iota_k \int dTC_{k,t}(j)\, dj = \sum_k E_k (\hat{P}_{k,t} + \tilde{A}_{k,t} - \hat{W}_t)$$

yields:

$$\begin{aligned} \hat{e}_t^t(i) - \sum_k s_k(i) \hat{Q}_{k,t} = & \frac{1 - \frac{1}{R}}{e(i) + \frac{\psi}{\sigma} Wn(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \frac{1}{R^s} \left(\frac{b(i)}{R} \left(\hat{R}_{t+s} - \pi_{cpi,t+s+1}^Q \right) + \left(1 + \frac{\psi}{\sigma} \right) Wn(i) \hat{\boldsymbol{y}}_{t+s} \right. \\ & \left. - \sum_{k=1}^{K} \left(e(i) (s_k(i) - \bar{s}_k) + \psi Wn(i) \left(\partial_e e_k(i) - \overline{\partial_e e_k} \right) \right) \hat{Q}_{k,t+s} + \tilde{\zeta}_{t+s}(i) \right) \\ & - \sigma \mathbb{E}_t \sum_{s=0}^{\infty} \frac{1}{R^{s+1}} \left(\hat{R}_{t+s} - \sum_k \partial_e e_k(i) \pi_{k,t+s+1}^Q \right). \end{aligned}$$

Averaging across households gives:

$$\tilde{\mathcal{E}}_{t-1}^{u,0}-\frac{1}{R}\tilde{\mathcal{E}}_{t}^{u,0}=-\frac{1}{R}\mathbb{E}_t\int\left(1-\varphi\left(i\right)\right)\left(\sigma\frac{e(i)}{E}+\psi\frac{Wn(i)}{WN}\right)\left(\hat{R}_{t-1}-\sum_{k=1}^{K}\partial_e e_k(i)\pi_{k,t}^{Q}\right)di$$
$$+\int\left(1-\varphi\left(i\right)\right)\left(1-\frac{1}{R}\right)\left(\frac{b(i)}{ER}\left(\hat{R}_{t-1}-\pi_{cpi,t}^{Q}\right)+\left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\hat{\boldsymbol{y}}_{t-1}\right.$$
$$\left.-\sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i)-\overline{\partial_e e}_k\right)\right)\hat{Q}_{k,t-1}+\frac{1}{E}\tilde{\zeta}_{t-1}\left(i\right)\right)di.$$

Letting $\mathcal{E}_t^{u,0}\equiv\tilde{\mathcal{E}}_t^{u,0}-\mathcal{E}_t^{u}$ gives:

$$\mathbb{E}_t\mathcal{E}_{t+1}^{u}=\mathcal{E}_t^{u}+\mathbb{E}_t\int\left(1-\varphi\left(i\right)\right)\left(\sigma\frac{e(i)}{E}+\psi\frac{Wn(i)}{WN}\right)\left(\hat{R}_{t}-\sum_{k=1}^{K}\partial_e e_k(i)\pi_{k,t+1}^{Q}\right)di$$
$$+\frac{\delta}{1-\delta}\mathbb{E}_t\mathcal{E}_{t+1}^{u,0},$$

$$\mathcal{E}_{t-1}^{u,0}-\frac{1}{R\left(1-\delta\right)}\mathcal{E}_{t}^{u,0}+\left(1-\frac{1}{R}\right)\mathcal{E}_{t-1}^{u}=\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\left(\frac{b(i)}{ER}\left(\hat{R}_{t-1}-\pi_{cpi,t}^{Q}\right)\right.$$
$$+\left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\hat{\boldsymbol{y}}_{t-1}-\sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i)-\overline{\partial_e e}_k\right)\right)\hat{Q}_{k,t-1}$$
$$\left.+\frac{1}{E}\tilde{\zeta}_{t-1}\left(i\right)\right)di.$$

Turning to the HtM contribution:

$$\mathbb{E}_t\mathcal{E}_{t+1}^{HtM}=\left(1-\delta\right)\mathcal{E}_t^{HtM}+\delta\mathbb{E}_t\tilde{\mathcal{E}}_{t+1}^{HtM,0}$$
$$+\left(1-\delta\right)\mathbb{E}_t\int\varphi\left(i\right)\left\{\Delta\hat{R}_{t+1}\frac{b(i)}{ER}+\left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\Delta\hat{\boldsymbol{y}}_{t+1}\right.$$
$$\left.-\sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i)-\overline{\partial_e e}_k\right)\right)\pi_{k,t+1}^{Q}+\frac{1}{E}\Delta\tilde{\zeta}_{t+1}\left(i\right)\right\}di$$
$$-\left(1-\delta\right)\mathbb{E}_t\int\varphi\left(i\right)\left\{\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\pi_{cpi,t+1}^{Q}\right\}di,$$

$$\tilde{\mathcal{E}}_{t-1}^{HtM,0}-\frac{1}{R}\tilde{\mathcal{E}}_{t}^{HtM,0}=-\frac{1}{R}\mathbb{E}_t\int\varphi\left(i\right)\left\{\Delta\hat{R}_{t}\frac{b(i)}{ER}+\left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\Delta\hat{\boldsymbol{y}}_{t}\right.$$
$$\left.-\sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i)-\overline{\partial_e e}_k\right)\right)\pi_{k,t}^{Q}+\frac{1}{E}\Delta\tilde{\zeta}_{t}\left(i\right)\right\}di$$
$$-\frac{1}{R}\mathbb{E}_t\int\varphi\left(i\right)\left\{-\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\pi_{cpi,t}^{Q}\right\}di$$
$$+\left(1-\frac{1}{R}\right)\mathbb{E}_t\int\varphi\left(i\right)\left\{\frac{b(i)}{ER}\left(\hat{R}_{t-1}-\pi_{cpi,t}^{Q}\right)+\left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\hat{\boldsymbol{y}}_{t-1}\right.$$
$$\left.-\sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i)-\overline{\partial_e e}_k\right)\right)\hat{Q}_{k,t-1}+\frac{1}{E}\tilde{\zeta}_{t-1}\left(i\right)\right\}di$$

Letting $\mathcal{E}_t^{HtM,0} \equiv \tilde{\mathcal{E}}_t^{HtM,0} - \mathcal{E}_t^{HtM}$ gives:

$$\begin{aligned}
\mathbb{E}_t \mathcal{E}_{t+1}^{HtM} &= \mathcal{E}_t^{HtM} + \frac{\delta}{1-\delta}\mathbb{E}_t \mathcal{E}_{t+1}^{HtM,0} \\
&\quad + \mathbb{E}_t \int \varphi(i) \left\{ \Delta \hat{R}_{t+1} \frac{b(i)}{ER} + \left(1 + \frac{\psi}{\sigma}\right) \frac{Wn(i)}{WN} \Delta \hat{\mathcal{Y}}_{t+1} \right. \\
&\quad - \sum_{k=1}^{K} \left(\frac{e(i)}{E} (s_k(i) - \bar{s}_k) + \psi \frac{Wn(i)}{WN} \left(\partial_e e_k(i) - \overline{\partial_e e_k} \right) \right) \pi_{k,t+1}^{Q} \\
&\quad \left. + \frac{1}{E} \Delta \tilde{\zeta}_{t+1}(i) \right\} di \\
&\quad - \mathbb{E}_t \int \varphi(i) \left\{ \left(1 - \frac{1}{R}\right) \frac{b(i)}{E} \pi_{cpi,t+1}^{Q} \right\} di,
\end{aligned}$$

$$\begin{aligned}
\mathcal{E}_{t-1}^{HtM,0} - \frac{1}{R(1-\delta)} \mathcal{E}_t^{HtM,0} + \left(1 - \frac{1}{R}\right) \mathcal{E}_{t-1}^{HtM} &= \left(1 - \frac{1}{R}\right) \int \varphi(i) \left\{ \frac{b(i)}{ER} \left(\hat{R}_{t-1} - \pi_{cpi,t}^{Q} \right) \right. \\
&\quad + \left(1 + \frac{\psi}{\sigma}\right) \frac{Wn(i)}{WN} \hat{\mathcal{Y}}_{t-1} \\
&\quad - \sum_{k=1}^{K} \left(\frac{e(i)}{E} (s_k(i) - \bar{s}_k) + \psi \frac{Wn(i)}{WN} \left(\partial_e e_k(i) - \overline{\partial_e e_k} \right) \right) \hat{Q}_{k,t-1} \\
&\quad \left. + \frac{1}{E} \tilde{\zeta}_{t-1}(i) \right\} di
\end{aligned}$$

Combining these results, and using the existing combined-gap notation $\tilde{\mathcal{E}}_t^0 \equiv \mathcal{E}_t^{u,0} + \mathcal{E}_t^{HtM,0}$, gives:

$$\begin{aligned}
\mathbb{E}_t \mathcal{E}_{t+1} &= \mathcal{E}_t + \mathbb{E}_t \int \left(1 - \varphi(i)\right) \left(\sigma \frac{e(i)}{E} + \psi \frac{Wn(i)}{WN} \right) \left(\hat{R}_t - \sum_{k=1}^{K} \partial_e e_k(i) \pi_{k,t+1}^{Q} \right) di \\
&\quad + \frac{\delta}{1-\delta} \mathbb{E}_t \tilde{\mathcal{E}}_{t+1}^0 + \mathbb{E}_t \int \varphi(i) \left\{ \Delta \hat{R}_{t+1} \frac{b(i)}{ER} + \left(1 + \frac{\psi}{\sigma}\right) \frac{Wn(i)}{WN} \Delta \hat{\mathcal{Y}}_{t+1} \right. \\
&\quad \left. - \sum_{k=1}^{K} \left(\frac{e(i)}{E} (s_k(i) - \bar{s}_k) + \psi \frac{Wn(i)}{WN} \left(\partial_e e_k(i) - \overline{\partial_e e_k} \right) \right) \pi_{k,t+1}^{Q} + \frac{1}{E} \Delta \tilde{\zeta}_{t+1}(i) \right\} di \\
&\quad - \mathbb{E}_t \int \varphi(i) \left\{ \left(1 - \frac{1}{R}\right) \frac{b(i)}{E} \pi_{cpi,t+1}^{Q} \right\} di,
\end{aligned}$$

$$\begin{aligned}
&\tilde{\mathcal{E}}_{t-1}^0 - \frac{1}{R(1-\delta)} \tilde{\mathcal{E}}_t^0 + \left(1 - \frac{1}{R}\right) \mathcal{E}_{t-1} \\
&\qquad = \left(1 - \frac{1}{R}\right) \left(\left(1 + \frac{\psi}{\sigma}\right) \hat{\mathcal{Y}}_{t-1} - \frac{T_{t-1}}{E} - \sum_{k=1}^{K} \psi \int \frac{Wn(i)}{WN} \left(\partial_e e_k(i) - \overline{\partial_e e_k} \right) di \hat{Q}_{k,t-1} \right)
\end{aligned}$$

The definitions of $\mathcal{E}_t$ and T_t then imply:

$$\tilde{\mathcal{E}}_{t-1}^0 - \beta \tilde{\mathcal{E}}_t^0 = \left(1 - \frac{1}{R}\right)(1-\delta) \left(\frac{B_{t-1}}{E} - \beta \frac{B_t}{E} \right).$$

Excluding an explosive homogeneous solution, it follows that $\tilde{\mathcal{E}}_t^0 = (1-1/R)(1-\delta)B_t/E$, and hence:

$$\begin{aligned}\mathbb{E}_t\mathcal{E}_{t+1} = \mathcal{E}_t + \mathbb{E}_t\int \left(1-\varphi\left(i\right)\right)\left(\sigma\frac{e(i)}{E}+\psi\frac{Wn(i)}{WN}\right)\left(\hat{R}_t - \sum_{k=1}^{K}\partial_e e_k(i)\pi^Q_{k,t+1}\right)di \\ + \left(1-\frac{1}{R}\right)\delta\mathbb{E}_t\frac{B_{t+1}}{E} + \mathbb{E}_t\int\varphi\left(i\right)\left\{\Delta\hat{R}_{t+1}\frac{b(i)}{ER} + \left(1+\frac{\psi}{\sigma}\right)\frac{Wn\left(i\right)}{WN}\Delta\hat{\mathcal{Y}}_{t+1}\right. \\ \left. - \sum_{k=1}^{K}\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)+\psi\frac{Wn\left(i\right)}{WN}\left(\partial_e e_k(i) - \overline{\partial_e e}_k\right)\right)\pi^Q_{k,t+1} + \frac{1}{E}\Delta\tilde{\zeta}_{t+1}\left(i\right)\right\}di \\ - \mathbb{E}_t\int\varphi\left(i\right)\left\{\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\pi^Q_{cpi,t+1}\right\}di.\end{aligned}$$

Applying the definition of $\mathcal{E}_t$ once more yields the Euler equation for the output gap:

$$\begin{aligned}\left(1-\varphi^N\right)\left(\mathbb{E}_t\tilde{\mathcal{Y}}_{t+1} - \tilde{\mathcal{Y}}_t\right) = \left(1-\varphi^N\right)\sigma\mathbb{E}_t\left(\hat{R}_t - \sum_{k=1}^{K}\overline{\partial_e e}_k\pi^Q_{k,t+1} - r_t^*\right) + \frac{\sigma}{\sigma+\psi}\left(1-\frac{1}{R}\right)\delta\mathbb{E}_t\frac{B_{t+1}}{E} \\ + \mathbb{E}_t\int\frac{\sigma\varphi\left(i\right)}{\sigma+\psi}\left\{\frac{b(i)}{RE}\left(\Delta\hat{R}_{t+1} - \sigma\left(R-1\right)\hat{R}_t\right)\right. \\ \left. - \frac{e(i)}{E}\sum_{k=1}^{K}\left(\left(s_k(i)-\bar{s}_k\right) - \sigma(\partial_e e_k(i) - \overline{\partial_e e}_k)\right)\pi^Q_{k,t+1} + \frac{1}{E}\Delta\tilde{\zeta}_{t+1}\left(i\right)\right\}di \\ - \mathbb{E}_t\int\frac{\sigma\varphi\left(i\right)}{\sigma+\psi}\left\{\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\left(\pi^Q_{cpi,t+1} - \sigma\pi^Q_{mcpi,t+1}\right)\right\}di,\end{aligned}$$

Here the existing natural-rate and marginal-consumption-price inflation notation is

$$r_t^* = \frac{1}{\sigma}\mathbb{E}_t(\hat{\mathcal{Y}}^*_{t+1} - \hat{\mathcal{Y}}^*_t), \qquad \pi^Q_{mcpi,t+1} = \sum_k\overline{\partial_e e}_k\pi^Q_{k,t+1}, \qquad \pi^Q_{cpi,t+1} = \sum_k\bar{s}_k\pi^Q_{k,t+1}.$$

B Loss function derivations

This appendix derives the second-order approximation to the social welfare function of the main text:

$$\mathcal{W} = (1-\delta)\int G\left(V_-\left(i\right),i\right)di + \delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int G\left(V_{t_0}\left(i\right),i\right)di.$$

Household i born at t_0 has value function:

$$\begin{aligned}V_{t_0}\left(i\right) =& \mathbb{E}_{t_0}\sum_{s=0}^{\infty}\left(\left(1-\delta\right)\beta\right)^s\left\{\left(1-\varphi\left(i\right)\right)\left[U_i\left(c^u_{1,t_0+s}\left(i\right),\ldots,c^u_{K,t_0+s}\left(i\right)\right) - \chi\left(\frac{n^u_{t_0+s}(i)}{\vartheta(i)}\right)\right]\right. \\ & \left. + \varphi\left(i\right)\left[U_i\left(c^{HtM}_{1,t_0+s}\left(i\right),\ldots,c^{HtM}_{K,t_0+s}\left(i\right)\right) - \chi\left(\frac{n^{HtM}_{t_0+s}(i)}{\vartheta(i)}\right)\right]\right\}.\end{aligned}$$

Consumption and labor supply, $\left\{\boldsymbol{c}^u_{k,t_0+s}\left(i\right),\boldsymbol{c}^{HtM}_{1,t_0+s}\left(i\right),n^u_{t_0+s}(i),n^{Htm}_{t_0+s}(i)\right\}_{s\geq 0}$, are chosen optimally (see the derivation appendix). The envelope condition gives the derivative of the value function:

$$\begin{aligned}dV_{t_0}\left(i\right) = \mathbb{E}_{t_0}\sum_{s=0}^{\infty}\left(\left(1-\delta\right)\beta\right)^s\partial_e v_{t_0+s}\left\{\frac{b_{t_0+s+1}\left(i\right)}{R_{t_0+s}}\hat{R}_{t_0+s} + \hat{W}_{t_0+s}W_{t_0+s}n_{t_0+s}\left(i\right)\right. \\ \left. - \sum_{l=1}^{K}c^{t_0}_{l,t_0+s}(i)\left(\left(1-\tau_{l,t_0+s}\right)\int\left(\frac{p_{l,t_0+s}\left(j\right)}{P_{l,t_0+s}}\right)^{-\epsilon_l}dp_{l,t_0+s}\left(j\right)dj - P_{l,t_0+s}d\tau_{l,t_0+s}\right) + \varsigma\left(i\right)dDiv_{t_0+s} + \zeta_{t_0+s}\left(i\right)\right\} \\ + \left(1-\varphi\left(i\right)\right)\partial_e v_{t_0}b_{t_0}\left(i\right)\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0} \\ + \varphi\left(i\right)\mathbb{E}_{t_0}\sum_{s=0}^{\infty}\left(\left(1-\delta\right)\beta\right)^s\partial_e v_{t_0+s}\left(1-\frac{1}{R_{t_0+s}}\right)b_{t_0}\left(i\right)\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0},\end{aligned}$$

Net of the government subsidy, the change in dividends, $\widetilde{dDiv_t} \equiv dDiv_t - \sum_k \iota_k \int dTC_k\left(j\right) dj$, is given by:

$$\begin{aligned}\widetilde{dDiv_t} = \sum_{k=1}^{K} \int y_k\left(j\right) \Bigg(dp_{k,t}\left(j\right) - \frac{1}{A_{k,t}} \Bigg(\mathcal{N}_k(\boldsymbol{P}_t, W_t) dW_t + \sum_{l=1}^{K} \mathcal{Y}_{l,k}(\boldsymbol{P}_t, W_t) dP_{l,t} \\ - \left(\mathcal{N}_k(\boldsymbol{P}_t, W_t) W_t + \sum_{l=1}^{K} \mathcal{Y}_{l,k}(\boldsymbol{P}_t, W_t) P_{l,t} \right) \hat{A}_{k,t} \Bigg) \Bigg) dj \\ + \sum_{k=1}^{K} \int dy_k\left(j\right) \left(p_{k,t}\left(j\right) - \frac{1}{A_{k,t}} \left(\mathcal{N}_k(\boldsymbol{P}_t, W_t) W_t + \sum_{l=1}^{K} \mathcal{Y}_{l,k}(\boldsymbol{P}_t, W_t) P_{l,t} \right) \right) dj,\end{aligned}$$

where the change in demand for variety j and total demand for intermediary $d\tilde{Y}_{k,t}$ are given by:

$$\begin{aligned}dy_{k,t}\left(j\right) = \left(\frac{p_{k,t}\left(j\right)}{P_{k,t}}\right)^{-\epsilon_k} \mathbb{E}_{\delta,t} \Bigg(\int -\epsilon_k \left(\frac{dp_{k,t}\left(j\right)}{p_{k,t}\left(j\right)} - \int \left(\frac{p_{k,t}\left(j^*\right)}{P_{k,t}}\right)^{1-\epsilon_k} \frac{dp_{k,t}\left(j^*\right)}{p_{k,t}\left(j^*\right)} dj^* \right) \\ \times \left(\left(1-\varphi\left(i\right)\right) c_{k,t}^{t_0,u}\left(i\right) + \varphi\left(i\right) c_{k,t}^{t_0,HtM} \right) + \left(\left(1-\varphi\left(i\right)\right) dc_{k,t}^{t_0,u}\left(i\right) + \varphi\left(i\right) dc_{k,t}^{t_0,HtM} \right) di \Bigg) \\ + \left(\frac{p_{k,t}\left(j\right)}{P_{k,t}}\right)^{-\epsilon_k} \left(-\epsilon_k \left(\frac{dp_{k,t}\left(j\right)}{p_{k,t}\left(j\right)} - \int \left(\frac{p_{k,t}\left(j^*\right)}{P_{k,t}}\right)^{1-\epsilon_k} \frac{dp_{k,t}\left(j^*\right)}{p_{k,t}\left(j^*\right)} dj^* \right) \tilde{Y}_{k,t} + d\tilde{Y}_{k,t} \right),\end{aligned}$$

$$d\tilde{Y}_{k,t} = \sum_l \frac{Y_{l,t}}{A_{l,t}} \left(\partial_W \mathcal{Y}_{k,l}(\boldsymbol{P}_t, W_t) dW_t + \sum_{m=1}^{K} \partial_{P_m} \mathcal{Y}_{k,l}(\boldsymbol{P}_t, W_t) dP_{m,t} \right) + \sum_l \frac{\mathcal{Y}_{k,l}(\boldsymbol{P}_t, W_t)}{A_{l,t}} dY_{l,t} - \sum_l \frac{Y_{l,t}}{A_{l,t}} \mathcal{Y}_{k,l}(\boldsymbol{P}_t, W_t) \hat{A}_{l,t},$$

$$dY_{k,t} = \int dy_{k,t}\left(j\right) dj.$$

In a steady state where prices, consumption, wealth, and labor supply are constant (and equal across generations and across constrained and unconstrained households of the same type i), and with $p_k\left(j\right) = \frac{1}{A_k}\left(\mathcal{N}_k(\boldsymbol{P}, W) W + \sum_{l=1}^{K} \mathcal{Y}_{l,k}(\boldsymbol{P}, W) P_l\right)$, this reduces to

$$dV_{t_0}\left(i\right) = \partial_e v\left(i\right) \mathbb{E}_{t_0} \sum_{s=0}^{\infty} \left(\left(1-\delta\right)\beta\right)^s \left\{ E\mathcal{I}_{t_0+s}\left(i\right) + \left(1 - \frac{1}{R}\right) b\left(i\right) \sum_{l=1}^{K} \bar{s}_l \hat{Q}_{l,t_0} + Wn\left(i\right) \sum_{l=1}^{K} \bar{s}_l \tilde{A}_{l,t_0+s} \right\},$$

with the expenditure-normalized income term:

$$\mathcal{I}_{t_0+s}\left(i\right) = \frac{b\left(i\right)}{RE} \hat{R}_{t_0+s} - \sum_k \left(\frac{e(i)}{E} \left(s_k(i) - \bar{s}_k\right) \right) \hat{Q}_{k,t_0+s} - \sum_k \left(1 - \frac{1}{R}\right) \frac{b\left(i\right)}{E} \bar{s}_k \hat{Q}_{k,t_0+s} + \frac{\tilde{\zeta}_{t_0+s}\left(i\right)}{E}$$

$$\zeta_t\left(i\right) = \tilde{\zeta}_t\left(i\right) + Wn\left(i\right) \sum \bar{s}_k \left(\hat{Q}_{k,t} - \hat{P}_{k,t} \right) - \frac{Wn\left(i\right)}{WN} \sum_{k=1}^{K} \iota_k \int dTC_{k,t}\left(j\right) dj.$$

We now turn to the second-order approximation of the social welfare function itself, again using that in steady state prices, consumption, wealth, and labor supply are constant (and equal across generations and across constrained and unconstrained households of the same type i), that $G'\left(V_{t_0}\left(i\right), i\right) \partial_e v_{t_0} = 1$, and that $p_k\left(j\right) = \frac{1}{A_k}\left(\mathcal{N}_k(\boldsymbol{P}, W) W + \sum_{l=1}^{K} \mathcal{Y}_{l,k}(\boldsymbol{P}, W) P_l\right)$.

$$
\begin{aligned}
d^2\mathcal{W} &= (1-\delta)\int G''(i)\,(dV_-(i))^2\,di + \delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int G''(i)\left(dV_{t_0}(i)\right)^2 di \\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mathbb{E}_{\delta,t}\int\left(1-\varphi(i)\right)\left(\frac{1}{\sigma}\left(\hat{e}_t^{t_0,u}-\sum_k s_k(i)\,\hat{Q}_{k,t}\right)+\sum_k \partial_e e_k(i)\,\hat{Q}_{k,t}\right) \\
&\times\left\{E\mathcal{I}_t(i)+Wn(i)\sum_{l=1}^{K}\bar{s}_l\tilde{A}_{l,t}+\mathbb{1}_{t=t_0}b(i)\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right\}di \\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mathbb{E}_{\delta,t}\int\varphi(i)\left(\frac{1}{\sigma}\left(\hat{e}_t^{t_0,HtM}-\sum_k s_k(i)\,\hat{Q}_{k,t}\right)+\sum_k \partial_e e_k(i)\,\hat{Q}_{k,t}\right) \\
&\times\left\{E\mathcal{I}_t(i)+Wn(i)\sum_{l=1}^{K}\bar{s}_l\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)b(i)\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right\}di \\
&+ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{dW_t dN_t-\sum_k dC_{k,t}dP_{k,t}-\sum_{l=1}^{K}C_{l,t}\int d\left\{\left(\frac{p_{l,t}(j)}{P_{l,t}}\right)^{-\epsilon_l}\right\}dp_{l,t}(j)\,dj+d^2\widetilde{Div_t}+\frac{\hat{R}_t}{R}B_{t+1}-\sum_k\tau_{k,t}P_{k,t}dC_{k,t}\right\} \\
&+ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mathbb{E}_{\delta,t}\left(\int\varphi(i)\frac{\hat{R}_t}{R}b(i)di\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right),
\end{aligned}
$$

with

$$
\int d\left\{\left(\frac{p_{l,t}(j)}{P_{l,t}}\right)^{-\epsilon_l}\right\}dp_{l,t}(j)\,dj=-\epsilon_l\int\left(\frac{p_{l,t}(j)}{P_{l,t}}\right)^{-\epsilon_l}dp_{l,t}(j)\left(\hat{p}_{l,t}(j)-\hat{P}_{l,t}\right)dj,
$$

and, for dividends net of the production subsidy,

$$
\begin{aligned}
d^2\widetilde{Div_t} &= -dW_t dN_t^d-\sum_{k=1}^{K}d\tilde{Y}_{k,t}^d dP_{k,t}-\sum_{k=1}^{K}\tilde{Y}_{k,t}^d d^2P_{k,t}+\sum_{k=1}^{K}\int dy_k(j)\left(dp_{k,t}(j)+P_k\hat{A}_{k,t}\right)dj \\
&+\sum_{k=1}^{K}\frac{Y_k}{A_{k,t}}\left(\mathcal{N}_k(\boldsymbol{P}_t,W_t)dW_t+\sum_{l=1}^{K}\mathcal{Y}_{l,k}(\boldsymbol{P}_t,W_t)dP_{l,t}-\left(\mathcal{N}_k(\boldsymbol{P}_t,W_t)W_t+\sum_{l=1}^{K}\mathcal{Y}_{l,k}(\boldsymbol{P}_t,W_t)P_{l,t}\right)\hat{A}_{k,t}\right)\hat{A}_{k,t} \\
&+\sum_{k=1}^{K}\int dy_k(j)\left(dp_{k,t}(j)-\frac{1}{A_{k,t}}\left(\mathcal{N}_k(\boldsymbol{P}_t,W_t)dW_t+\sum_{l=1}^{K}\mathcal{Y}_{l,k}(\boldsymbol{P}_t,W_t)dP_{l,t}\right)+p_{k,t}(j)\,\hat{A}_{k,t}\right)dj.
\end{aligned}
$$

Simplifying with the market-clearing conditions for labor, bonds and goods and steady-state properties, and dropping terms independent of monetary policy, we get:

$$
\begin{aligned}
dW_t dN_t-\sum_k dC_{k,t}dP_{k,t}-\sum_{l=1}^{K}C_{l,t}\int d\left\{\left(\frac{p_{l,t}(j)}{P_{l,t}}\right)^{-\epsilon_l}\right\}dp_{l,t}(j)\,dj& \\
+d^2\widetilde{Div_t}-\sum_k\tau_{k,t}P_{k,t}dC_{k,t}=& \\
-\sum_k P_k dC_{k,t}\left(\hat{W}_t-\hat{Q}_{k,t}-2\mathbf{A}_{k,t}\right)& \\
-\left(\sum_{k,l}\frac{1}{A_l}P_kY_lW\partial_W\mathcal{Y}_{k,l}\hat{W}_t\left(\hat{W}_t-\hat{P}_{k,t}-2\tilde{A}_{k,t}\right)\right.& \\
\left.+\sum_{k,l,m}\frac{1}{A_l}P_kP_mY_l\hat{P}_{m,t}\partial_{P_m}\mathcal{Y}_{k,l}\left(\hat{W}_t-\hat{P}_{k,t}-2\mathbf{A}_{k,t}\right)\right)& \\
+\hat{W}_t\sum_k E_k\mathbf{A}_{k,t}-\sum_{k=1}^{K}\epsilon_kP_kY_k\int\left(\hat{p}_{k,t}(j)-\hat{P}_{k,t}\right)\hat{p}_{k,t}(j)\,dj.&
\end{aligned}
$$

Letting $\hat{p}^*_{k,t}$ denote the reset price at t, we have

$$\begin{aligned}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \left(\hat{p}_{k,t}(j) - \hat{P}_{k,t}\right) \hat{p}_{k,t}(j)\, dj &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1-\theta_k}{1-\beta\theta_k} \left(\hat{p}^*_{k,t}\right)^2 - \left(\hat{P}_{k,t}\right)^2 \right) \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{(1-\beta\theta_k)(1-\theta_k)} (\pi_{k,t})^2 + 2\frac{1}{(1-\beta\theta_k)} \pi_{k,t} \hat{P}_{k,t-1} \right. \\
&\quad \left. + \frac{(1-\theta_k)}{1-\beta\theta_k} \left(\hat{P}_{k,t-1}\right)^2 - \left(\hat{P}_{k,t}\right)^2 \right\} \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\theta_k}{(1-\beta\theta_k)(1-\theta_k)} (\pi_{k,t})^2 - \frac{\theta_k}{1-\beta\theta_k} \left(\hat{P}_{k,t-1}\right)^2 + \frac{\beta\theta_k}{(1-\beta\theta_k)} \left(\hat{P}_{k,t}\right)^2 \right\} \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\theta_k}{(1-\beta\theta_k)(1-\theta_k)} (\pi_{k,t})^2 .
\end{aligned}$$

Since unit demand for inputs is homogeneous of degree 0 in prices and its derivatives satisfy cost-function symmetry, the expression inside the preceding negative bracket can be rewritten as follows, up to terms involving only productivity shocks:

$$\begin{aligned}
&\left(\sum_{k,l} \frac{1}{A_l} P_k Y_l W \partial_W \mathcal{Y}_{k,l} \hat{W}_t \left(\hat{W}_t - \hat{P}_{k,t} - 2\mathbf{A}_{k,t} \right) + \sum_{k,l,m} \frac{1}{A_l} P_k P_m Y_l \hat{P}_{m,t} \partial_{P_m} \mathcal{Y}_{k,l} \left(\hat{W}_t - \hat{P}_{k,t} - 2\mathbf{A}_{k,t} \right) \right) = \\
&\qquad - \sum_l \frac{1}{A_l} Y_l W^2 \partial_W \mathcal{N}_l \hat{W}_t^{\,2} - \sum_{k,l} \frac{1}{A_l} P_k Y_l W \partial_W \mathcal{Y}_{k,l} \hat{W}_t \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \\
&\qquad\quad - \sum_{k,l} \frac{1}{A_l} W Y_l \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) P_k \partial_{P_k} \mathcal{N}_l \hat{W}_t - \sum_{k,l,m} \frac{1}{A_l} P_m P_k Y_l \left(\hat{P}_{m,t} + \mathbf{A}_{m,t} \right) \partial_{P_m} \mathcal{Y}_{k,l} \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right).
\end{aligned}$$

We define

$$\begin{aligned}
\mathcal{N}(W, \boldsymbol{P}) &\equiv \sum_l \frac{1}{A_l} Y_l \mathcal{N}_l (W, \boldsymbol{P}), \\
\mathcal{Y}_k (W, \boldsymbol{P}) &= \sum_l \frac{1}{A_l} Y_l \mathcal{Y}_{k,l} (W, \boldsymbol{P}).
\end{aligned}$$

This lets us rewrite

$$\begin{aligned}
&- \sum_l \frac{1}{A_l} Y_l W^2 \partial_W \mathcal{N}_l \hat{W}_t^{\,2} - \sum_{k,l} \frac{1}{A_l} P_k Y_l W \partial_W \mathcal{Y}_{k,l} \hat{W}_t \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \\
&\qquad - \sum_{k,l} \frac{1}{A_l} W Y_l \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) P_k \partial_{P_k} \mathcal{N}_l \hat{W}_t - \sum_{k,l,m} \frac{1}{A_l} P_m P_k Y_l \left(\hat{P}_{m,t} + \mathbf{A}_{m,t} \right) \partial_{P_m} \mathcal{Y}_{k,l} \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \\
&\qquad\quad = -W^2 \partial_W \mathcal{N} \hat{W}_t^{\,2} - \sum_k P_k W \partial_W \mathcal{Y}_k \hat{W}_t \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \\
&\qquad\qquad - \sum_k W P_k \partial_{P_k} \mathcal{N} \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \hat{W}_t - \sum_{k,l} P_l P_k \partial_{P_l} \mathcal{Y}_k \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \left(\hat{P}_{l,t} + \mathbf{A}_{l,t} \right).
\end{aligned}$$

In addition

$$\begin{aligned}
P_k dC_{k,t} &= \mathbb{E}_{\delta,t} \left(\int \left((1-\varphi(i)) P_k dc^{t_0,u}_{k,t}(i) + \varphi(i) P_k dc^{t_0,HtM}_{k,t}(i) \right) di \right), \\
P_k dc^{t_0}_{k,t}(i) &= e(i) \partial_e e_k(i) \left(\hat{e}^{t_0}_t(i) - \sum_l s_l(i) \hat{Q}_{l,t} \right) + e_k(i) \sum_l \rho_{k,l}(i) \hat{Q}_{l,t}.
\end{aligned}$$

Rearranging then gives:

$$\begin{aligned}
\sum_k P_k dC_{k,t} \left(\hat{W}_t - \hat{Q}_{k,t} - 2\mathbf{A}_{k,t} \right) &= \sum_k \mathbb{E}_{\delta,t} \int \frac{1}{\sigma} \left(\hat{e}^{t_0}_t(i) - \sum_l s_l(i) \hat{Q}_{l,t} \right) \sigma e(i) \partial_e e_k(i)\, di \left(\hat{W}_t - \hat{Q}_{k,t} - 2\tilde{A}_{k,t} \right) \\
&\quad - \sum_k E_k \sum_l \bar{\rho}_{k,l} \left(\hat{Q}_{l,t} + \tilde{A}_{l,t} \right) \left(\hat{Q}_{k,t} + \tilde{A}_{k,t} \right).
\end{aligned}$$

So we can rewrite the welfare loss as:

$$d^2 \mathcal{W} = (1-\delta) \int G''(i) (dV_-(i))^2\, di + \delta \mathbb{E}_0 \sum_{t_0=0}^{\infty} \beta^{t_0} \int G''(i) \left(dV_{t_0}(i) \right)^2 di$$

$$
\begin{aligned}
&- E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{\delta,t} \int \left(1-\varphi\left(i\right)\right) \left(\frac{1}{\sigma} \left(\hat{e}_t^{t_0,u} - \sum_k s_k\left(i\right) \hat{Q}_{k,t} \right) + \sum_k \partial_e e_k\left(i\right) \hat{Q}_{k,t} \right) \\
&\times \left\{ \mathcal{I}_t\left(i\right) + \sum_{l=1}^{K} \left(\frac{Wn\left(i\right)}{WN} \bar{s}_l - \sigma \frac{e\left(i\right)}{E} \partial_e e_l\left(i\right) \right) \tilde{A}_{l,t} + \mathbb{1}_{t=t_0} \frac{b\left(i\right)}{E} \sum_{l=1}^{K} \bar{s}_l \hat{Q}_{l,t_0} \right\} di \\
&- E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{\delta,t} \int \left(1-\varphi\left(i\right)\right) \left(\frac{1}{\sigma} \left(\hat{e}_t^{t_0,u} - \sum_k s_k\left(i\right) \hat{Q}_{k,t} \right) + \sum_k \partial_e e_k\left(i\right) \hat{Q}_{k,t} \right) \\
&\times \left\{ \sum_{k=1}^{K} \sigma \frac{e\left(i\right)}{E} \partial_e e_k\left(i\right) \left(\hat{W}_t - \hat{Q}_{k,t} - \tilde{A}_{k,t} \right) \right\} di \\
&- E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{\delta,t} \int \varphi\left(i\right) \left(\frac{1}{\sigma} \left(\hat{e}_t^{t_0,HtM} - \sum_k s_k\left(i\right) \hat{Q}_{k,t} \right) + \sum_k \partial_e e_k\left(i\right) \hat{Q}_{k,t} \right) \\
&\times \left\{ \mathcal{I}_t\left(i\right) + \sum_{l=1}^{K} \left(\frac{Wn\left(i\right)}{WN} \bar{s}_l - \sigma \frac{e\left(i\right)}{E} \partial_e e_l\left(i\right) \right) \tilde{A}_{l,t} + \left(1-\frac{1}{R}\right) \frac{b\left(i\right)}{E} \sum_{l=1}^{K} \bar{s}_l \hat{Q}_{l,t_0} \right\} di \\
&\quad - E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{\delta,t} \int \varphi\left(i\right) \left(\frac{1}{\sigma} \left(\hat{e}_t^{t_0,HtM} - \sum_k s_k\left(i\right) \hat{Q}_{k,t} \right) + \sum_k \partial_e e_k\left(i\right) \hat{Q}_{k,t} \right) \\
&\quad \times \left\{ \sum_{k=1}^{K} \sigma \frac{e\left(i\right)}{E} \partial_e e_k\left(i\right) \left(\hat{W}_t - \hat{Q}_{k,t} - \tilde{A}_{k,t} \right) \right\} di \\
&\qquad + E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{k,l}^{K} \int \sigma \frac{e\left(i\right)}{E} \partial_e e_k\left(i\right) \partial_e e_l\left(i\right) di \hat{Q}_{l,t} \left(\hat{W}_t - \hat{Q}_{k,t} - 2\tilde{A}_{k,t} \right) \\
&\qquad + E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_k \bar{s}_k \hat{W}_t \mathbf{A}_{k,t} \\
&- E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg(- \frac{W \partial_W \boldsymbol{\mathcal{N}}}{N} \hat{W}_t{}^2 - \sum_k \frac{P_k \partial_W \boldsymbol{\mathcal{Y}}_k}{N} \hat{W}_t \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \\
&\quad - \sum_k \frac{P_k \partial_{P_k} \boldsymbol{\mathcal{N}}}{N} \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \hat{W}_t - \sum_{k,l} \frac{P_l P_k \partial_{P_l} \boldsymbol{\mathcal{Y}}_k}{E} \left(\hat{P}_{k,t} + \mathbf{A}_{k,t} \right) \left(\hat{P}_{l,t} + \mathbf{A}_{l,t} \right) \Bigg) \qquad (21) \\
&\qquad + E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_k \bar{s}_k \sum_l \bar{\rho}_{k,l} \left(\hat{Q}_{l,t} + \tilde{A}_{l,t} \right) \left(\hat{Q}_{k,t} + \tilde{A}_{k,t} \right) \\
&\qquad - E \sum_k \frac{P_k Y_k}{E} \frac{\theta_k \epsilon_k}{\left(1-\beta\theta_k\right)\left(1-\theta_k\right)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_{k,t}\right)^2 \\
&\qquad\qquad + E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\hat{R}_t}{R} \frac{B_{t+1}}{E} \\
&\qquad + E\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{\delta,t} \left(\int \varphi\left(i\right) \frac{\hat{R}_t}{R} \frac{b\left(i\right)}{E} \sum_{l=1}^{K} \bar{s}_l \hat{Q}_{l,t_0} di \right).
\end{aligned}
$$

Simplification of the terms involving unconstrained households

In the following product identities we suppress the common factor E multiplying the allocation terms in welfare. It is restored in the final expression; the normalization $G'\partial_e v = 1$ and the $G''(dV)^2$ terms are unchanged.

We start by reexpressing the terms concerning unconstrained households (second and third line). To do so, we introduce a

variable R_t such that $\hat{R}_t = \mathsf{R}_{t+1} - \mathsf{R}_t$. Using the Euler equation for the output gap, we have:

$$\begin{aligned}\left(\left(1-\varphi^E\right)\sigma + \left(1-\varphi^N\right)\psi\right)\mathsf{R}_t = \left(1-\varphi^N\right)\psi\hat{W}_t + \sigma\sum_{k=1}^{K}\int \left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}\partial_e e_k(i)di\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\\ + \int \left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_t\left(i\right) + \sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l - \sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right)di\\ - \left(1-\frac{1}{R}\right)\delta\mathcal{B}_t + \frac{T_t}{E}, \qquad \frac{B_t}{E} = \mathcal{B}_t - \mathcal{B}_{t-1}.\end{aligned}$$

Since unconstrained households' expenditures grow at rate $\sigma\left(\hat{R}_t - \sum_k \partial_e e_k\left(i\right)\pi^Q_{k,t+1}\right)$, we have for all t:

$$\begin{aligned}\mathbb{E}_t\left(\frac{1}{\sigma}\left(\hat{e}^{t_0,u}_{t+1} - \sum_k s_k\left(i\right)\hat{Q}_{k,t+1}\right) + \sum_k \partial_e e_k\left(i\right)\hat{Q}_{k,t+1} - \mathsf{R}_{t+1}\right) = &\frac{1}{\sigma}\left(\hat{e}^{t_0,u}_{t} - \sum_k s_k\left(i\right)\hat{Q}_{k,t}\right)\\ &+ \sum_k \partial_e e_k\left(i\right)\hat{Q}_{k,t} - \mathsf{R}_t\end{aligned}$$

Using the lifetime budget constraint of household i, we have for all t:

$$\begin{aligned}\frac{1}{\sigma}\left(\hat{e}^{t_0,u}_{t} - \sum_k s_k\left(i\right)\hat{Q}_{k,t}\right) + \sum_k \partial_e e_k\left(i\right)\hat{Q}_{k,t} - \mathsf{R}_t = &-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathsf{R}_{t_0+s} - \sum_k \partial_e e_k\left(i\right)\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)\\ &+ \left(1-\frac{1}{R}\right)\frac{E}{\left(\sigma e\left(i\right)+Wn\left(i\right)\psi\right)}\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathcal{I}_{t_0+s}\left(i\right)\right.\\ &\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l - \sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s} + \mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)\\ &+ \left(1-\frac{1}{R}\right)\frac{\psi Wn\left(i\right)}{\left(\sigma e\left(i\right)+Wn\left(i\right)\psi\right)}\sum_{s=0}^{\infty}\frac{1}{R^s}\\ &\times\left(\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_{t_0+s} - \hat{Q}_{k,t_0+s} - \tilde{A}_{k,t_0+s}\right)\right).\end{aligned}$$

so we have:

$$\begin{aligned}&\sum_{s=0}^{\infty}R^{-s}\left(\frac{1}{\sigma}\left(\hat{e}^{t_0,u}_{t_0+s} - \sum_k s_k\left(i\right)\hat{Q}_{k,t_0+s}\right) + \sum_k \partial_e e_k\left(i\right)\hat{Q}_{k,t_0+s} - \mathsf{R}_{t_0+s}\right)\\ &\times\left\{\mathcal{I}_{t_0+s}\left(i\right) + \sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l - \sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s} + \mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0} + \sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_e e_k\left(i\right)\left(\hat{W}_{t_0+s} - \hat{Q}_{k,t_0+s} - \tilde{A}_{k,t_0+s}\right)\right\}\\ &= \left(1-\frac{1}{R}\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathcal{I}_{t_0+s}\left(i\right)\right.\right.\\ &\left.\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l - \sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s} + \mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)\right)^2\\ &+\left(1-\frac{1}{R}\right)\frac{\sigma\psi Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\frac{e\left(i\right)}{E}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_{t_0+s} - \hat{Q}_{k,t_0+s} - \tilde{A}_{k,t_0+s}\right)\right)\right)^2\\ &-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathsf{R}_{t_0+s} - \hat{W}_{t_0+s}\right)\\ &\times\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathcal{I}_{t_0+s}\left(i\right) + \sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l - \sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s} + \mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)\end{aligned}$$

$$-\left(1-\frac{1}{R}\right)\sigma\frac{e\left(i\right)}{E}\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\sum_{k}\partial_{e}e_{k}\left(i\right)\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)$$
$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\sum_{k=1}^{K}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right).$$

Averaging across unconstrained households born at t_0, we obtain:

$$\sum_{s=0}^{\infty}R^{-s}\int\left(1-\varphi\left(i\right)\right)\left(\frac{1}{\sigma}\left(\hat{e}_{t_0+s}^{t_0,u}-\sum_{k}s_{k}\left(i\right)\hat{Q}_{k,t_0+s}\right)+\sum_{k}\partial_{e}e_{k}\left(i\right)\hat{Q}_{k,t_0+s}-\mathsf{R}_{t_0+s}\right)$$
$$\times\left\{\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right\}di$$
$$=\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathcal{I}_{t_0+s}\left(i\right)\right.\right.$$
$$\left.\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}\right)\right)^{2}di$$
$$+\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{\sigma\psi Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\frac{e\left(i\right)}{E}\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\sum_{k=1}^{K}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right)\right)^{2}di$$
$$-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)$$
$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}\right)di$$
$$-\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\sigma\frac{e\left(i\right)}{E}\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\sum_{k}\partial_{e}e_{k}\left(i\right)\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)$$
$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\sum_{k=1}^{K}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right)di.$$

Plugging the definition of R_t we can rewrite the last two lines as:

$$-\frac{1}{\left(1-\varphi^{E}\right)\sigma+\left(1-\varphi^{N}\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left\{\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}\right)\right.\right.$$
$$\left.\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}+\frac{T_{t_0+s}}{E}\right\}\right)^{2}$$
$$-\frac{\left(1-\varphi^{N}\right)\psi\left(1-\varphi^{E}\right)\sigma}{\left(1-\varphi^{E}\right)\sigma+\left(1-\varphi^{N}\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{W}_{t_0+s}-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}(i)di\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)\right)^{2}$$
$$-\sigma\left(1-\varphi^{E}\right)\left(1-\frac{1}{R}\right)\sum_{k,l}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}\left(i\right)\partial_{e}e_{l}\left(i\right)di$$
$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{l,t_0+s}+\tilde{A}_{l,t_0+s}\right)$$
$$+\sigma\left(1-\varphi^{E}\right)\left(1-\frac{1}{R}\right)\sum_{k,l}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{l}(i)di$$
$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{l,t_0+s}+\tilde{A}_{l,t_0+s}\right)$$

$$-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)\int\left(1-\varphi\left(i\right)\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}di$$

$$-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}-\frac{T_{t_0+s}}{E}\right\}.$$

So putting everything together, and again preserving the original row order for the line references that follow, we have:

$$\sum_{s=0}^{\infty}R^{-s}\int\left(1-\varphi\left(i\right)\right)\left(\frac{1}{\sigma}\left(\hat{e}_{t_0+s}^{t_0,u}-\sum_{k}s_{k}\left(i\right)\hat{Q}_{k,t_0+s}\right)+\sum_{k}\partial_{e}e_{k}\left(i\right)\hat{Q}_{k,t_0+s}\right)$$

$$\times\left\{\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right\}di$$

$$=\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathcal{I}_{t_0+s}\left(i\right)\right.\right.$$

$$\left.\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}\right)\right)^{2}di$$

$$+\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{\sigma\psi Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\frac{e\left(i\right)}{E}\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\sum_{k=1}^{K}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right)\right)^{2}di$$

$$-\frac{\left(1-\varphi^{N}\right)\psi\left(1-\varphi^{E}\right)\sigma}{\left(1-\varphi^{E}\right)\sigma+\left(1-\varphi^{N}\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{W}_{t_0+s}-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}(i)di\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)\right)^{2}$$

$$-\sigma\left(1-\varphi^{E}\right)\left(1-\frac{1}{R}\right)\sum_{k,l}\left\{\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}\left(i\right)\partial_{e}e_{l}\left(i\right)di-\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{k}(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^{E}\right)}\partial_{e}e_{l}(i)di\right\}$$

$$\times\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\hat{Q}_{l,t_0+s}+\tilde{A}_{l,t_0+s}\right)$$

$$-\frac{1}{\left(1-\varphi^{E}\right)\sigma+\left(1-\varphi^{N}\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left\{\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}\right)\right.\right.$$

$$\left.\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}+\frac{T_{t_0+s}}{E}di\right\}\right)^{2}$$

$$-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)\int\left(1-\varphi\left(i\right)\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}di$$

$$-\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^{s}}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}-\frac{T_{t_0+s}}{E}\right\}$$

$$+\sum_{s=0}^{\infty}R^{-s}\mathsf{R}_{t_0+s}\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_{l}-\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{l}\left(i\right)\right)\tilde{A}_{l,t_0+s}\right.$$

$$\left.+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_{e}e_{k}\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right\}di. \tag{22}$$

Simplification of the terms involving Hand-to-Mouth households

We turn next to the terms involving HtM households (fourth and fifth line). Recall:

$$\frac{1}{\sigma}\left(\hat{e}_t^{t_0,HtM}-\sum_k s_k(i)\,\hat{Q}_{k,t}\right)+\sum_k \partial_e e_k(i)\,\hat{Q}_{k,t}=\frac{E}{\sigma e(i)+\psi Wn(i)}\Bigg(\mathcal{I}_t(i)$$
$$+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{k=1}^{K}\bar{s}_k\hat{Q}_{k,t_0}\Bigg)$$
$$+\frac{\sigma e(i)}{\sigma e(i)+\psi Wn(i)}\sum_{l=1}^{K}\partial_e e_l(i)\left(\tilde{A}_{l,t}+\hat{Q}_{l,t}\right)+\frac{Wn(i)\psi}{\sigma e(i)+\psi Wn(i)}\hat{W}_t.$$

Direct algebra then gives:

$$\left(\frac{1}{\sigma}\left(\hat{e}_t^{t_0,HtM}-\sum_k s_k(i)\,\hat{Q}_{k,t}\right)+\sum_k \partial_e e_k(i)\,\hat{Q}_{k,t}\right)$$
$$\times\left\{\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e(i)}{E}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}$$
$$=\frac{E}{\sigma e(i)+\psi Wn(i)}\left(\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)^2$$
$$+\frac{Wn(i)\psi\sigma}{\sigma e(i)+\psi Wn(i)}\frac{e(i)}{E}\left\{\sum_{k=1}^{K}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}^2$$
$$+\sigma\frac{e(i)}{E}\sum_{l=1}^{K}\partial_e e_l(i)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)\sum_{k=1}^{K}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)$$
$$+\left(\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)\hat{W}_t$$

Averaging across types i and generation t_0 (again preserving row order for the line references that follow), we obtain:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mathbb{E}_{\delta,t}\int\varphi(i)\left(\frac{1}{\sigma}\left(\hat{e}_t^{t_0,HtM}-\sum_k s_k(i)\,\hat{Q}_{k,t}\right)+\sum_k\partial_e e_k(i)\,\hat{Q}_{k,t}\right)$$
$$\times\left\{\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e(i)}{E}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}di$$
$$=(1-\delta)\,\mathbb{E}_0\sum_{t=0}^{\infty}R^{-t}\int\varphi(i)\frac{E}{\sigma e(i)+\psi Wn(i)}\left(\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,-}\right)^2di$$
$$+\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\sum_{t=0}^{\infty}R^{-t}\int\varphi(i)\frac{E}{\sigma e(i)+\psi Wn(i)}\left(\mathcal{I}_{t_0+t}(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t_0+t}+\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)^2di$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int\varphi(i)\frac{Wn(i)\psi\sigma}{\sigma e(i)+\psi Wn(i)}\frac{e(i)}{E}\left\{\sum_{k=1}^{K}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}^2di$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int\varphi(i)\,\sigma\frac{e(i)}{E}\sum_{l=1}^{K}\partial_e e_l(i)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)\sum_{k=1}^{K}\partial_e e_k(i)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)di$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int\varphi(i)\left(\mathcal{I}_t(i)+\sum_{l=1}^{K}\left(\frac{Wn(i)}{WN}\bar{s}_l-\sigma\frac{e(i)}{E}\partial_e e_l(i)\right)\tilde{A}_{l,t}\right)di\hat{W}_t$$
$$+\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int\varphi(i)\frac{b(i)}{E}di\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\left(1-\frac{1}{R}\right)\sum_{t=0}^{\infty}R^{-t}\hat{W}_{t_0+t}\tag{23}$$

To simplify further, we first rewrite one term from equation 22 — the last line:

$$\sum_{s=0}^{\infty} R^{-s}\mathsf{R}_{t_0+s}\int\left(1-\varphi\left(i\right)\right)$$
$$\times\left\{\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_e e_k\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right\}di.$$

Averaging this term across generations gives:

$$\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right.$$
$$\left.+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_e e_k\left(i\right)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}di$$
$$+\delta\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\int\left(1-\varphi\left(i\right)\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t}di.$$

Focusing on the first term, and using the expression for R_t, gives:

$$\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right.$$
$$\left.+\sum_{k=1}^{K}\sigma\frac{e\left(i\right)}{E}\partial_e e_k\left(i\right)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}di$$
$$=\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\sum_{t=0}^{\infty}\beta^t\left\{\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right\}di\right.$$
$$\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\frac{T_t}{E}\right\}^2$$
$$+\sum_{t=0}^{\infty}\beta^t\hat{W}_t\left\{\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right\}di-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\frac{T_t}{E}\right\}$$
$$+\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\left(\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t-\frac{T_t}{E}\right)$$
$$+\sum_{t=0}^{\infty}\beta^t\frac{\left(1-\varphi^N\right)\psi\left(1-\varphi^E\right)\sigma}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(\hat{W}_t-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\right)^2$$
$$+\sum_{t=0}^{\infty}\beta^t\sum_{k=1}^{K}\int\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}\sigma\partial_e e_k(i)di\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\hat{W}_t$$
$$-\sigma\left(1-\varphi^E\right)\sum_{t=0}^{\infty}\beta^t\left(\sum_{k=1}^{K}\int\frac{(1-\varphi(i))e(i)/E}{1-\varphi^E}\partial_e e_k(i)\,di\,(\hat{Q}_{k,t}+\tilde{A}_{k,t})\right)^2. \tag{24}$$

Combining terms in equations 21, 22, 23, 24 lets us show that they cancel. First, combining the second line in 24 with the last in 23 gives:

$$\sum_{t=0}^{\infty}\beta^t\hat{W}_t\left\{\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right\}di-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\frac{T_t}{E}\right\}$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\hat{W}_t\int\varphi\left(i\right)\left(\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right)di$$
$$=-\sum_{t=0}^{\infty}\beta^t\hat{W}_t\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\sum_{t=0}^{\infty}\beta^t\hat{W}_t\sum_{l=1}^{K}\left(\bar{s}_l-\sigma\overline{\partial_e e_l}\right)\tilde{A}_{l,t}.$$

Next, averaging across generations the last term in the second to last line of 22 gives:

$$(1-\delta)\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\hat{W}_s\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_s-\frac{T_s}{E}\right\}$$
$$+\delta\sum_{t_0=0}^{\infty}\beta^{t_0}\left(1-\frac{1}{R}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\hat{W}_{t_0+s}\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}-\frac{T_{t_0+s}}{E}\right\}. \tag{25}$$

Direct algebra gives:

$$\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}-\frac{T_{t_0+s}}{E}\right\}=\mathcal{B}_{t_0}-(1-\delta)\,\mathcal{B}_{t_0-1}.$$

Since $\mathcal{B}_0=\mathcal{B}_{-1}=0$, the expression in 25 simplifies to:

$$\delta\sum_{t_0=0}^{\infty}\beta^{t_0}\left(1-\frac{1}{R}\right)\left(\mathcal{B}_{t_0}-(1-\delta)\,\mathcal{B}_{t_0-1}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\hat{W}_{t_0+s}$$
$$=\delta\left(1-\frac{1}{R}\right)\sum_{t\geq 1}\beta^t\hat{W}_t\mathcal{B}_t.$$

Next, combining terms in the last line of 24, the second to last of 23, and the sixth line of 21, together with the remaining wage–productivity term from the preceding cancellation, and dropping terms independent of policy, gives:

$$\sum_{t=0}^{\infty}\beta^t\sum_{k=1}^{K}\int\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}\sigma\partial_e e_k(i)di\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\hat{W}_t$$
$$-\sigma\left(1-\varphi^E\right)\sum_{t=0}^{\infty}\beta^t\left(\sum_{k=1}^{K}\int\frac{(1-\varphi(i))e(i)/E}{1-\varphi^E}\partial_e e_k(i)\,di\,(\hat{Q}_{k,t}+\tilde{A}_{k,t})\right)^2$$
$$+\sum_{t=0}^{\infty}\beta^t\int\varphi\left(i\right)\sigma\frac{e\left(i\right)}{E}\sum_{l=1}^{K}\partial_e e_l\left(i\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)di$$
$$-\sum_{t=0}^{\infty}\beta^t\sum_{k,l}^{K}\int\sigma\frac{e\left(i\right)}{E}\partial_e e_k\left(i\right)\partial_e e_l\left(i\right)di\hat{Q}_{l,t}\left(\hat{W}_t-\hat{Q}_{k,t}-2\tilde{A}_{k,t}\right)-\sum_{t=0}^{\infty}\beta^t\sum_k\bar{s}_k\hat{W}_t\mathbf{A}_{k,t}$$
$$+\sum_{t=0}^{\infty}\beta^t\hat{W}_t\sum_{k=1}^{K}(\bar{s}_k-\sigma\overline{\partial_e e}_k)\tilde{A}_{k,t}$$
$$=\sigma\left(1-\varphi^E\right)\sum_{t=0}^{\infty}\beta^t\sum_{k,l}^{K}\left\{\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k\left(i\right)\partial_e e_l\left(i\right)di\right.$$
$$\left.-\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_l\left(i\right)di\right\}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right).$$

Next, we combine the terms involving R_t, $\hat{R}_t$ and $\mathcal{B}_t$ — namely, the last term in the second to last line of equation 22, the term in the fourth line of equation 24, and the term in the second to last line of equation 21:

$$-\left(1-\frac{1}{R}\right)(1-\delta)\sum_{s=0}^{\infty}\frac{1}{R^s}\mathsf{R}_s\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_s-\frac{T_s}{E}\right\}$$
$$-\left(1-\frac{1}{R}\right)\delta\sum_{t_0=0}^{\infty}\beta^{t_0}\sum_{s=0}^{\infty}\frac{1}{R^s}\mathsf{R}_{t_0+s}\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}-\frac{T_{t_0+s}}{E}\right\}$$
$$+\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\left(\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t-\frac{T_t}{E}\right)-\sum_{t=0}^{\infty}\beta^t\frac{\hat{R}_t}{R}\frac{B_{t+1}}{E}$$

Using the same derivations as before we can rewrite the expressions as:

$$-\delta\left(1-\frac{1}{R}\right)\sum_{t\geq 1}\beta^t\mathsf{R}_t\mathcal{B}_t+\sum_{t=0}^{\infty}\beta^t\mathsf{R}_t\left(\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t-\frac{T_t}{E}\right)+\sum_{t=0}^{\infty}\beta^t\frac{T_t}{E}\mathsf{R}_t=0.$$

Finally, we collect the terms stemming from the revaluation of newborns' wealth:

$$
\begin{aligned}
&-\left(1-\frac{1}{R}\right)\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathsf{R}_{t_0+s}-\hat{W}_{t_0+s}\right)\int\left(1-\varphi\left(i\right)\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}di\\
&+\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int\varphi\left(i\right)\frac{b\left(i\right)}{E}di\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\left(1-\frac{1}{R}\right)\sum_{t=0}^{\infty}R^{-t}\hat{W}_{t_0+t}\\
&+\delta\sum_{t=0}^{\infty}\beta^{t}\mathsf{R}_t\int\left(1-\varphi\left(i\right)\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t}di\\
&-\delta\sum_{t_0=0}^{\infty}\beta^{t_0}\sum_{s=0}^{\infty}\frac{1}{R^s}\frac{\hat{R}_{t_0+s}}{R}\int\varphi\left(i\right)\frac{b\left(i\right)}{E}di\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}=0,
\end{aligned}
$$

using that $\int\frac{b(i)}{E}di=0$ and $\hat{R}_t=\mathsf{R}_{t+1}-\mathsf{R}_t$.

Combining all of the derivations above, the loss function can be written as:

$$
\begin{aligned}
d^2\mathcal{W}=&E\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\\
&-E\sum_k\frac{P_kY_k}{E}\frac{\theta_k\epsilon_k}{\left(1-\beta\theta_k\right)\left(1-\theta_k\right)}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(\pi_{k,t}\right)^2\\
&-E\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg(-\frac{W\partial_W\boldsymbol{\mathcal{N}}}{N}\hat{W}_t^{\,2}-\sum_k\frac{P_k\partial_W\boldsymbol{\mathcal{Y}}_k}{N}\hat{W}_t\left(\hat{P}_{k,t}+\mathbf{A}_{k,t}\right)\\
&-\sum_k\frac{P_k\partial_{P_k}\boldsymbol{\mathcal{N}}}{N}\left(\hat{P}_{k,t}+\mathbf{A}_{k,t}\right)\hat{W}_t-\sum_{k,l}\frac{P_lP_k\partial_{P_l}\boldsymbol{\mathcal{Y}}_k}{E}\left(\hat{P}_{k,t}+\mathbf{A}_{k,t}\right)\left(\hat{P}_{l,t}+\mathbf{A}_{l,t}\right)\Bigg)\\
&-E\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\left(1-\varphi^N\right)\psi\left(1-\varphi^E\right)\sigma}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(\hat{W}_t-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\right)^2\\
&-E\left(1-\delta\right)\mathbb{E}_0\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{\sigma\psi Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\frac{e\left(i\right)}{E}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_s-\hat{Q}_{k,s}-\tilde{A}_{k,s}\right)\right)\right)^2di\\
&-E\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\left(1-\frac{1}{R}\right)\int\left(1-\varphi\left(i\right)\right)\frac{\sigma\psi Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\frac{e\left(i\right)}{E}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_{t_0+s}-\hat{Q}_{k,t_0+s}-\tilde{A}_{k,t_0+s}\right)\right)\right)^2di\\
&+E\left(1-\delta\right)\mathbb{E}_0\frac{\left(1-\varphi^N\right)\psi\left(1-\varphi^E\right)\sigma}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{W}_s-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\left(\hat{Q}_{k,s}+\tilde{A}_{k,s}\right)\right)\right)^2\\
&+E\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\frac{\left(1-\varphi^N\right)\psi\left(1-\varphi^E\right)\sigma}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{W}_{t_0+s}-\sum_{k=1}^{K}\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\right)\right)^2\\
&-E\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int\varphi\left(i\right)\frac{\psi\sigma Wn(i)}{\sigma e(i)+\psi Wn(i)}\frac{e\left(i\right)}{E}\left\{\sum_{k=1}^{K}\partial_e e_k\left(i\right)\left(\hat{W}_t-\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\right\}^2di\\
&-E\sigma\left(1-\varphi^E\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\sum_{k,l}^{K}\Bigg\{\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k\left(i\right)\partial_e e_l\left(i\right)di\\
&-\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_l\left(i\right)di\Bigg\}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+E\left(1-\delta\right)\mathbb{E}_0\sigma\left(1-\varphi^E\right)\left(1-\frac{1}{R}\right)\sum_{k,l}\left\{\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k\left(i\right)\partial_e e_l\left(i\right)di\right.\\
&\left.-\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_l(i)di\right\}\\
&\times\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{Q}_{k,s}+\tilde{A}_{k,s}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)\\
&+E\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\sigma\left(1-\varphi^E\right)\left(1-\frac{1}{R}\right)\sum_{k,l}\left\{\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k\left(i\right)\partial_e e_l\left(i\right)di\right.\\
&\left.-\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_k(i)di\int\frac{\left(1-\varphi\left(i\right)\right)\frac{e(i)}{E}}{\left(1-\varphi^E\right)}\partial_e e_l(i)di\right\}\\
&\times\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{Q}_{k,t_0+s}+\tilde{A}_{k,t_0+s}\right)\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\hat{Q}_{l,t_0+s}+\tilde{A}_{l,t_0+s}\right)\\
&+\left(1-\delta\right)\int G''\left(i\right)\left(dV_-\left(i\right)\right)^2di+\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int G''\left(i\right)\left(dV_{t_0}\left(i\right)\right)^2di\\
&-E\left(1-\delta\right)\left(1-\frac{1}{R}\right)\mathbb{E}_0\int\left(1-\varphi\left(i\right)\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathcal{I}_s\left(i\right)\right.\right.\\
&\left.\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,-}\right)\right)^2di\\
&-E\delta\left(1-\frac{1}{R}\right)\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\int\left(1-\varphi\left(i\right)\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left(\mathcal{I}_{t_0+s}\left(i\right)\right.\right.\\
&\left.\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s}+\mathbb{1}_{s=0}\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)\right)^2di\\
&-E\left(1-\delta\right)\mathbb{E}_0\sum_{t=0}^{\infty}R^{-t}\int\varphi\left(i\right)\frac{E}{\sigma e(i)+\psi Wn(i)}\left(\mathcal{I}_t\left(i\right)\right.\\
&\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}+\left(1-\frac{1}{R}\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,-}\right)^2di\\
&-E\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\sum_{t=0}^{\infty}R^{-t}\int\varphi\left(i\right)\frac{E}{\sigma e(i)+\psi Wn(i)}\left(\mathcal{I}_{t_0+t}\left(i\right)\right.\\
&\left.+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+t}+\left(1-\frac{1}{R}\right)\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_l\hat{Q}_{l,t_0}\right)^2di\\
&-E\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right\}di\right.\\
&\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\frac{T_t}{E}\right\}^2\\
&+E\left(1-\delta\right)\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(1-\frac{1}{R}\right)\mathbb{E}_0\left(\sum_{t=0}^{\infty}\frac{1}{R^t}\left\{\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_t\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t}\right)\right.\right.\\
&\left.\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_t+\frac{T_t}{E}di\right\}\right)^2
\end{aligned}
$$

$$+E\delta\mathbb{E}_0\sum_{t_0=0}^{\infty}\beta^{t_0}\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\left(1-\frac{1}{R}\right)\left(\sum_{s=0}^{\infty}\frac{1}{R^s}\left\{\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_{t_0+s}\left(i\right)+\sum_{l=1}^{K}\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,t_0+s}\right)\right.\right.$$
$$\left.\left.-\left(1-\frac{1}{R}\right)\delta\mathcal{B}_{t_0+s}+\frac{T_{t_0+s}}{E}di\right\}\right)^2$$

Rewriting directly then yields the formula given in appendix C.

C Model summary

In this section of the appendix, we summarize the evolution of aggregate variables and provide the expression of the loss function in the general version of the model with non random Hand-to-Mouth households, Input-Output linkages across sectors and government debt.

Evolution of Aggregate variables

The evolution of government debt, B_t, announced at time 0 constrains the aggregate tax levied on housholds $T_t = -\int\zeta_t\left(i\right)di - \sum_{k=1}^{K}\left(\tau_{k,t}P_kC_k+\iota_k\int dTC_{k,t}\left(j\right)dj\right)$.

$$\mathbb{E}_t B_{t+1} = R\left(\left(1-\delta\right)B_t+T_t\right).$$

Imposing $B_t = 0$ for all t gives the model without government debt.
Sectoral Phillips curves determine sectoral inflation:

$$\pi_{k,t} = \lambda_k\left\{\Omega_{N,k}\left(\frac{1}{\sigma}+\frac{1}{\psi}\right)\tilde{\boldsymbol{\mathcal{Y}}}_t+\Omega_{N,k}\left(\sum_l\overline{\partial_e e}_l\left(\hat{Q}_{l,t}-\hat{P}_{l,t}\right)\right)+\Omega_{N,k}\mathcal{NH}_t-\mathcal{P}_{k,t}\right\}+\tilde{\beta}\mathbb{E}_t\pi_{k,t+1},$$

with $\Omega_{N,k}$ the share of labor in the total cost of production for firms in sector k, $\Omega_{k,l}$ is the share of intermediate good l in the production cost of k and where the wedges are defined by:

$$\mathcal{NH}_t \equiv \sum_{l=1}^{K}(\overline{\partial_e e_l}-\bar{s}_l)(\hat{P}_{l,t}+\tilde{A}_{l,t}),$$
$$\bar{\mathcal{P}}_{k,t} \equiv (\hat{P}_{k,t}-\sum_{l=1}^{K}\bar{s}_l\hat{P}_{l,t})+(\tilde{A}_{k,t}-\sum_{l=1}^{K}\bar{s}_l\tilde{A}_{l,t}),$$
$$\mathcal{P}_{k,t} \equiv \Omega_{N,k}\bar{\mathcal{P}}_{k,t}-\sum_{l=1}^{K}\Omega_{k,l}\left(\bar{\mathcal{P}}_{l,t}-\bar{\mathcal{P}}_{k,t}\right).$$

Finally, the aggregate Euler equation determine the evolution of the output gap $\tilde{\boldsymbol{\mathcal{Y}}}_t$:

$$\left(1-\varphi^N\right)\left(\mathbb{E}_t\tilde{\boldsymbol{\mathcal{Y}}}_{t+1}-\tilde{\boldsymbol{\mathcal{Y}}}_t\right) = \left(1-\varphi^N\right)\sigma\mathbb{E}_t\left(\hat{R}_t-\sum_{k=1}^{K}\overline{\partial_e e}_k\pi^Q_{k,t+1}-r^*_t\right)+$$
$$\mathbb{E}_t\int\frac{\sigma\varphi\left(i\right)}{\sigma+\psi}\left\{\frac{b(i)}{RE}\left(\Delta\hat{R}_{t+1}-\sigma\left(R-1\right)\hat{R}_t\right)-\frac{e(i)}{E}\sum_{k=1}^{K}\left(\left(s_k(i)-\bar{s}_k\right)-\sigma(\partial_e e_k(i)-\overline{\partial_e e}_k)\right)\pi^Q_{k,t+1}+\frac{\Delta\tilde{\zeta}_{t+1}}{E}\left(i\right)\right\}di$$
$$-\mathbb{E}_t\int\frac{\sigma\varphi\left(i\right)}{\sigma+\psi}\left\{\left(1-\frac{1}{R}\right)\frac{b(i)}{E}\left(\pi^Q_{cpi,t+1}-\sigma\pi^Q_{mcpi,t+1}\right)\right\}di$$
$$+\frac{\sigma}{\sigma+\psi}\mathbb{E}_t\delta\left(1-\frac{1}{R}\right)\frac{B_{t+1}}{E}$$

with $r^*_t \equiv \mathbb{E}_t\frac{1}{\sigma+\psi}\left(\sum_{k=1}^{K}\left(\psi\overline{\partial_e e}_k+\bar{s}_k\right)\left(\tilde{A}_{k,t+1}-\tilde{A}_{k,t}\right)\right)$.

Loss Function

We define the operator $\mathbb{E}_\beta\left(Z_{t_0}\right)$ which is the social average over the population of the variable $Z_{t_0}$with

$$\mathbb{E}_\beta\left(Z_{t_0}\right) \equiv \mathbb{E}_0\left(1-\beta\right)\left(\frac{1}{1-\frac{1}{R}}\left(1-\delta\right)Z_- + \delta\frac{1}{1-\frac{1}{R}}\sum_{t_0}\beta^{t_0}Z_{t_0}\right).$$

We can express the loss function as:

$$
\begin{aligned}
&\mathcal{L} = \\
&(1-\beta)\sum \beta^s \frac{\left(1-\varphi^N\right)\psi}{1+\frac{\left(1-\varphi^N\right)\psi}{\left(1-\varphi^E\right)\sigma}} \underbrace{\int \frac{\left(1-\varphi\left(i\right)\right)e\left(i\right)}{\int\left(1-\varphi\left(i\right)\right)e\left(i\right)di}\left(\frac{\sigma+\psi}{\sigma\psi}\tilde{\boldsymbol{y}}_s - \sum_l\left(\partial_e e_l\left(i\right)-\overline{\partial_e e_l}\right)\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)\right)^2 di}_{\text{Labor Distortions with average wealth effect}} \\
&-(1-\beta)\sum \beta^s \frac{\left(1-\varphi^N\right)\psi}{1+\frac{\left(1-\varphi^N\right)\psi}{\left(1-\varphi^E\right)\sigma}} \underbrace{\mathbb{E}_\beta\left(\int\left(1-\tilde{\beta}\right)\frac{\frac{\left(1-\varphi(i)\right)b(i)}{\left(1-\varphi^E\right)E}-\frac{\left(1-\varphi(i)\right)Wn(i)}{\left(1-\varphi^N\right)WN}\frac{\int\left(1-\varphi(i)\right)b(i)di}{\left(1-\varphi^E\right)E}}{1+\frac{Wn(i)\psi}{\sigma e(i)}}\left(\left(1-\tilde{\beta}\right)\sum_{u\geq t_0}\tilde{\beta}^{u-t_0}\left(\frac{\sigma+\psi}{\sigma\psi}\tilde{\boldsymbol{y}}_t-\sum_l\left(\partial_e e_l\left(i\right)-\overline{\partial_e e_l}\right)\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)\right)\right)^2 di\right)}_{\text{Correction for the dispersion in wealth effects}} \\
&+(1-\beta)\sum\beta^s\underbrace{\int\varphi\left(i\right)\frac{Wn\left(i\right)}{WN}\frac{\psi}{1+\frac{Wn(i)\psi}{\sigma e(i)}}\left(\frac{\sigma+\psi}{\sigma\psi}\tilde{\boldsymbol{y}}_s-\sum_l\left(\partial_e e_l\left(i\right)-\overline{\partial_e e_l}\right)\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)\right)^2 di}_{\text{Labor distortion HtM}} \\
&+(1-\beta)\sum\beta^s\frac{\left(1-\varphi^E\right)\sigma}{\left(1+\frac{\left(1-\varphi^N\right)\psi}{\left(1-\varphi^E\right)\sigma}\right)}\underbrace{\sum_{k,l}\mathcal{E}_{k,l}\left\{\left(\hat{Q}_{k,s}+\tilde{A}_{k,s}\right)\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)-\mathbb{E}_\beta\left(\left(1-\tilde{\beta}\right)\left(\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\left(\hat{Q}_{k,s}+\tilde{A}_{k,s}\right)\right)\left(\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)\right)\right)\right\}}_{\text{Intertemporal misallocation of expenditures}} \\
&-(1-\beta)\sum\beta^s\underbrace{\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}\left(\hat{Q}_{k,s}+\tilde{A}_{k,s}\right)\left(\hat{Q}_{l,s}+\tilde{A}_{l,s}\right)}_{\text{Intratemporal misallocation of expenditures}}+(1-\beta)\sum\beta^s\underbrace{\left(\hat{\mathrm{W}}_s,\hat{P}_s+\mathbf{A}_s\right)^T O\left(\hat{\mathrm{W}}_s,\hat{P}_s+\mathbf{A}_s\right)}_{\text{Input misallocation}} \\
&+(1-\beta)\sum\beta^s\underbrace{\sum_k\frac{P_kY_k}{E_k}\bar{s}_k\bar{e}_k\frac{\theta_k}{\left(1-\beta\theta_k\right)\left(1-\theta_k\right)}\left(\pi_{k,s}\right)^2}_{\text{Within Sector misallocation}} \\
&\underbrace{+\mathbb{E}_\beta\left(\int g\left(i\right)\left(\nu_{t_0}(i)+\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\sum_l\frac{Wn\left(i\right)}{WN}\bar{s}_l\tilde{A}_{l,s}\right)^2 di\right)}_{\text{Redistribution}}\underbrace{-2\mathbb{E}_\beta\left(\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\int\left(\nu_{t_0}(i)\tilde{\beta}^{s-t_0}\sum_l\frac{\partial_e e_l\left(i\right)}{1+\frac{Wn(i)\psi}{\sigma e(i)}}\tilde{A}_{l,s}\right)di\right)}_{\text{"Bang for Buck "}} \\
&\underbrace{+\mathbb{E}_\beta\left(\left(1-\tilde{\beta}\right)\sum\tilde{\beta}^{s-t_0}\int\varphi\left(i\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\mathcal{I}_s\left(i\right)+\sum_l\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,s}\right\}^2 di\right.}_{\text{Redistribution, net HtM contribution}} \\
&\qquad\underbrace{\left.-\int\varphi\left(i\right)\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\left(\mathcal{I}_s\left(i\right)+\sum_l\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,s}\right)\right\}^2 di\right)}_{\text{Redistribution, net HtM contribution}} \\
&+\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\underbrace{\mathbb{E}_\beta\left\{\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\left(\int\left(1-\varphi\left(i\right)\right)\left\{\mathcal{I}_s\left(i\right)+\sum_l\frac{Wn\left(i\right)}{WN}\bar{s}_l\tilde{A}_{l,s}-\sum_l\frac{\sigma e\left(i\right)}{E}\partial_e e_l\left(i\right)\tilde{A}_{l,s}\right\}di+\left(1-\delta\right)\left(\beta B_{s+1}-B_s\right)/E-\delta\left(1-\tilde{\beta}\right)\mathcal{B}_s\right)^2\right\}}_{\text{Interest Rate Distortions}} \\
&-\frac{1}{\left(1-\varphi^E\right)\sigma+\left(1-\varphi^N\right)\psi}\underbrace{\mathbb{E}_\beta\left\{\left(\left(1-\tilde{\beta}\right)\sum_{s\geq t_0}\tilde{\beta}^{s-t_0}\left(\int\left(1-\varphi\left(i\right)\right)\left(\mathcal{I}_s\left(i\right)+\sum_l\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\frac{\sigma e\left(i\right)}{E}\partial_e e_l\left(i\right)\right)\tilde{A}_{l,s}\right)di+\left(1-\delta\right)\left(\beta B_{s+1}-B_s\right)/E-\delta\left(1-\tilde{\beta}\right)\mathcal{B}_s\right)\right)^2\right\}}_{\text{Interest Rate Distortions}}
\end{aligned}
$$

With O the aggregate input substitution matrix which is of dimension $(K+1)\times(K+1)$, and given by:

$$
O=\begin{bmatrix} -\frac{W\partial_W\mathcal{N}}{N} & -\left(\frac{P_k\partial_{P_k}\mathcal{N}}{N}\right)^T \\ -\frac{P_k\partial_{P_k}\mathcal{N}}{N} & -\frac{P_k\boldsymbol{y}_k}{E}\frac{P_l\partial_{P_l}\boldsymbol{y}_k}{\boldsymbol{y}_k} \end{bmatrix}
$$

and

$$1-\varphi^{E}=\int\left(1-\varphi(i)\right)\frac{e(i)}{E}di$$

$$1-\varphi^{N}=\int\left(1-\varphi(i)\right)\frac{Wn(i)}{WN}di$$

$$\mathcal{E}_{k,l}=\int\frac{\left(1-\varphi\left(i\right)\right)e\left(i\right)}{\int\left(1-\varphi\left(i\right)\right)e\left(i\right)di}\partial_{e}e_{l}\left(i\right)\partial_{e}e_{k}\left(i\right)di-\frac{\int\left(1-\varphi\left(i\right)\right)e\left(i\right)\partial_{e}e_{k}\left(i\right)di\int\left(1-\varphi\left(i\right)\right)e\left(i\right)\partial_{e}e_{l}\left(i\right)di}{\left(\int\left(1-\varphi\left(i\right)\right)e\left(i\right)di\right)^{2}},$$

$$g(i)=-\frac{G''\left(V(i)\right)v'\left(e\left(i\right)\right)E}{\left(1-\tilde{\beta}\right)G'\left(V(i)\right)}+\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi},$$

$$\mathcal{I}_{s}\left(i\right)=\frac{b\left(i\right)}{RE}\hat{R}_{s}-\sum_{k}\left(\frac{e(i)}{E}\left(s_{k}(i)-\bar{s}_{k}\right)\right)\hat{Q}_{k,s}-\sum_{k}\left(1-\frac{1}{R}\right)\frac{b\left(i\right)}{E}\bar{s}_{k}\hat{Q}_{k,s}+\frac{\tilde{\zeta}_{s}\left(i\right)}{E},$$

$$v_{t_{0}}(i)=\left(1-\tilde{\beta}\right)\left(\left(\sum_{s\geq t_{0}}\tilde{\beta}^{s-t_{0}}\mathcal{I}_{s}\left(i\right)\right)+\frac{b\left(i\right)}{E}\sum_{l=1}^{K}\bar{s}_{l}\hat{Q}_{l,t_{0}}\right),$$

$$v_{-}(i)=\left(1-\tilde{\beta}\right)\left(\sum_{s\geq0}\tilde{\beta}^{s}\mathcal{I}_{s}\left(i\right)\right),$$

$$\mathcal{B}_{s}=\sum_{u=0}^{s}\frac{B_{u}}{E}.$$

D Proofs of Propositions 1-4

In this section we provide the proofs of Propositions 1 to 4.

Proof of Proposition 1

Consider a subsidy to the price of sector k good, $\hat{Q}_{k,t}=\hat{P}_{k,t}-\tau_{k,t}$. Under Assumption A, we have $\pi_{mcpi,t}=\pi_{cpi,t}$ following any sequence of subsidy so the evolution of the output gap and inflation is summarized by:

$$\tilde{\boldsymbol{y}}_{t}=-\sigma\left(\hat{R}_{t}-\mathbb{E}_{t}\pi_{cpi,t+1}-\mathcal{F}_{t}+\mathbb{E}_{t}\mathcal{F}_{t+1}-\hat{r}_{t}^{*}\right)+\mathbb{E}_{t}\tilde{\boldsymbol{y}}_{t+1}$$

$$\pi_{cpi,t}=\lambda\left((\sigma^{-1}+\psi^{-1})\tilde{\boldsymbol{y}}_{t}-\mathcal{F}_{t}\right)+\frac{1}{R}\mathbb{E}_{t}\pi_{cpi,t+1}$$

with $\mathcal{F}_{t}=-\overline{\partial_{e}e_{k}}\tau_{k,t}$, and where we used $(1-\delta)\beta=1/R$. Under the monetary rule (10), $\hat{R}_{t}=\hat{r}_{t}^{*}+\phi_{y}\tilde{\boldsymbol{y}}_{t}+\phi_{\pi}\pi_{cpi,t}$ with $\phi_{y},\phi_{\pi}\geq0$ we can rewrite the system as:

$$\mathbb{E}_{t}\pi_{cpi,t+2}-\left(1+\sigma\phi_{y}+R\left(1+\lambda\left(1+\frac{\sigma}{\psi}\right)\right)\right)\mathbb{E}_{t}\pi_{cpi,t+1}+\left(1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}\right)R\pi_{cpi,t}$$
$$=R\lambda\left(\left(\frac{\sigma}{\psi}-\sigma\phi_{y}\right)\overline{\partial_{e}e}_{k}\tau_{k,t}-\frac{\sigma}{\psi}\mathbb{E}_{t}\overline{\partial_{e}e}_{k}\tau_{k,t+1}\right)$$

The roots of the associated characteristic polynomial are:

$$\rho_{\pm}=$$
$$\frac{\left(1+\sigma\phi_{y}+R\left(1+\lambda\left(1+\frac{\sigma}{\psi}\right)\right)\right)\pm\sqrt{\left(\sigma\phi_{y}+R\left(1+\lambda\left(1+\frac{\sigma}{\psi}\right)\right)-1\right)^{2}-4R\left(\left(1-\frac{1}{R}\right)\sigma\phi_{y}-\lambda\left(1+\frac{\sigma}{\psi}\right)\left(1-\phi_{\pi}\right)\right)}}{2}$$

Under the determinacy condition $\left(1-\frac{1}{R}\right)\sigma\phi_{y}-\lambda\left(1+\frac{\sigma}{\psi}\right)\left(1-\phi_{\pi}\right)>0$, both roots lie outside the unit circle, $\left|\rho_{\pm}\right|>1$. The roots may be real or complex conjugates: since $\rho_{+}\rho_{-}=R\left(1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}\right)>1$, complex roots always satisfy this property, while if the roots are real the condition (which is $P(1)>0$, with P the characteristic polynomial defined below) together with $\rho_{+}+\rho_{-}>2$ implies $\rho_{+}\geq\rho_{-}>1$. The formulas below hold in both cases as long as $\rho_{+}\neq\rho_{-}$. Solving forward, we obtain:

$$\rho_{+}\rho_{-}\pi_{cpi,t}=\sum_{s\geq0}\sum_{u\geq0}\rho_{+}^{-s}\rho_{-}^{-u}R\lambda\left(\left(\frac{\sigma}{\psi}-\sigma\phi_{y}\right)\overline{\partial_{e}e}_{k}\tau_{k,t+s+u}-\frac{\sigma}{\psi}\mathbb{E}_{t}\overline{\partial_{e}e}_{k}\tau_{k,t+s+u+1}\right).$$

We can simplify the formula to:

$$\pi_{cpi,t} = -\frac{\lambda}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\sigma\phi_y-\frac{\sigma}{\psi}\right)\overline{\partial_e e}_k\tau_{k,t}$$
$$-\frac{\lambda}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\sum_{s\geq 1}\frac{\rho_-^{-s}\left(\frac{\sigma}{\psi}\left(1-\rho_-^{-1}\right)+\rho_-^{-1}\sigma\phi_y\right)-\rho_+^{-s}\left(\frac{\sigma}{\psi}\left(1-\rho_+^{-1}\right)+\rho_+^{-1}\sigma\phi_y\right)}{\rho_-^{-1}-\rho_+^{-1}}\overline{\partial_e e}_k\tau_{k,t+s}.$$

Similarly, we have for the output gap:

$$\tilde{\mathcal{Y}}_t = \frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,t}$$
$$-\frac{\sigma\psi}{\sigma+\psi}\frac{1}{R\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\sum_{s\geq 1}\frac{\left(R\rho_-^{-1}-1\right)\rho_-^{-s+1}\left(\frac{\sigma}{\psi}\left(1-\rho_-^{-1}\right)+\rho_-^{-1}\sigma\phi_y\right)}{\rho_-^{-1}-\rho_+^{-1}}\overline{\partial_e e}_k\tau_{k,t+s}$$
$$-\frac{\sigma\psi}{\sigma+\psi}\frac{1}{R\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\sum_{s\geq 1}\frac{\left(1-\rho_+^{-1}R\right)\rho_+^{-s+1}\left(\frac{\sigma}{\psi}\left(1-\rho_+^{-1}\right)+\rho_+^{-1}\sigma\phi_y\right)}{\rho_-^{-1}-\rho_+^{-1}}\overline{\partial_e e}_k\tau_{k,t+s}.$$

Inspecting the formulas, we have for a transitory shock $\tau_{k,t+s}=0$ for $s>0$:

$$\pi_{cpi,0} = -\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y-1\right)\overline{\partial_e e}_k\tau_{k,0}$$
$$\tilde{\mathcal{Y}}_0 = \frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}$$
$$\pi_{cpi,t},\ \tilde{\mathcal{Y}}_t = 0 \quad for\ t>0.$$

We directly have $\pi_{cpi,0}>0$ if and only if $\psi\phi_y<1$ and since $\sigma\phi_y-\frac{\sigma}{\psi}<1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi$ with $\phi_y,\phi_\pi\geq 0$, $\tilde{\mathcal{Y}}_0>0$ for all ϕ_y,ϕ_π. Finally using the monetary rule, we have:

$$\hat{R}_0 = -\frac{\lambda\frac{\sigma}{\psi}\phi_\pi}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y-1\right)\overline{\partial_e e}_k\tau_{k,0}$$
$$+\frac{\sigma\psi}{\sigma+\psi}\phi_y\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}$$
$$\hat{R}_t = 0 \quad for\ t>0.$$

We can rewrite $\hat{R}_0$ as

$$\hat{R}_0 = \frac{\lambda\frac{\sigma}{\psi}\phi_\pi+\sigma\phi_y}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\overline{\partial_e e}_k\tau_{k,0}>0$$

which concludes the proof.

Finally consider the extended rule

$$\hat{R}_t = r_t^* + \phi_y\tilde{\mathcal{Y}}_t + \phi_\pi\pi_{cpi,t} + \phi_\tau\left(\mathcal{F}_t-\mathbb{E}_t\mathcal{F}_{t+1}\right),$$

in that case, we have:

$$\pi_{cpi,0} = \frac{\lambda\frac{\sigma}{\psi}\left(1-\left(1+\frac{\psi}{\sigma}\right)\phi_\tau-\psi\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\overline{\partial_e e}_k\tau_{k,0}$$

$$\tilde{\mathcal{Y}}_0 = \frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\frac{\sigma}{\psi}\left(\left(1+\frac{\psi}{\sigma}\right)\phi_\tau+\psi\phi_y-1\right)}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}$$

$$\hat{R}_0 = \frac{\sigma\phi_y+\lambda\frac{\sigma}{\psi}\phi_\pi+\phi_\tau}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\overline{\partial_e e}_k\tau_{k,0}$$

We now generalize the results of Proposition 1 by considering the impact of a subsidy of good k at time $t_f \geq 0$ announced at 0 on the path of inflation and output gap $\pi_{cpi,t}\left[t_f\right]$, $\tilde{\mathcal{Y}}_t\left[t_f\right]$. Note first that since we only consider rules with determinacy, we have, in response to the subsidy, $\pi_{cpi,t} = \tilde{\mathcal{Y}}_t = 0$ for $t > t_f$. Second, denoting the horizon of the subsidy as $h(t,t_f) \equiv t_f - t$ we have $\pi_{cpi,t}\left[t_f\right] = \pi_{cpi,t'}\left[t'_f\right]$ if $h(t',t'_f) = h(t,t_f)$ (and similarly for the output gap): the response of the output gap and inflation only depends on $h(t,t_f) \equiv t_f - t$ and we denote the response of inflation and the output gap as $\pi_{cpi}[h]$ and $\tilde{\mathcal{Y}}[h]$.

Proposition A1. *Assume* $1 + R\lambda\left(1+\frac{\sigma}{\psi}\right) \geq R$, *that the monetary rule has determinacy, and that the roots* $\rho_\pm$ *of the characteristic polynomial are real and distinct, i.e.*

$$\Delta \equiv \left(\sigma\phi_y + R\left(1+\lambda\left(1+\frac{\sigma}{\psi}\right)\right)-1\right)^2 - 4R\left(\left(1-\frac{1}{R}\right)\sigma\phi_y - \lambda\left(1+\frac{\sigma}{\psi}\right)(1-\phi_\pi)\right) > 0.$$

First, the response of inflation at t *to a subsidy* τ_k *occurring at* $t+h$, $\pi_{cpi}[h]$, *satisfies:*

- *If* $\psi\phi_y \geq 1$ *then* $\pi_{cpi}[h] \leq 0$ *for all* h.
- *If* $\psi\phi_y < 1$, *then* $\pi_{cpi}[0] > 0$ *and there exists a maximum horizon* $h^*\left(\phi_y,\phi_\pi\right) < \infty$ *such that* $\pi_{cpi}[h] \geq 0$ *if and only if* $h \leq h^*$.

Second, the response of the output gap at t *to a subsidy occurring at* $t+h$, $\tilde{\mathcal{Y}}[h]$, *satisfies:*

- $\tilde{\mathcal{Y}}[0] > 0$.
- *If* $\phi_\pi \leq R$ *then* $\tilde{\mathcal{Y}}[h] < 0$ *for all* $h > 0$.
- *If* $\phi_\pi > R$ *there exists a horizon* $1 \leq h^*\left(\phi_y,\phi_\pi\right) < \infty$ *such that* $\tilde{\mathcal{Y}}[h] < 0$ *if and only if* $1 \leq h < h^*\left(\phi_y,\phi_\pi\right)$.
- $\tilde{\mathcal{Y}}[h] > 0$ *at all* h *if and only if* $\phi_\pi > R\left(1+\lambda^{-1}\right)$ *and* $\psi\phi_y > \left((1+R)\phi_\pi - R\right)/\left(\phi_\pi - R\left(1+\lambda^{-1}\right)\right)$.

Remark. The assumption $\Delta > 0$ is not implied by determinacy and fails for standard calibrations of the rule (e.g. $\sigma = \psi = 1$, $\lambda = 0.1$, $R = 1.01$, $\phi_\pi = 1.5$, $\phi_y = 0.125$). When $\Delta < 0$ the roots are complex conjugates with modulus larger than one, the impulse responses $\pi_{cpi}[h]$ and $\tilde{\mathcal{Y}}[h]$ oscillate in h, and the sign patterns above need not hold beyond $h = 0$. The results of Proposition 1 (transitory subsidy, $h = 0$) do not rely on this assumption.

Proof. Using the derivations of Proposition 1, we directly have, for $h \geq 1$:

$$\pi[h] = -\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\frac{\rho_-^{-h}\left(\left(1-\rho_-^{-1}\right)+\rho_-^{-1}\psi\phi_y\right)-\rho_+^{-h}\left(\left(1-\rho_+^{-1}\right)+\rho_+^{-1}\psi\phi_y\right)}{\rho_-^{-1}-\rho_+^{-1}}\overline{\partial_e e}_k\tau_k,$$

and at $h = 0$

$$\pi[0] = -\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y-1\right)\overline{\partial_e e}_k\tau_k,$$

which coincides with the general formula evaluated at $h = 0$. Similarly

$$\tilde{\mathcal{Y}}[h] = -\frac{\sigma\psi}{\sigma+\psi}\frac{\frac{\sigma}{\psi}}{R\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\frac{\left(R\rho_-^{-1}-1\right)\rho_-^{-h+1}\left(\left(1-\rho_-^{-1}\right)+\rho_-^{-1}\psi\phi_y\right)+\left(1-\rho_+^{-1}R\right)\rho_+^{-h+1}\left(\left(1-\rho_+^{-1}\right)+\rho_+^{-1}\psi\phi_y\right)}{\rho_-^{-1}-\rho_+^{-1}}\overline{\partial_e e}_k\tau_k, \quad h \geq 1,$$

$$\tilde{\mathcal{Y}}[0] = \frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma}{\psi}\frac{\psi\phi_y-1}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_k = \frac{\sigma\left(1+\lambda\phi_\pi\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\overline{\partial_e e}_k\tau_k > 0.$$

Since the roots are real, distinct and larger than one (determinacy), we have $1 > \rho_-^{-1} > \rho_+^{-1} > 0$, so that $g_\pm \equiv \left(1-\rho_\pm^{-1}\right)+\rho_\pm^{-1}\psi\phi_y = 1+\rho_\pm^{-1}\left(\psi\phi_y-1\right) > 0$. If $\psi\phi_y \geq 1$ then $g_- \geq g_+$ and $\rho_-^{-h} \geq \rho_+^{-h}$, which implies that $\pi[h] \leq 0$ for all h. If $\psi\phi_y < 1$, we have

$0 < g_- < g_+$, which implies that $\pi[h] \geq 0$ if and only if

$$h \leq \ln\left(\frac{g_+}{g_-}\right)/\ln\left(\frac{\rho_+}{\rho_-}\right),$$

so that h^* is the integer part of the right-hand side, and $\pi[0] > 0$. This proves the first set of results. For the output gap, recall that ρ_-, ρ_+ are the roots of:

$$P(x) = x^2 - \left(1 + \sigma\phi_y + R\left(1 + \lambda\left(1 + \frac{\sigma}{\psi}\right)\right)\right)x + \left(1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)R.$$

Since $\rho_- + \rho_+ > 2R$ (by the assumption $1 + R\lambda\left(1 + \frac{\sigma}{\psi}\right) \geq R$) and $\rho_+ > \rho_-$, we have $\rho_+ > R$, so $\left(1 - \rho_+^{-1}R\right)g_+ > 0$. In addition, since R does not exceed the mid-point of the roots, $\rho_- \geq R$ if and only if $P(R) \geq 0$; and since $P(R) = R\lambda\frac{\sigma+\psi}{\psi}\left(\phi_\pi - R\right)$, we have $\rho_- \geq R$ if and only if $\phi_\pi \geq R$. Therefore $\left(R\rho_-^{-1} - 1\right)g_- \geq 0$ if and only if $\phi_\pi \leq R$. Write the numerator of $\tilde{\mathcal{Y}}[h]$ as $\rho_-^{-h+1}\left\{\left(R\rho_-^{-1} - 1\right)g_- + \left(1 - \rho_+^{-1}R\right)g_+\left(\rho_-/\rho_+\right)^{h-1}\right\}$. If $\phi_\pi \leq R$ the curly bracket is strictly positive, so $\tilde{\mathcal{Y}}[h] < 0$ for all $h > 0$. If $\phi_\pi > R$, the first term in the bracket is negative and the second is positive and strictly decreasing in h, converging to zero; hence the bracket changes sign at most once and is negative for h large enough, so there exists a horizon $1 \leq h^*\left(\phi_y, \phi_\pi\right) < \infty$ such that $\tilde{\mathcal{Y}}[h] < 0$ if and only if $1 \leq h < h^*\left(\phi_y, \phi_\pi\right)$. By the same argument, $\tilde{\mathcal{Y}}[h] > 0$ for all $h \geq 1$ if and only if the bracket is negative at $h = 1$, i.e. if and only if $\left(R\rho_-^{-1} - 1\right)g_- + \left(1 - \rho_+^{-1}R\right)g_+ < 0$. Dividing by $\rho_-^{-1} - \rho_+^{-1} > 0$ and using $\rho_-^{-1} + \rho_+^{-1} = \left(\rho_- + \rho_+\right)/\left(\rho_-\rho_+\right)$, this expression has the same sign as $\left(1 - \psi\phi_y\right) + R - \left(1 - \psi\phi_y\right)\frac{1+\sigma\phi_y+R\left(1+\lambda\left(1+\frac{\sigma}{\psi}\right)\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}$ which itself has the same sign as $-\psi\phi_y\left(\lambda\phi_\pi - R(1+\lambda)\right) + \lambda\left((1+R)\phi_\pi - R\right)$. For $\phi_\pi > R$ the second term is positive, so the expression is negative only if $\lambda\phi_\pi > R(1+\lambda)$. Therefore, since $\tilde{\mathcal{Y}}[0] > 0$, $\tilde{\mathcal{Y}}[h] > 0$ at all h if and only if $\phi_\pi > R\left(1 + \lambda^{-1}\right)$ and $\psi\phi_y > \left((1+R)\phi_\pi - R\right)/\left(\phi_\pi - R\left(1 + \lambda^{-1}\right)\right)$.

Proof of Proposition 2

Let us rewrite the loss function under Assumptions A and B. Recall that under Assumption B $\partial_e e_l(i) = \overline{\partial_e e}_l$ for all l so $\mathcal{E}_{k,l} = 0$. Under Assumption A, we have no input-output linkages so $O = 0$. Throughout the proofs we normalize the loss by $(1-\beta)$, i.e. the expressions below are $(1-\beta)$ times the ones in Proposition 2 of the main text. The loss function becomes:

$$\begin{aligned}
\mathcal{L} = {} & (1-\beta)\sum_t \beta^t (1-\varphi)\left(\frac{1}{\psi} + \frac{1}{\sigma}\right)\tilde{\mathcal{Y}}_t^2 \\
& - (1-\varphi)\left(\frac{1}{\psi} + \frac{1}{\sigma}\right)\int (1-\beta)\frac{\frac{b(i)}{E}}{1 + \frac{Wn(i)\psi}{\sigma e(i)}}di\left((1-\beta)\sum_t \beta^t \tilde{\mathcal{Y}}_t\right)^2 \\
& + \varphi\int \frac{Wn(i)}{WN}\frac{\psi}{1 + \frac{Wn(i)\psi}{\sigma e(i)}}di\,(1-\beta)\sum \beta^t\left(\left(\frac{1}{\psi} + \frac{1}{\sigma}\right)\tilde{\mathcal{Y}}_t\right)^2 \\
& + (1-\beta)\sum \beta^t \sum_k \bar{s}_k \sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t} + \tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t} + \tilde{A}_{l,t}\right) + (1-\beta)\frac{\bar{\epsilon}}{\lambda}\sum \beta^t \sum_k \bar{s}_k\left(\pi_{k,t}\right)^2 \\
& + (1-\varphi)\int \frac{E}{\sigma e(i) + Wn(i)\psi}\left((1-\beta)\sum_t \beta^t\left(\tilde{\mathcal{I}}_t(i) + \sum_l\left(\frac{Wn(i)}{WN}\bar{s}_l - \sigma\frac{e(i)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right)\right)^2 di \\
& + \varphi(1-\beta)\sum \beta^t \int \frac{E}{\sigma e(i) + Wn(i)\psi}\left\{\tilde{\mathcal{I}}_t(i) + \sum_l\left(\frac{Wn(i)}{WN}\bar{s}_l - \sigma\frac{e(i)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right\}^2 di \\
& + \frac{1-\varphi}{\sigma+\psi}(1-\beta)\sum_t \beta^t\left(\int \tilde{\mathcal{I}}_t(i) + \sum_l\left(\frac{Wn(i)}{WN}\bar{s}_l - \sigma\frac{e(i)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}di\right)^2 \\
& - \frac{1-\varphi}{\sigma+\psi}\left((1-\beta)\sum_t \beta^t \int\left(\tilde{\mathcal{I}}_t(i) + \sum_l\left(\frac{Wn(i)}{WN}\bar{s}_l - \sigma\frac{e(i)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right)di\right)^2
\end{aligned}$$

First, we rewrite the output gap terms ($\mathcal{L}^{\tilde{\mathcal{Y}}}$, first three lines). We have:

$$\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\int \frac{Wn\left(i\right)}{WN}\frac{\psi}{1+\frac{Wn(i)\psi}{\sigma e(i)}}di = \int \frac{e\left(i\right)}{E}\frac{\left(\psi+\sigma\right)Wn\left(i\right)}{\sigma e\left(i\right)+Wn\left(i\right)\psi}di$$

$$= 1-\int \left(1-\beta\right)\frac{\frac{b(i)}{E}}{1+\frac{Wn(i)\psi}{\sigma e(i)}}di$$

where we used $e\left(i\right)=\left(1-\beta\right)b\left(i\right)+Wn\left(i\right)$. Note in addition that we have:

$$\int \left(1-\beta\right)\frac{\frac{b(i)}{E}}{1+\frac{Wn(i)\psi}{\sigma e(i)}}di = \frac{\sigma\psi}{\sigma+\psi}\int \left(\left(1-\beta\right)\frac{b\left(i\right)}{E}\right)^2\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}di > 0.$$

So the output gap loss can be rewritten:

$$\mathcal{L}^{\tilde{\mathcal{Y}}} = \left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\left\{\left(1-\beta\right)\sum_t \beta^t \tilde{\mathcal{Y}}_t^2 - \eta\left(\varphi\left(1-\beta\right)\sum_t \beta^t \tilde{\mathcal{Y}}_t^2 + \left(1-\varphi\right)\left(\left(1-\beta\right)\sum_t \beta^t \tilde{\mathcal{Y}}_t\right)^2\right)\right\},$$

with $\eta \geq 0$ (strictly positive as soon as $b\left(i\right)\neq 0$ for a positive mass of households). Secondly, the between inflation loss is directly given by:

$$\mathcal{L}^{\pi,\text{between}} = \left(1-\beta\right)\sum \beta^t \sum_k \bar{s}_k \sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right).$$

Third, we simplify the within inflation term. We have:

$$\mathcal{L}^{\pi,\text{within}} = \left(1-\beta\right)\frac{\bar{\epsilon}}{\lambda}\sum \beta^t \sum_k \bar{s}_k\left(\pi_{k,t}\right)^2$$

$$= \left(1-\beta\right)\frac{\bar{\epsilon}}{\lambda}\sum \beta^t\left(\pi_{cpi,t}^2 + \sum_k \bar{s}_k\left(\pi_{k,t}-\pi_{cpi,t}\right)^2\right)$$

Under our homogenous price stickiness assumption, we have that $\pi_{k,t}-\pi_{cpi,t}$ is independent from monetary and fiscal policy (see Olivi, Sterk & Xhani (2026) Proposition 1 for a formal treatment). Indeed:

$$\pi_{k,t}-\pi_{cpi,t} = -\lambda\left(\hat{P}_{k,t}-\hat{P}_{cpi,t}+\tilde{A}_{k,t}-\tilde{A}_{cpi,t}\right)+\beta\mathbb{E}_t\left(\pi_{k,t+1}-\pi_{cpi,t+1}\right)$$

so relative producer prices are a function of (relative) productivity shocks only. Therefore,

$$\mathcal{L}^{\pi,\text{within}} = \left(1-\beta\right)\frac{\bar{\epsilon}}{\lambda}\sum \beta^t\left(\pi_{cpi,t}\right)^2.$$

Finally, we rewrite the redistribution terms. First note that $\int \tilde{\mathcal{I}}_t\left(i\right)di = 0$ (indeed $\int b\left(i\right)di = 0$ since bonds are in zero net supply, $\int \zeta\left(i\right)di = 0$ since debt is not used and $\zeta_t\left(i\right)$ in $\tilde{\mathcal{I}}_t\left(i\right)$ denotes the transfer net of the lump-sum taxes levied, in proportion to labor income, to finance the price subsidy (see Appendix B), and $\int \frac{e(i)}{E}\left(s_k\left(i\right)-\bar{s}_k\right)di = 0$). This implies that the terms in the last two lines of the loss function are independent from monetary and fiscal policy. In addition, we have:

$$\int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\tilde{\mathcal{I}}_t\left(i\right)+\sum_l\left(\frac{Wn\left(i\right)}{WN}\bar{s}_l-\sigma\frac{e\left(i\right)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right\}^2 di$$

$$= \int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\tilde{\mathcal{I}}_t\left(i\right)+\sum_l\left(\frac{Wn\left(i\right)}{WN}\left(\bar{s}_l+\psi\overline{\partial_e e}_l\right)-\left(\sigma\frac{e\left(i\right)}{E}+\frac{Wn\left(i\right)}{WN}\psi\right)\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right\}^2 di$$

$$= \int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\tilde{\mathcal{I}}_t\left(i\right)\right.$$

$$\left.-\sum_l\left(\frac{\sigma}{\sigma+\psi}\left(1-\beta\right)\frac{b\left(i\right)}{E}\left(\bar{s}_l+\psi\overline{\partial_e e}_l\right)+\left(\sigma\frac{e\left(i\right)}{E}+\frac{Wn\left(i\right)}{WN}\psi\right)\left(\overline{\partial_e e}_l-\frac{1}{\sigma+\psi}\left(\bar{s}_l+\psi\overline{\partial_e e}_l\right)\right)\right)\tilde{A}_{l,t}\right\}^2 di$$

$$= \int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left\{\tilde{\mathcal{I}}_t\left(i\right)-\left(1-\beta\right)\frac{b\left(i\right)}{E}\tilde{\mathcal{Y}}_t^*\right\}^2 di$$

$$+\left(\sigma+\psi\right)\left(\sum_l\left(\overline{\partial_e e}_l-\frac{1}{\sigma+\psi}\left(\bar{s}_l+\psi\overline{\partial_e e}_l\right)\right)\tilde{A}_{l,t}\right)^2.$$

where the derivation uses again $\int \tilde{\mathcal{I}}_t\left(i\right)di = 0$, $e\left(i\right)=\left(1-\beta\right)b\left(i\right)+Wn\left(i\right)$ and $\tilde{\mathcal{Y}}_t^* = \frac{\sigma}{\sigma+\psi}\sum_l\left(\bar{s}_l+\psi\overline{\partial_e e}_l\right)\tilde{A}_{l,t}$. Since the second

term in the last line is independent from policy, we can remove it from the loss function. Similarly, we have:

$$\int \frac{E}{\sigma e(i) + Wn(i)\psi}\left(\left(1-\beta\right)\sum_t \beta^t\left(\tilde{\mathcal{I}}_t(i) + \sum_l\left(\frac{Wn(i)}{WN}\bar{s}_l - \sigma\frac{e(i)}{E}\overline{\partial_e e}_l\right)\tilde{A}_{l,t}\right)\right)^2 di =$$

$$\int \frac{E}{\sigma e(i) + Wn(i)\psi}\left(\left(1-\beta\right)\sum_t \beta^t\left(\tilde{\mathcal{I}}_t(i) - \left(1-\beta\right)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*\right)\right)^2 di$$

$$+ \left(\sigma+\psi\right)\left(\left(1-\beta\right)\sum_t \beta^t\left(\sum_l\left(\overline{\partial_e e}_l - \frac{1}{\sigma+\psi}\left(\bar{s}_l + \psi\overline{\partial_e e}_l\right)\right)\tilde{A}_{l,t}\right)\right)^2$$

so the redistribution term can be rewritten:

$$\mathcal{L}^{\text{redist}} = \left(1-\varphi\right)\int \frac{E}{\sigma e(i) + Wn(i)\psi}\left(\left(1-\beta\right)\sum_t \beta^t\left(\tilde{\mathcal{I}}_t(i) - \left(1-\beta\right)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*\right)\right)^2 di$$

$$+ \varphi\left(1-\beta\right)\sum_t \beta^t \int \frac{E}{\sigma e(i) + Wn(i)\psi}\left\{\tilde{\mathcal{I}}_t(i) - \left(1-\beta\right)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*\right\}^2 di.$$

We therefore obtain

$$\mathcal{L} = \mathcal{L}^{\tilde{\mathcal{Y}}} + \mathcal{L}^{\pi,\text{within}} + \mathcal{L}^{\pi,\text{between}} + \mathcal{L}^{\text{redist}}$$

as in the main text.

Proof of Proposition 3

Consider a transitory productivity shock in necessity sector k (with $\bar{s_k} > \overline{\partial_e e_k}$), $-\tilde{A}_{k,t}$ with $\tilde{A}_{k,0} > 0$, $\tilde{A}_{k,t} = 0$ for $t > 0$. Consider in addition a transitory subsidy to the same sector $\hat{Q}_{k,t} = \hat{P}_{k,t} - \tau_{k,t}$ with $\tau_{k,0} > 0$ and $\tau_{k,t} = 0$ for $t > 0$. In this proof, $\tilde{A}_{k,t}$ denotes the size of the negative productivity shock, i.e. $\hat{A}_{k,t} = -\tilde{A}_{k,t}$, so that the terms $\hat{Q}_{k,t} + \hat{A}_{k,t}$ of the loss function read $\hat{Q}_{k,t} - \tilde{A}_{k,t}$; l denotes any sector $l \neq k$. We consider a small subsidy: for each component of the loss we compute $\Delta\mathcal{L}$ exactly and sign it to first order in $\tau_{k,0}$, i.e. we sign $\partial\Delta\mathcal{L}/\partial\tau_{k,0}$ at $\tau_{k,0} = 0$. When the sign holds for any $\tau_{k,0} > 0$ we say so, and otherwise we report the exact condition on $\tau_{k,0}$. We have:

$$\begin{aligned}
\tilde{\mathcal{Y}}_t &= -\sigma\left(\phi_y\tilde{\mathcal{Y}}_t + \phi_\pi\pi_{cpi,t} - \mathbb{E}_t\pi_{cpi,t+1} + \mathbb{E}_t\left(\pi_{cpi,t+1} - \pi_{mcpi,t+1}\right) + \overline{\partial_e e_k}\left(\mathbb{E}_t\tau_{k,t+1} - \tau_{k,t}\right)\right) + \mathbb{E}_t\tilde{\mathcal{Y}}_{t+1} \\
\pi_{cpi,t} &= \lambda\left(\left(\sigma^{-1}+\psi^{-1}\right)\tilde{\mathcal{Y}}_t + \mathcal{NH}_t - \overline{\partial_e e_k}\tau_{k,t}\right) + \beta\mathbb{E}_t\pi_{cpi,t+1} \\
\pi_{k,t} - \pi_{cpi,t} &= -\lambda\left(\hat{P}_{k,t} - \hat{P}_{cpi,t} - \left(1-\bar{s_k}\right)\tilde{A}_{k,t}\right) + \beta\mathbb{E}_t\left(\pi_{k,t+1} - \pi_{cpi,t+1}\right) \\
\pi_{l,t} - \pi_{cpi,t} &= -\lambda\left(\hat{P}_{l,t} - \hat{P}_{cpi,t} + \bar{s_k}\tilde{A}_{k,t}\right) + \beta\mathbb{E}_t\left(\pi_{l,t+1} - \pi_{cpi,t+1}\right)
\end{aligned}$$

where the non-homotheticity wedge is $\mathcal{NH}_t = \left(\bar{s_k} - \overline{\partial_e e_k}\right)\left(\tilde{A}_{k,t} - \left(\hat{P}_{k,t} - \hat{P}_{l,t}\right)\right)$.

Note first from the last two equations that relative producer prices are independent from the subsidy. Defining:

$$\mu_\pm = \frac{R\left(1+\lambda\right) + 1 \pm \sqrt{\left(R\left(1+\lambda\right)-1\right)^2 + 4\lambda R}}{2},$$

with $0 < \mu_- < 1 < \mu_+$, we directly have

$$\begin{aligned}
\hat{P}_{k,t} - \hat{P}_{cpi,t} &= \left(1-\bar{s_k}\right)\lambda\mu_-^{t+1}\tilde{A}_{k,0} > 0 \\
\hat{P}_{l,t} - \hat{P}_{cpi,t} &= -\bar{s_k}\lambda\mu_-^{t+1}\tilde{A}_{k,0} < 0 \\
\hat{P}_{k,0} - \hat{P}_{cpi,0} - \tilde{A}_{k,0} + \tilde{A}_{cpi,0} &= \left(1-\bar{s_k}\right)\left(\lambda\mu_- - 1\right)\tilde{A}_{k,0} < 0 \\
\hat{P}_{l,0} - \hat{P}_{cpi,0} - \tilde{A}_{l,0} + \tilde{A}_{cpi,0} &= -\bar{s_k}\left(\lambda\mu_- - 1\right)\tilde{A}_{k,0} > 0
\end{aligned}$$

both with and without the price subsidy. In particular $\hat{P}_{k,t} - \hat{P}_{l,t} = \lambda\mu_-^{t+1}\tilde{A}_{k,0}$, so that $\mathcal{NH}_0 = \left(\bar{s_k} - \overline{\partial_e e_k}\right)\left(1-\lambda\mu_-\right)\tilde{A}_{k,0}$ and $\mathcal{NH}_t = -\left(\bar{s_k} - \overline{\partial_e e_k}\right)\lambda\mu_-^{t+1}\tilde{A}_{k,0}$ for $t \geq 1$. To understand the signs of the expressions, note that we have:

$$\begin{aligned}
\mu_-^2 - \left(R\left(1+\lambda\right)+1\right)\mu_- + R &= 0 \\
\left(\lambda\mu_- - 1\right) &= -\frac{\mu_-}{R}\left(R + 1 - \mu_-\right) < 0
\end{aligned}$$

since $\mu_- < 1$. With this derivation we can characterize:

$$\Delta\mathcal{L}^{\pi,\text{between}} = (1-\beta)\sum\beta^t\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}-\tilde{A}_{l,t}\right)^{\text{fiscal}}$$
$$-(1-\beta)\sum\beta^t\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{P}_{k,t}-\tilde{A}_{k,t}\right)\left(\hat{P}_{l,t}-\tilde{A}_{l,t}\right)^{\text{no fiscal}}.$$

First using $\sum_k \bar{s}_k\bar{\rho}_{k,l}=0$ and $\sum_l \bar{\rho}_{k,l}=0$ (using homogeneity of degree 0 and symmetry of Hicksian demand function), we have:

$$\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t}-\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}-\tilde{A}_{l,t}\right) =$$
$$\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,t}-\tilde{A}_{k,t}-\left(\hat{Q}_{cpi,t}-\tilde{A}_{cpi,t}\right)\right)\left(\hat{Q}_{l,t}-\tilde{A}_{l,t}-\left(\hat{Q}_{cpi,t}-\tilde{A}_{cpi,t}\right)\right)$$

$$\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{P}_{k,t}-\tilde{A}_{k,t}\right)\left(\hat{P}_{l,t}-\tilde{A}_{l,t}\right) =$$
$$\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{P}_{k,t}-\tilde{A}_{k,t}-\left(\hat{P}_{cpi,t}-\tilde{A}_{cpi,t}\right)\right)\left(\hat{P}_{l,t}-\tilde{A}_{l,t}-\left(\hat{P}_{cpi,t}-\tilde{A}_{cpi,t}\right)\right)$$

we therefore have:

$$\Delta\mathcal{L}^{\pi,\text{between}} = (1-\beta)\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{Q}_{k,0}-\tilde{A}_{k,0}-\left(\hat{Q}_{cpi,0}-\tilde{A}_{cpi,0}\right)\right)\left(\hat{Q}_{l,0}-\tilde{A}_{l,0}-\left(\hat{Q}_{cpi,0}-\tilde{A}_{cpi,0}\right)\right)$$
$$-(1-\beta)\sum_k \bar{s}_k\sum_l \bar{\rho}_{k,l}\left(\hat{P}_{k,0}-\tilde{A}_{k,0}-\left(\hat{P}_{cpi,0}-\tilde{A}_{cpi,0}\right)\right)\left(\hat{P}_{l,0}-\tilde{A}_{l,0}-\left(\hat{P}_{cpi,0}-\tilde{A}_{cpi,0}\right)\right),$$

since $\hat{P}_{l,t}-\hat{P}_{cpi,t}=\hat{Q}_{l,t}-\hat{Q}_{cpi,t}$ for all $l, t>0$. Using again the property of Hicksian demand functions, we can rewrite the loss as:

$$\begin{aligned}\Delta\mathcal{L}^{\pi,\text{between}} &= (1-\beta)\,\bar{s}_k\bar{\rho}_{k,k}\left(\left(\hat{Q}_{k,0}-\tilde{A}_{k,0}-\left(\hat{Q}_{l,0}-\tilde{A}_{l,0}\right)\right)^2-\left(\hat{P}_{k,0}-\tilde{A}_{k,0}-\left(\hat{P}_{l,0}-\tilde{A}_{l,0}\right)\right)^2\right)\\ &= (1-\beta)\,\bar{s}_k\bar{\rho}_{k,k}\left(\tau_{k,0}^2+2\tau_{k,0}\left(1-\lambda\mu_-\right)\tilde{A}_{k,0}\right)>0\end{aligned}$$

since $\bar{\rho}_{k,k}>0$. Therefore the fiscal intervention increases the loss due to misallocation across sectors, for any $\tau_{k,0}>0$ (not only to first order).
In addition, note that without wealth heterogeneity, the redistributive terms only depends on relative prices, indeed we have:

$$\mathcal{L}^{\text{redist}} = (1-\varphi)\int\frac{E}{\sigma e(i)+Wn(i)\psi}\left((1-\beta)\sum_t\beta^t\frac{e(i)}{E}\sum_l\left(s_l(i)-\bar{s}_l\right)\hat{Q}_{l,t}\right)^2 di$$
$$+\varphi(1-\beta)\sum_t\beta^t\int\frac{E}{\sigma e(i)+Wn(i)\psi}\left\{\frac{e(i)}{E}\sum_l\left(s_l(i)-\bar{s}_l\right)\hat{Q}_{l,t}\right\}^2 di$$

Using again that $\hat{P}_{l,t}-\hat{P}_{cpi,t}=\hat{Q}_{l,t}-\hat{Q}_{cpi,t}$ for all $l, t>0$, we have:

$$\begin{aligned}\Delta\mathcal{L}^{\text{redist}} &= (1-\varphi)\int\frac{E\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)^2}{\sigma e(i)+Wn(i)\psi}di\left\{\left((1-\beta)\left(-\tau_{k,0}+\frac{\lambda R\mu_-}{R-\mu_-}\tilde{A}_{k,0}\right)\right)^2-\left((1-\beta)\frac{\lambda R\mu_-}{R-\mu_-}\tilde{A}_{k,0}\right)^2\right\}\\ &+\varphi(1-\beta)\int\frac{E\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)^2}{\sigma e(i)+Wn(i)\psi}di\left\{\left(\left(-\tau_{k,0}+\lambda\mu_-\tilde{A}_{k,0}\right)\right)^2-\left(\lambda\mu_-\tilde{A}_{k,0}\right)^2\right\}\\ &= (1-\varphi)\int\frac{E\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)^2}{\sigma e(i)+Wn(i)\psi}di\left\{\left((1-\beta)\tau_{k,0}\right)^2-2(1-\beta)^2\tau_{k,0}\frac{\lambda R\mu_-}{R-\mu_-}\tilde{A}_{k,0}\right\}\\ &+\varphi(1-\beta)\int\frac{E\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)^2}{\sigma e(i)+Wn(i)\psi}di\left\{\tau_{k,0}^2-2\tau_{k,0}\left(\lambda\mu_-\tilde{A}_{k,0}\right)\right\}\end{aligned}$$

To first order in $\tau_{k,0}$, $\Delta\mathcal{L}^{\text{redist}}=-2(1-\beta)\int\frac{E\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)^2}{\sigma e(i)+Wn(i)\psi}di\left\{(1-\varphi)(1-\beta)\frac{\lambda R\mu_-}{R-\mu_-}+\varphi\lambda\mu_-\right\}\tilde{A}_{k,0}\tau_{k,0}<0$ as soon as consumption baskets are heterogeneous. Exactly, $\Delta\mathcal{L}^{\text{redist}}<0$ if and only if $\tau_{k,0}<\frac{2\left\{(1-\varphi)(1-\beta)\frac{\lambda R\mu_-}{R-\mu_-}+\varphi\lambda\mu_-\right\}}{(1-(1-\varphi)\beta)}\tilde{A}_{k,0}$. That is, the subsidy reduces the redistribution loss as long as it does not exceed twice a weighted average of the (current and discounted future) increases in the

relative price of k.
Next, we turn to the inflation loss. CPI inflation solves:

$$\mathbb{E}_t \pi_{cpi,t+2} - \left(1 + \sigma\phi_y + R\left(1 + \lambda\left(1 + \frac{\sigma}{\psi}\right)\right)\right)\mathbb{E}_t \pi_{cpi,t+1} + \left(1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right) R\pi_{cpi,t}$$
$$= R\lambda\left(\left(\frac{\sigma}{\psi} - \sigma\phi_y\right)\overline{\partial_e e}_k \tau_{k,t} - \frac{\sigma}{\psi}\mathbb{E}_t \overline{\partial_e e}_k \tau_{k,t+1}\right)$$
$$+ R\lambda\left(\left(1 + \sigma\phi_y\right)\mathcal{NH}_t - \mathbb{E}_t \mathcal{NH}_{t+1}\right) - R\lambda\left(1 + \frac{\sigma}{\psi}\right)\mathbb{E}_t\left(\pi_{cpi,t+1} - \pi_{mcpi,t+1}\right)$$

with

$$\pi_{cpi,t+1} - \pi_{mcpi,t+1} = -\left(\bar{s_k} - \overline{\partial_e e_k}\right)\lambda\left(1 - \mu_-\right)\mu_-^{t+1}\tilde{A}_{k,0}, \quad t \geq 0.$$

The roots of the associated characteristic polynomial $P(x)$ (defined in the proof of Proposition A1) still are (they may be real or complex; we only use $|\rho_\pm| > 1$, which holds under determinacy):

$$\rho_\pm = \frac{\left(1 + \sigma\phi_y + R\left(1 + \lambda\left(1 + \frac{\sigma}{\psi}\right)\right)\right) \pm \sqrt{\left(\sigma\phi_y + R\left(1 + \lambda\left(1 + \frac{\sigma}{\psi}\right)\right) - 1\right)^2 - 4R\left(\left(1 - \frac{1}{R}\right)\sigma\phi_y - \lambda\left(1 + \frac{\sigma}{\psi}\right)\left(1 - \phi_\pi\right)\right)}}{2}$$

Using the same step as in the proof of Proposition 1, we have:

$$\pi_{cpi,0} = \left\{\frac{R\lambda^2\frac{\sigma}{\psi}}{\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right)}\left(1 - \psi\phi_y - \mu_-\right)\mu_- + \frac{\lambda\left(1 + \sigma\phi_y\right)}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k} - \overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$- \frac{\lambda\frac{\sigma}{\psi}}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y - 1\right)\overline{\partial_e e}_k \tau_{k,0}$$
$$\pi_{cpi,t} = \frac{R\lambda^2\frac{\sigma}{\psi}}{\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right)}\left(1 - \psi\phi_y - \mu_-\right)\mu_-^{t+1}\left(\bar{s_k} - \overline{\partial_e e_k}\right)\tilde{A}_{k,0}.$$

For a transitory subsidy, $\pi_{cpi,t}$ only depends on the productivity shock for $t > 0$, therefore we have:

$$\Delta\mathcal{L}^{\pi,\text{within}} = \left(1 - \beta\right)\frac{\bar{\epsilon}}{\lambda}\left(\left(\pi_{cpi,0}^{fiscal}\right)^2 - \left(\pi_{cpi,0}^{nofiscal}\right)^2\right)$$
$$= \left(1 - \beta\right)\frac{\bar{\epsilon}}{\lambda}\left(\frac{\lambda\frac{\sigma}{\psi}}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y - 1\right)\overline{\partial_e e}_k \tau_{k,0}\right)^2$$
$$-2\left(1 - \beta\right)\frac{\bar{\epsilon}}{\lambda}\frac{\lambda\frac{\sigma}{\psi}}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y - 1\right)\overline{\partial_e e}_k \tau_{k,0}\left\{\frac{R\lambda^2\frac{\sigma}{\psi}}{\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right)}\left(1 - \psi\phi_y - \mu_-\right)\mu_- + \frac{\lambda\left(1 + \sigma\phi_y\right)}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k} - \overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

To characterize the inflation loss, we need to sign the term $\frac{R\lambda^2\frac{\sigma}{\psi}}{\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right)}\left(1 - \psi\phi_y - \mu_-\right)\mu_- + \frac{\lambda\left(1 + \sigma\phi_y\right)}{1 + \sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi}$. Using the characteristic equations determining μ_- and ρ_+, ρ_-, we have:

$$\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right) = -\left(\sigma\phi_y + R\lambda\frac{\sigma}{\psi}\right)\mu_- + \left(\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)R > 0$$

where the inequality holds because $\left(\rho_+ - \mu_-\right)\left(\rho_- - \mu_-\right) = P\left(\mu_-\right)$, which is positive since either the roots are real and larger than

one while $\mu_- < 1$, or the roots are complex and $P > 0$ on the real line. We then have:

$$\begin{aligned}
&\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\mu_- + \frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi} \\
&\quad= \frac{R\lambda^2\frac{\sigma}{\psi}\left(1-\psi\phi_y-\mu_-\right)\mu_-\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)+\lambda\left(1+\sigma\phi_y\right)\left(\sigma\phi_y\left(R-\mu_-\right)+R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi-R\lambda\frac{\sigma}{\psi}\mu_-\right)}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\quad= \frac{\lambda\left(\sigma\phi_y\right)^2\left(R-\mu_--R\lambda\mu_-\right)+R\lambda\sigma\phi_y\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\left(1-\lambda\mu_-\right)+\sigma\phi_y\lambda\left(R-\mu_--R\lambda\left(\mu_-+\frac{\sigma}{\psi}\mu_-^2\right)\right)}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad+\frac{\left(\lambda^2\frac{\sigma}{\psi}\left(1-\mu_-\right)\mu_-+\lambda\right)R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi-R\lambda^2\frac{\sigma}{\psi}\mu_-^2}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\quad= \frac{\lambda\left(\sigma\phi_y\right)^2\left(R-\mu_-\right)\mu_-+\lambda\sigma\phi_y\lambda\frac{\sigma+\psi}{\psi}\phi_\pi R\mu_--\sigma\phi_y\lambda R\lambda\left(\mu_-+\frac{\sigma}{\psi}\mu_-^2\right)}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad+\frac{\lambda\sigma\phi_y\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\left(\left(1-\mu_-\right)\mu_-\right)+\left(\lambda^2\frac{\sigma}{\psi}\left(1-\mu_-\right)\mu_-\right)R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad+\frac{\sigma\phi_y\lambda\left(R-\mu_-\right)+R\lambda^2\frac{\sigma+\psi}{\psi}\phi_\pi-R\lambda^2\frac{\sigma}{\psi}\mu_-^2}{(\rho_+-\mu_-)(\rho_--\mu_-)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}
\end{aligned}$$

Under the assumption that the rule satisfies determinacy, we have $\left(1-\frac{1}{R}\right)\sigma\phi_y+\lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi > \lambda\left(1+\frac{\sigma}{\psi}\right)$. Since $\mu_- < 1$, $\phi_y, \phi_\pi \geq 0$ we therefore have:

$$\begin{aligned}
&\lambda\left(\sigma\phi_y\right)^2\left(R-\mu_-\right)\mu_-+\lambda\sigma\phi_y\lambda\frac{\sigma+\psi}{\psi}\phi_\pi R\mu_--\sigma\phi_y\lambda R\lambda\left(\mu_-+\frac{\sigma}{\psi}\mu_-^2\right) \\
&\qquad\geq \lambda\sigma\phi_y R\mu_-\left(\left(1-\frac{1}{R}\right)\sigma\phi_y+\lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi-\lambda\left(1+\frac{\sigma}{\psi}\right)\right)>0
\end{aligned}$$

$$\begin{aligned}
&\sigma\phi_y\lambda\left(R-\mu_-\right)+R\lambda^2\frac{\sigma+\psi}{\psi}\phi_\pi-R\lambda^2\frac{\sigma}{\psi}\mu_-^2 \\
&\qquad\geq R\lambda\left(\left(1-\frac{1}{R}\right)\sigma\phi_y+\lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi-\lambda\left(1+\frac{\sigma}{\psi}\right)\right)>0
\end{aligned}$$

$$\lambda\sigma\phi_y\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\left(\left(1-\mu_-\right)\mu_-\right)+\left(\lambda^2\frac{\sigma}{\psi}\left(1-\mu_-\right)\mu_-\right)R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi>0.$$

These derivations shows that under any policy rules satisfying determinacy, inflation increases at $t=0$ following a shock in a necessity sector (without fiscal intervention). We therefore have that

$$\left\{\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\mu_-+\frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}>0$$

which implies that, to first order in $\tau_{k,0}$,

$$\Delta\mathcal{L}^{\pi,\text{within}}=-2\left(1-\beta\right)\frac{\bar{\epsilon}}{\lambda}\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y-1\right)\overline{\partial_e e}_k\tau_{k,0}\,\pi_{cpi,0}^{nofiscal},$$

so that, for a small subsidy, $\Delta\mathcal{L}^{\pi,\text{within}}>0$ if and only if $\psi\phi_y<1$ (and $\Delta\mathcal{L}^{\pi,\text{within}}=0$ if $\psi\phi_y=1$). Exactly, $\Delta\mathcal{L}^{\pi,\text{within}}>0$ for any

$\tau_{k,0} > 0$ if $\psi\phi_y < 1$, while if $\psi\phi_y > 1$ we have $\Delta\mathcal{L}^{\pi,\text{within}} < 0$ if and only if

$$0 < \tau_{k,0} < \frac{2\left\{\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\mu_- + \frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}}{\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\psi\phi_y-1\right)\overline{\partial_e e_k}}.$$

We now turn to the output gap losses. Having solved for the path of inflation direct algebra gives:

$$\begin{aligned}\tilde{\mathcal{Y}}_0 &= \sigma\lambda\phi_\pi\left\{\frac{R\lambda\mu_-}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)} - \frac{1}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\\ &\quad+\frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}\\ \tilde{\mathcal{Y}}_t &= \frac{R\lambda^2\sigma\phi_\pi}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}\mu_-^{t+1}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\end{aligned}$$

Therefore, the output gap loss is

$$\begin{aligned}\Delta\mathcal{L}^{\tilde{\mathcal{Y}}} &= \left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\left(1-\beta\right)\left(1-\eta\varphi\right)\left(\left(\tilde{\mathcal{Y}}_0^{fiscal}\right)^2-\left(\tilde{\mathcal{Y}}_0^{nofiscal}\right)^2\right)\\ &\quad-\eta\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\left(1-\varphi\right)\left(\left(\left(1-\beta\right)\sum_t\beta^t\tilde{\mathcal{Y}}_t^{fiscal}\right)^2-\left(\left(1-\beta\right)\sum_t\beta^t\tilde{\mathcal{Y}}_t^{nofiscal}\right)^2\right)\end{aligned}$$

It is direct to show that $\eta < 1$: $\eta = \int\frac{e(i)}{E}\frac{\sigma\left(1-\beta\right)b(i)}{\sigma e(i)+Wn(i)\psi}di$ and $\sigma\left(1-\beta\right)b\left(i\right) = \sigma\left(e\left(i\right)-Wn\left(i\right)\right) < \sigma e\left(i\right)+\psi Wn\left(i\right)$ since $Wn\left(i\right) > 0$. This implies that $A \equiv \left(1-\beta\right)\left(1-\eta\varphi\right)-\left(1-\varphi\right)\eta\left(1-\beta\right)^2 \geq \left(1-\beta\right)\left(1-\eta\right) > 0$. Note first that:

$$\begin{aligned}&\tilde{\mathcal{Y}}_0^{nofiscal}\\ &= \sigma\lambda\phi_\pi\left\{\frac{R\lambda\mu_-}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)} - \frac{1}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right\}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\\ &= -\sigma\lambda\phi_\pi\left\{\frac{\left(\left(1-\frac{1}{R}\right)\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)R\left(1-\lambda\mu_-\right)-R\lambda\mu_-\left(1+\frac{\sigma}{\psi}\right)+\sigma\phi_y\left(1-\lambda\mu_--\mu_-\right)}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right\}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\\ &\leq -\sigma\lambda\phi_\pi\left(1-\lambda\mu_--\mu_-\right)\frac{R\lambda\frac{\sigma+\psi}{\psi}+\sigma\phi_y}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \leq 0\end{aligned}$$

where the last line uses again $\left(1-\frac{1}{R}\right)\sigma\phi_y+\lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi > \lambda\left(1+\frac{\sigma}{\psi}\right)$ and $\left(1-\lambda\mu_--\mu_-\right) = \frac{\mu_-}{R}\left(1-\mu_-\right) > 0$. Hence $\tilde{\mathcal{Y}}_0^{nofiscal} < 0$ if $\phi_\pi > 0$, while if $\phi_\pi = 0$ the necessity shock does not move the output gap, $\tilde{\mathcal{Y}}_t^{nofiscal} = 0$ for all t. In addition, we have:

$$\tilde{\mathcal{Y}}_t^{nofiscal} = \frac{R\lambda^2\sigma\phi_\pi}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}\mu_-^{t+1}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

where $\sum_{t\geq1}\beta^t\tilde{\mathcal{Y}}_t^{nofiscal} = \frac{R\lambda^2\sigma\phi_\pi}{(\rho_+-\mu_-)(\rho_--\mu_-)}\frac{\mu_-^2}{R-\mu_-}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \geq 0$. Since the subsidy only affects $\tilde{\mathcal{Y}}_0$, we therefore have:

$$\begin{aligned}\Delta\mathcal{L}^{\tilde{\mathcal{Y}}} &= \left(\left(1-\beta\right)\left(1-\eta\varphi\right)-\left(1-\varphi\right)\eta\left(1-\beta\right)^2\right)\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}\\ &\quad\times\left(\frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}+2\tilde{\mathcal{Y}}_0^{nofiscal}\right)\\ &\quad-2\eta\left(1-\varphi\right)\left(1-\beta\right)^2\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0}\left(\frac{R\lambda^2\sigma\phi_\pi}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}\frac{\mu_-^2}{R-\mu_-}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\end{aligned}$$

To first order in $\tau_{k,0}$, the term in $\tau_{k,0}^2$ vanishes and $\Delta\mathcal{L}^{\tilde{\mathcal{Y}}}$ has the sign of $A\,\tilde{\mathcal{Y}}_0^{nofiscal}-\eta\left(1-\varphi\right)\left(1-\beta\right)^2\sum_{t\geq1}\beta^t\tilde{\mathcal{Y}}_t^{nofiscal} \leq 0$

(using $A > 0$, $\tilde{\mathcal{Y}}_0^{nofiscal} \leq 0$ and $\sum_{t\geq 1} \beta^t \tilde{\mathcal{Y}}_t^{nofiscal} \geq 0$), with strict inequality if and only if $\phi_\pi > 0$. So $\Delta\mathcal{L}^{\tilde{\mathcal{Y}}} < 0$ for $\tau_{k,0} > 0$ small enough if $\phi_\pi > 0$. If $\phi_\pi = 0$, $\tilde{\mathcal{Y}}_t^{nofiscal} = 0$ for all t and $\Delta\mathcal{L}^{\tilde{\mathcal{Y}}} > 0$ for any $\tau_{k,0} > 0$. For a finite subsidy

$$\begin{aligned}(\text{formally, we need} \quad & \left(\left(1-\beta\right)\left(1-\eta\varphi\right)-\left(1-\varphi\right)\eta\left(1-\beta\right)^2\right)\frac{\sigma\psi}{\sigma+\psi}\left(1-\frac{\sigma\phi_y-\frac{\sigma}{\psi}}{\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\right)\overline{\partial_e e}_k\tau_{k,0} \\ & \leq -2\left(\left(1-\beta\right)\left(1-\eta\varphi\right)-\left(1-\varphi\right)\eta\left(1-\beta\right)^2\right)\tilde{\mathcal{Y}}_0^{nofiscal} \\ & \quad + 2\eta\left(1-\varphi\right)\left(1-\beta\right)^2\left(\frac{R\lambda^2\sigma\phi_\pi}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}\frac{\mu_-^2}{R-\mu_-}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}.)\end{aligned}$$

Finally, we turn to the redistribution terms without consumption basket heterogeneity. We start with the welfare impact on unconstrained households:

$$\begin{aligned}\Delta\mathcal{L}^{\text{redist},u} = & \left(1-\varphi\right)\int\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\frac{b\left(i\right)}{E}\right)^2 di\left(\left(1-\beta\right)\sum_t\beta^{t+1}\left(\hat{R}_t^{fiscal}-\left(R-1\right)\left(\hat{Q}_{cpi,t}^{fiscal}+\mathcal{Y}_t^*\right)\right)\right)^2 \\ & -\left(1-\varphi\right)\int\frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\frac{b\left(i\right)}{E}\right)^2 di\left(\left(1-\beta\right)\sum_t\beta^{t+1}\left(\hat{R}_t^{nofiscal}-\left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)\right)^2\end{aligned}$$

Using our previous derivations and the policy rule, and noting that $r_0^* = \frac{\bar{s}_k+\psi\overline{\partial_e e_k}}{\sigma+\psi}\tilde{A}_{k,0}$ and $\mathcal{Y}_0^* = -\frac{\sigma\left(\bar{s}_k+\psi\overline{\partial_e e_k}\right)}{\sigma+\psi}\tilde{A}_{k,0}$ (both zero for $t \geq 1$), so that $r_0^* - (R-1)\mathcal{Y}_0^* > 0$, we have:

$$\begin{aligned}\sum_t\beta^{t+1}\left(\hat{R}_t^{nofiscal}-\left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right) = & \left(\frac{1}{R}+\sigma\left(1-\frac{1}{R}\right)\right)\frac{\bar{s_k}+\psi\overline{\partial_e e_k}}{\sigma+\psi}\tilde{A}_{k,0} \\ & +\lambda\left(\frac{\frac{\sigma}{\psi}\left(\phi_\pi-R\right)\left(1-\mu_-\right)^2+R\sigma\phi_y\left(1-\mu_-\right)}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}-\frac{1}{R}\frac{\left(R\left(1+\sigma\phi_y\right)-\phi_\pi\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}\end{aligned}$$

We first examine the term $\frac{\frac{\sigma}{\psi}\left(\phi_\pi-R\right)\left(1-\mu_-\right)^2+R\sigma\phi_y\left(1-\mu_-\right)}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)} - \frac{1}{R}\frac{\left(R\left(1+\sigma\phi_y\right)-\phi_\pi\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}$. Using again $\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right) = -R\lambda\frac{\sigma}{\psi}\mu_- + \left(R-\mu_-\right)\sigma\phi_y + R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi$, we have

$$\begin{aligned}& \frac{\frac{\sigma}{\psi}\left(\phi_\pi-R\right)\left(1-\mu_-\right)^2+R\sigma\phi_y\left(1-\mu_-\right)}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)}-\frac{1}{R}\frac{\left(R\left(1+\sigma\phi_y\right)-\phi_\pi\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi} \\ & = \frac{\phi_\pi\sigma\phi_y\left(R\frac{\sigma}{\psi}\left(1-\mu_-\right)^2+\left(R-\mu_-\right)-R^2\lambda\frac{\sigma+\psi}{\psi}\mu_-\right)+\phi_\pi^2\left(R\frac{\sigma}{\psi}\lambda\frac{\sigma+\psi}{\psi}\left(1-\mu_-\right)^2+R\lambda\frac{\sigma+\psi}{\psi}\right)-R\left(R-1\right)\mu_-\left(\sigma\phi_y\right)^2}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\ & \quad + \frac{\left(-R\left(R-1\right)\mu_-+R^2\frac{\sigma}{\psi}\left(R-1\right)\left(1-\mu_--\lambda\mu_-\right)\right)\sigma\phi_y+R^2\frac{\sigma}{\psi}\left(R-1\right)\left(1-\mu_--\lambda\mu_-\right)}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\ & \quad + \frac{\phi_\pi\left(\left(R\frac{\sigma}{\psi}-\frac{\sigma}{\psi}R^2\lambda\frac{\sigma+\psi}{\psi}\right)\left(1-\mu_-\right)^2-R^2\lambda\frac{\sigma+\psi}{\psi}-R\lambda\frac{\sigma}{\psi}\mu_-\right)}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\end{aligned}$$

where we used $\lambda\mu_- - (1-\mu_-)^2 = (R-1)(1-\mu_- - \lambda\mu_-) \geq 0$. Using $\left(1-\frac{1}{R}\right)\sigma\phi_y\phi_\pi + \lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi^2 > \lambda\left(1+\frac{\sigma}{\psi}\right)\phi_\pi$, we have:

$$\begin{aligned}
&\frac{\frac{\sigma}{\psi}(\phi_\pi - R)(1-\mu_-)^2 + R\sigma\phi_y(1-\mu_-)}{(\rho_+ - \mu_-)(\rho_- - \mu_-)} - \frac{1}{R}\frac{(R(1+\sigma\phi_y) - \phi_\pi)}{1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi} \\
&\qquad \geq \frac{\phi_\pi\sigma\phi_y\left(\frac{\sigma}{\psi}(1-\mu_-)^2 + (1-\mu_-) - R^2\lambda\frac{\sigma+\psi}{\psi}\mu_-\right) - R(R-1)\mu_-(\sigma\phi_y)^2}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad + \frac{\left(-R(R-1)\mu_- + R^2\frac{\sigma}{\psi}(R-1)(1-\mu_- - \lambda\mu_-)\right)\sigma\phi_y + R^2\frac{\sigma}{\psi}(R-1)(1-\mu_- - \lambda\mu_-)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad - \frac{\phi_\pi R(R-1)\left(\frac{\sigma}{\psi}\lambda\frac{\sigma+\psi}{\psi}(1-\mu_-)^2 + \lambda\frac{\sigma+\psi}{\psi} + \frac{\sigma}{\psi}(1-\mu_- - \lambda\mu_-)\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}
\end{aligned}$$

We therefore have:

$$\begin{aligned}
&\frac{\frac{\sigma}{\psi}(\phi_\pi - R)(1-\mu_-)^2 + R\sigma\phi_y(1-\mu_-)}{(\rho_+ - \mu_-)(\rho_- - \mu_-)} - \frac{1}{R}\frac{(R(1+\sigma\phi_y) - \phi_\pi)}{1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi} \\
&\qquad \geq -\frac{\phi_\pi R(R-1)\left(\frac{\sigma}{\psi}\lambda\frac{\sigma+\psi}{\psi}(1-\mu_-)^2 + \lambda\frac{\sigma+\psi}{\psi} + \frac{\sigma}{\psi}(1-\mu_- - \lambda\mu_-)\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad - \frac{\phi_\pi\sigma\phi_y\left(R(R-\mu_-)(1-\mu_-)\frac{\sigma+\psi}{\psi} - \frac{\sigma}{\psi}(1-\mu_-)^2 - (1-\mu_-)\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad - \frac{R(R-1)\mu_-\left((\sigma\phi_y)^2 + \sigma\phi_y\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad \geq -\frac{\phi_\pi R(R-1)\left(\frac{\sigma}{\psi}\lambda\frac{\sigma+\psi}{\psi}(1-\mu_-)^2 + \lambda\frac{\sigma+\psi}{\psi} + \frac{\sigma}{\psi}(1-\mu_- - \lambda\mu_-)\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)} \\
&\qquad - \frac{\phi_\pi\sigma\phi_y\left(R(R-1)(1-\mu_-) + (R-1)(1-\mu_-)^2\right)\frac{\sigma+\psi}{\psi} + \lambda^{-1}(R-1)(R-\mu_-)(1-\mu_-)\left((\sigma\phi_y)^2 + \sigma\phi_y\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}
\end{aligned}$$

where we used $(R-\mu_-)(1-\mu_-) = \lambda R\mu_-$.

Next we have:

$$\left(\frac{1}{R} + \sigma\left(1-\frac{1}{R}\right)\right)\frac{\bar{s_k} + \psi\overline{\partial_e e_k}}{\sigma+\psi}\tilde{A}_{k,0} \geq \frac{(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}{R(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\frac{\bar{s_k} - \overline{\partial_e e_k}}{\sigma+\psi}\tilde{A}_{k,0}$$

We have:

$$\begin{aligned}
&(\rho_+ - \mu_-)(\rho_- - \mu_-)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right) \\
&\qquad = \left(-R\lambda\frac{\sigma}{\psi}\mu_- + (R-\mu_-)\sigma\phi_y + R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)\left(1+\sigma\phi_y + \lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right) \\
&\qquad \geq R(1-\mu_-)\left(\sigma\phi_y + (\sigma\phi_y)^2\right) \\
&\qquad + (R+1)(1-\mu_-)\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\sigma\phi_y \\
&\qquad + R\lambda(1-\mu_-)\frac{\sigma+\psi}{\psi}\phi_\pi + R\lambda\frac{\sigma}{\psi}(1-\mu_-)\lambda\frac{\sigma+\psi}{\psi}\phi_\pi + R\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\lambda
\end{aligned}$$

where we repeatedly used $\lambda\frac{\sigma+\psi}{\psi}\phi_\pi + \left(1-\frac{1}{R}\right)\sigma\phi_y \geq \lambda\frac{\sigma+\psi}{\psi}$. Direct algebra then shows that if $\left(\sigma+\psi\right)\left(R-1\right)\leq 1$ then:

$$\begin{aligned}
&\left(\lambda\frac{\frac{\sigma}{\psi}\left(\phi_\pi - R\right)\left(1-\mu_-\right)^2 + R\sigma\phi_y\left(1-\mu_-\right)}{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)} - \lambda\frac{1}{R}\frac{\left(R\left(1+\sigma\phi_y\right)-\phi_\pi\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \\
&\qquad + \frac{\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\frac{\bar{s_k}-\overline{\partial_e e_k}}{\sigma+\psi}\tilde{A}_{k,0} \\
&\qquad \geq \frac{\phi_\pi R\lambda\left(\frac{\sigma}{\psi}\lambda\frac{\sigma+\psi}{\psi}\mu_-\left(1-\mu_-\right)+\frac{\sigma}{\psi}\lambda\mu_-\right)}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \\
&\qquad + \frac{\left(\left(\sigma\phi_y\right)^2+\sigma\phi_y\right)\left(R\left(1-\mu_-\right)-\left(R-\mu_-\right)\left(1-\mu_-\right)\right)}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \\
&\qquad + \frac{\lambda\left(\left(R+1\right)\left(1-\mu_-\right)-R\left(1-\mu_-\right)-\left(1-\mu_-\right)^2\right)\frac{\sigma+\psi}{\psi}\phi_\pi\sigma\phi_y}{R\left(\rho_+-\mu_-\right)\left(\rho_--\mu_-\right)\left(1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right)}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0} \geq 0
\end{aligned}$$

So we have $\sum_t \beta^{t+1}\left(\hat{R}_t^{nofiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal} + \mathcal{Y}_t^*\right)\right) > 0$. In addition we have:

$$\begin{aligned}
\sum_t \beta^{t+1}\left(\hat{R}_t^{fiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{fiscal} + \mathcal{Y}_t^*\right)\right) &= \sum_t \beta^{t+1}\left(\hat{R}_t^{nofiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal} + \mathcal{Y}_t^*\right)\right) \\
&\quad + \frac{1}{R}\frac{\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left\{\lambda\phi_\pi + \left(1+R\lambda\right)\psi\phi_y - R\lambda\right\}\overline{\partial_e e}_k\tau_{k,0} \\
&\quad + \left(1-\frac{1}{R}\right)\bar{s_k}\tau_{k,0}
\end{aligned}$$

Since k is a necessity sector, we have $\bar{s_k} \geq \overline{\partial_e e}_k$ so

$$\begin{aligned}
&\frac{1}{R}\frac{\frac{\sigma}{\psi}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left\{\lambda\phi_\pi + \left(1+R\lambda\right)\psi\phi_y - R\lambda\right\}\overline{\partial_e e}_k\tau_{k,0} + \left(1-\frac{1}{R}\right)\bar{s_k}\tau_{k,0} \\
&\qquad \geq \frac{1}{R}\frac{\frac{\sigma}{\psi}\left\{\lambda\phi_\pi + \left(1+R\lambda\right)\psi\phi_y - R\lambda\right\} + \left(R-1\right)\left\{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right\}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\overline{\partial_e e}_k\tau_{k,0} \\
&\qquad \geq \frac{1}{R}\frac{\frac{\sigma}{\psi}\left\{\left(\left(1+R\lambda\right)-\left(1-\frac{1}{R}\right)\frac{\sigma}{\psi+\sigma}\right)\psi\phi_y - \left(R-1\right)\lambda\right\} + \left(R-1\right)\left\{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi\right\}}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\overline{\partial_e e}_k\tau_{k,0} \geq 0
\end{aligned}$$

where both the second and last line use the fact the rule satisfies determinacy. Using

$$\sum_t \beta^{t+1}\left(\hat{R}_t^{nofiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal} + \mathcal{Y}_t^*\right)\right) > 0$$

and

$$\sum_t \beta^{t+1}\left(\hat{R}_t^{fiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{fiscal} + \mathcal{Y}_t^*\right)\right) - \sum_t \beta^{t+1}\left(\hat{R}_t^{nofiscal} - \left(R-1\right)\left(\hat{P}_{cpi,t}^{nofiscal} + \mathcal{Y}_t^*\right)\right) > 0,$$

we therefore have:

$$\boldsymbol{\Delta}\mathcal{L}^{\text{redist},\mathbf{u}} > 0$$

For the Hand-to-Mouth, the loss is

$$\begin{aligned}
\Delta\mathcal{L}^{\text{redist},HtM} &= \varphi\frac{\left(1-\beta\right)}{R^2}\int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\frac{b\left(i\right)}{E}\right)^2 di\left(\left(\hat{R}_0^{fiscal} - \left(R-1\right)\left(\hat{Q}_{cpi,0}^{fiscal} + \mathcal{Y}_0^*\right)\right)\right)^2 \\
&\quad - \varphi\frac{\left(1-\beta\right)}{R^2}\int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\frac{b\left(i\right)}{E}\right)^2 di\left(\hat{R}_0^{nofiscal} - \left(R-1\right)\left(\hat{P}_{cpi,0}^{nofiscal} + \mathcal{Y}_0^*\right)\right)^2 \\
&\quad + \varphi\frac{\left(1-\beta\right)}{R^2}\int \frac{E}{\sigma e\left(i\right)+Wn\left(i\right)\psi}\left(\frac{b\left(i\right)}{E}\right)^2 di\sum_{t\geq 1}\beta^t\left\{\left(\hat{R}_t^{fiscal} - \left(R-1\right)\hat{P}_{cpi,t}^{fiscal}\right)^2 - \left(\hat{R}_t^{nofiscal} - \left(R-1\right)\hat{P}_{cpi,t}^{nofiscal}\right)^2\right\}
\end{aligned}$$

So we need to sign:

$$\left\{\left(\left(\hat{R}_0^{fiscal}-(R-1)\left(\hat{Q}_{cpi,0}^{fiscal}+\mathcal{Y}_0^*\right)\right)\right)^2-\left(\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)\right)\right)^2\right\}$$
$$+\sum_{t\geq 1}\beta^t\left\{\left(\hat{R}_t^{fiscal}-(R-1)\hat{P}_{cpi,t}^{fiscal}\right)^2-\left(\hat{R}_t^{fiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}\right)\right)^2\right\}$$
$$=\left(\left(\hat{R}_0^{fiscal}-(R-1)\left(\hat{Q}_{cpi,0}^{fiscal}\right)\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}\right)\right)\right)^2$$
$$+2\left(\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)\right)\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)\right)$$
$$+(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)^2$$
$$-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)$$

where we used the fact that $\left(\hat{R}_t^{fiscal}-(R-1)\hat{P}_{cpi,t}^{fiscal}\right)-\left(\hat{R}_t^{nofiscal}-(R-1)\hat{P}_{cpi,t}^{nofiscal}\right)=-(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)$ for $t\geq 1$ and $\sum_{t\geq 1}\beta^t=1/(R-1)$, since both the nominal rate and inflation do not move in response to the subsidy for $t>0$ (and $\mathcal{Y}_t^*=0$ for $t\geq 1$). We have:

$$\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)$$
$$=r_0^*-(R-1)\mathcal{Y}_0^*+\left(\frac{R\lambda^2\left(\frac{\sigma}{\psi}\left(\phi_\pi-(R-1)\right)(1-\mu_-)+(R-1)\sigma\phi_y\right)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_-+\frac{\lambda\left(\phi_\pi-(R-1)\left(1+\sigma\phi_y\right)\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

$$\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)$$
$$=\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_-^{t+1}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$-(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\mu_-\frac{1-\mu_-^{t+1}}{1-\mu_-}+\frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

To sign $\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)$, we use our previous derivation. Note that:

$$\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)=r_0^*-(R-1)\mathcal{Y}_0^*+\frac{\lambda\phi_\pi}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$+\left(\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_--(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\mu_-\frac{1-\mu_-}{1-\mu_-}+\frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

Denote by a the second line of this expression (the bracket in parentheses times $\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$). If $a\geq 0$ then $\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)>0$, since the first line is positive. If $a<0$, we show below that $\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)<0$ for all $t\geq 1$; since $\sum_t\beta^{t+1}\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)>0$, this implies $\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)>0$. For $t\geq 1$ we have:

$$\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)$$
$$=\mu_-^t\left(\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_-+(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)\frac{\mu_-^2}{1-\mu_-}\right)\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$-(R-1)\left(\frac{\lambda\frac{\sigma}{\psi}(R-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\left(1-\psi\phi_y-\mu_-\right)+\frac{\lambda\left(1+\sigma\phi_y\right)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

We have

$$\frac{\lambda\frac{\sigma}{\psi}\left(R-\mu_{-}\right)}{\left(\rho_{+}-\mu_{-}\right)\left(\rho_{-}-\mu_{-}\right)}\left(1-\psi\phi_{y}-\mu_{-}\right)+\frac{\lambda\left(1+\sigma\phi_{y}\right)}{1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}}$$

$$=\frac{\lambda\left(\frac{\sigma}{\psi}\left(R-\mu_{-}\right)\left(1-\mu_{-}\right)\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}+R\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}+\mu_{-}\sigma\phi_{y}\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}\right)}{\left(\rho_{+}-\mu_{-}\right)\left(\rho_{-}-\mu_{-}\right)\left(1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}\right)}>0$$

Denote by $C>0$ this term times $\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$, and by α the coefficient multiplying μ_{-}^{t} in the expression for $X_t\equiv\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)$, $t\geq 1$, so that $X_t=\mu_{-}^{t}\alpha-(R-1)C$. Then $X_t+(R-1)C=\mu_{-}^{t-1}\left(X_1+(R-1)C\right)$ for $t\geq 1$, so that X_t lies between X_1 and $-(R-1)C<0$. Moreover, using $\lambda\left(R-\mu_{-}\right)\left(1-\mu_{-}\right)=\lambda^2R\mu_{-}$, direct algebra gives

$$X_1=\mu_{-}a-(R-1)\left(1-\mu_{-}\right)C<0$$

when $a<0$. Therefore $X_t<0$ for all $t\geq 1$ and $\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^{*}\right)>0$.

For the fiscal contribution we have;

$$\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)=\frac{1}{1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}}\left\{\frac{\sigma}{\psi}\lambda\phi_{\pi}+\left(1+(R-1)\lambda\right)\sigma\phi_{y}-\frac{\sigma}{\psi}(R-1)\lambda\right\}\overline{\partial_e e}_k\tau_{k,0}+(1$$

$$\left(\hat{R}_t^{fiscal}-(R-1)\hat{Q}_{cpi,t}^{fiscal}\right)-\left(\hat{R}_t^{nofiscal}-(R-1)\hat{P}_{cpi,t}^{nofiscal}\right)=-(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)$$

$$=(R-1)\frac{\lambda\frac{\sigma}{\psi}}{1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}}\left(\psi\phi_{y}-1\right)\overline{\partial_e e}_k\tau_{k,0}$$

First from our previous derivation, we directly have that:

$$\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)\geq 0$$

$$\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)-\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\geq 0$$

We therefore have:

$$\left(\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)\right)\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^{*}\right)\right)>0.$$

To sign the full expression, we need to take into account

$$-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right).$$

If $-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)>0$ then $\Delta\mathcal{L}^{\text{redist},HtM}>0$. So we focus on the case where $-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)<0$. We have two cases, either $\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)<0$ and $\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)<0$ or $\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)>0$ and $\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)>0$. In the first case note that we have:

$$\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)$$

$$=\frac{1}{1+\sigma\phi_{y}+\lambda\frac{\sigma+\psi}{\psi}\phi_{\pi}}\left\{\frac{\sigma}{\psi}\lambda\phi_{\pi}+\sigma\phi_{y}\right\}\overline{\partial_e e}_k\tau_{k,0}+(R-1)\bar{s_k}\tau_{k,0}$$

$$-(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)$$

$$>-(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)$$

So

$$+2\left(\left(\hat{R}_0^{fiscal}-(R-1)\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\hat{P}_{cpi,0}^{nofiscal}\right)\right)\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^{*}\right)\right)$$

$$-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)$$

$$\geq -2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 0}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^{*}\right)\right)\geq 0$$

where we used the fact that we previously showed $\sum_{t\geq 0}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)\geq 0$. In the second case, we have:

$$\left(\hat{R}_0^{fiscal}-(R-1)\,\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\,\hat{P}_{cpi,0}^{nofiscal}\right)>\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)$$

Since

$$\sum\beta^t\left(\hat{R}_t^{fiscal}-(R-1)\,\hat{Q}_{cpi,t}^{fiscal}\right)-\left(\hat{R}_t^{nofiscal}-(R-1)\,\hat{P}_{cpi,t}^{nofiscal}\right)$$
$$=\left(\hat{R}_0^{fiscal}-(R-1)\,\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\,\hat{P}_{cpi,0}^{nofiscal}\right)-\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\geq 0.$$

and

$$\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)\geq(R-1)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)$$

Indeed we have:

$$(R-1)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)$$
$$=-(R-1)\left(\frac{\lambda\frac{\sigma}{\psi}(R-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}(1-\psi\phi_y-\mu_-)+\frac{\lambda(1+\sigma\phi_y)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$+\frac{\mu_-(R-1)}{R-\mu_-}\left(\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)\mu_-}{(\rho_+-\mu_-)(\rho_--\mu_-)}+(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}(1-\psi\phi_y-\mu_-)\frac{\mu_-^2}{1-\mu_-}\right)\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

$$\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)$$
$$=-(R-1)\left(\frac{\lambda\frac{\sigma}{\psi}(R-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}(1-\psi\phi_y-\mu_-)+\frac{\lambda(1+\sigma\phi_y)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$+\left(\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_-+(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}(1-\psi\phi_y-\mu_-)\frac{\mu_-^2}{1-\mu_-}\right)\right)\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$
$$+r_0^*-(R-1)\,\mathcal{Y}_0^*+\frac{\lambda\phi_\pi}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}$$

Under the assumption that $(R-1)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)>0$, we necessarily have:

$$\frac{R\lambda^2\frac{\sigma}{\psi}\phi_\pi(1-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}\mu_-+(R-1)\left(\frac{R\lambda^2\frac{\sigma}{\psi}}{(\rho_+-\mu_-)(\rho_--\mu_-)}(1-\psi\phi_y-\mu_-)\frac{\mu_-^2}{1-\mu_-}\right)\geq 0,$$

since $\frac{\lambda\frac{\sigma}{\psi}(R-\mu_-)(1-\psi\phi_y-\mu_-)}{(\rho_+-\mu_-)(\rho_--\mu_-)}+\frac{\lambda(1+\sigma\phi_y)}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}>0$ (shown above). Therefore, since $\frac{\mu_-(R-1)}{R-\mu_-}<1$ (as $\mu_-<1$) and $r_0^*-(R-1)\,\mathcal{Y}_0^*+\frac{\lambda\phi_\pi}{1+\sigma\phi_y+\lambda\frac{\sigma+\psi}{\psi}\phi_\pi}\left(\bar{s_k}-\overline{\partial_e e_k}\right)\tilde{A}_{k,0}>0$, comparing the two expressions above term by term we have $\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)\geq(R-1)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)$. We therefore have:

$$2\left(\left(\hat{R}_0^{fiscal}-(R-1)\,\hat{Q}_{cpi,0}^{fiscal}\right)-\left(\hat{R}_0^{nofiscal}-(R-1)\,\hat{P}_{cpi,0}^{nofiscal}\right)\right)\left(\hat{R}_0^{nofiscal}-(R-1)\left(\hat{P}_{cpi,0}^{nofiscal}+\mathcal{Y}_0^*\right)\right)$$
$$-2(R-1)\left(\hat{P}_{cpi,0}^{fiscal}-\hat{P}_{cpi,0}^{nofiscal}\right)\sum_{t\geq 1}\beta^t\left(\hat{R}_t^{nofiscal}-(R-1)\left(\hat{P}_{cpi,t}^{nofiscal}+\mathcal{Y}_t^*\right)\right)\geq 0$$

which proves that $\Delta\mathcal{L}^{\text{redist},HtM}>0$. Since both $\Delta\mathcal{L}^{\text{redist},u}>0$ and $\Delta\mathcal{L}^{\text{redist},HtM}>0$ hold for any $\tau_{k,0}>0$, the result on redistribution without consumption basket heterogeneity does not require the subsidy to be small.

Summary. For a small transitory subsidy $\tau_{k,0}>0$: $\Delta\mathcal{L}^{\pi,\text{between}}>0$ (for any $\tau_{k,0}>0$); $\Delta\mathcal{L}^{\pi,\text{within}}>0$ if and only if $\psi\phi_y<1$; $\Delta\mathcal{L}^{\tilde{\mathcal{Y}}}<0$ if $\phi_\pi>0$; $\Delta\mathcal{L}^{\text{redist}}<0$ without wealth heterogeneity; and $\Delta\mathcal{L}^{\text{redist}}>0$ without consumption basket heterogeneity (for any $\tau_{k,0}>0$, under $(\sigma+\psi)(R-1)\leq 1$).

Proof of Proposition 4

We now derive optimal joint monetary and fiscal policy in our simple case, assuming that the Planner has access to sector specific consumption subsidies and type specific transfers. Recall that the loss function is given by:

$$\mathcal{L} = (1-\beta)\frac{\bar{\epsilon}}{\lambda}\sum\beta^t\left(\pi_{cpi,t}\right)^2 + (1-\beta)\sum\beta^t\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)$$
$$+\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\left\{(1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2 - \eta\left(\varphi(1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2 + (1-\varphi)\left((1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t\right)^2\right)\right\}$$
$$+(1-\varphi)\int\frac{E}{\sigma e(i)+Wn(i)\psi}\left((1-\beta)\sum_t\beta^t\left(\tilde{I}_t(i)-(1-\beta)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*\right)\right)^2 di$$
$$+\varphi(1-\beta)\sum_t\beta^t\int\frac{E}{\sigma e(i)+Wn(i)\psi}\left\{\tilde{I}_t(i)-(1-\beta)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*\right\}^2$$

With:

$$\tilde{\mathcal{Y}}_t = -\sigma\left(\hat{R}_t - r_t^* - \mathbb{E}_t\pi_{cpi,t+1} + \mathbb{E}_t\left(\pi_{cpi,t+1}-\pi_{mcpi,t+1}\right) + \sum_k\overline{\partial_e e_k}\left(\mathbb{E}_t\tau_{k,t+1}-\tau_{k,t}\right)\right) + \mathbb{E}_t\tilde{\mathcal{Y}}_{t+1}$$
$$\pi_{cpi,t} = \lambda\left((\sigma^{-1}+\psi^{-1})\tilde{\mathcal{Y}}_t + \mathcal{NH}_t - \sum_k\overline{\partial_e e_k}\tau_{k,t}\right) + \beta\mathbb{E}_t\pi_{cpi,t+1}$$

We first show that $\mathcal{L}\geq 0$. First the matrix $\left[\bar{s}_k\bar{\rho}_{k,l}\right]_{k,l}$ is symmetric and positive semi-definite (it is minus the expenditure-weighted matrix of compensated price elasticities, and the Slutsky matrix of Hicksian demand functions is symmetric negative semi-definite). In addition recall that we have:

$$\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}=0.$$

Next recall that we have $\eta<1$. A direct application of Jensen inequality gives:

$$\left((1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t\right)^2 \leq (1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2$$

Therefore

$$\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\left\{(1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2 - \eta\left(\varphi(1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2 + (1-\varphi)\left((1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t\right)^2\right)\right\}$$
$$\geq\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)(1-\eta)(1-\beta)\sum_t\beta^t\tilde{\mathcal{Y}}_t^2\geq 0.$$

Since the other terms are squares, we therefore have $\mathcal{L}\geq 0$. Therefore any policy implementing $\mathcal{L}=0$ is optimal. Note first that:

$$\tilde{I}_t(i)-(1-\beta)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^* = \frac{b(i)}{RE}\hat{R}_t - \sum_k\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)\hat{Q}_{k,t} - \left(1-\frac{1}{R}\right)\frac{b(i)}{E}\left(\sum_k\bar{s}_k\hat{Q}_{k,t}+\tilde{\mathcal{Y}}_t^*\right)+\zeta(i)$$

where $\zeta(i)$ is a lumpsum transfer. Consider the transfers:

$$\zeta(i) = -\left(\frac{b(i)}{RE}\hat{R}_t - \sum_k\left(\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)\right)\hat{Q}_{k,t} - \left(1-\frac{1}{R}\right)\frac{b(i)}{E}\left(\sum_k\bar{s}_k\hat{Q}_{k,t}+\tilde{\mathcal{Y}}_t^*\right)\right)$$

they imply

$$\tilde{I}_t(i)-(1-\beta)\frac{b(i)}{E}\tilde{\mathcal{Y}}_t^*=0$$
$$\int\zeta(i)\,di=0$$

since $b(i)$ is in zero net supply and $\int\frac{e(i)}{E}\left(s_k(i)-\bar{s}_k\right)di=0$, so the transfers are implementable (as in Proposition 2, $\zeta(i)$ is net of the lump-sum taxes, proportional to labor income, that finance the subsidies $\tau_{k,t}$) for any $\hat{Q}_{k,t},\hat{R}_t,\tilde{\mathcal{Y}}_t^*$. Next we have $\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)=0$ if $\hat{Q}_{k,t}+\tilde{A}_{k,t}=\hat{Q}_{l,t}+\tilde{A}_{l,t}$ for all l,k (since $\sum_l\bar{\rho}_{k,l}=0$), and only if when the matrix $\left[\bar{s}_k\bar{\rho}_{k,l}\right]$ has rank $K-1$. Therefore implementing

$$\tau_{k,t}=\hat{P}_{k,t}+\tilde{A}_{k,t}+\tau_t$$

gives $\sum_k\bar{s}_k\sum_l\bar{\rho}_{k,l}\left(\hat{Q}_{k,t}+\tilde{A}_{k,t}\right)\left(\hat{Q}_{l,t}+\tilde{A}_{l,t}\right)=0$ for any sequence $\hat{P}_{k,t},\tilde{A}_{k,t},\tau_t$. Finally to implement $\tilde{\mathcal{Y}}_t=\pi_{cpi,t}=0$ it is enough

to have:

$$\mathcal{NH}_t - \sum_k \overline{\partial_e e_k} \tau_{k,t} = 0$$

$$\hat{R}_t - r_t^* - \mathbb{E}_t \left(\pi_{mcpi,t+1} \right) + \sum_k \overline{\partial_e e_k} \left(\mathbb{E}_t \tau_{k,t+1} - \tau_{k,t} \right) = 0$$

which can be implemented with

$$\tau_t = \mathcal{NH}_t - \sum_k \overline{\partial_e e_k} \left(\hat{P}_{k,t} + \tilde{A}_{k,t} \right)$$

$$\hat{R}_t = r_t^* + \mathbb{E}_t \left(\pi_{mcpi,t+1} \right) - \sum_k \overline{\partial_e e_k} \left(\mathbb{E}_t \tau_{k,t+1} - \tau_{k,t} \right),$$

where the expression for τ_t uses $\sum_k \overline{\partial_e e_k} \tau_{k,t} = \sum_k \overline{\partial_e e_k} \left(\hat{P}_{k,t} + \tilde{A}_{k,t} \right) + \tau_t$. To ensure that $\tilde{\boldsymbol{y}}_t = \pi_{cpi,t} = 0$ is the unique equilibrium, the interest rate rule can be augmented with feedback terms that are zero in equilibrium, e.g. $+\phi_\pi \pi_{cpi,t} + \phi_y \tilde{\boldsymbol{y}}_t$ with (ϕ_π, ϕ_y) satisfying the determinacy condition. This achieves the proof.

E Empirical and Quantitative Details

E.1 Estimation of Welfare Weights

This section explains how individual welfare weights are recovered from UK data. It first lays out the assumptions needed to derive an identifying equation. It then details the data construction and each of the estimation steps. It concludes by presenting the empirical weights and comparing them with those implied by a linear social welfare function.

Identification approach. We estimate empirical welfare weights from observed business-cycle variations of pre and post-tax income at the household level. Intuitively, when the social function assigns a high welfare weight to a given household, transfers adjust following shocks to smooth their real income hit. The degree of observed smoothing in the data depends on: (a) how finely targeted household-level transfers can be, and (b) the degree to which shocks create a trade-off between redistribution and efficiency (captured in our 'Bang for buck' term). For these reasons, a structural approach is required to estimating these welfare weights from observed data.

We assume that following an unanticipated shock in period t, the government can make lump-sum transfers $\tau_{t+s}(\omega)$ to group ω in all future periods $s \geq 0$. The granularity of these groups captures how targeted the transfers can be. Let $\hat{\theta}_t(i)$ denote the change in labour productivity at the household level, which captures both changes in aggregate productivity and a potentially household-specific incidence of the aggregate shock. We assume that all households that belong to a given group ω, experience the same productivity shock $\hat{\theta}_t(\omega)$. The optimality condition for group-specific transfer $\tau_{t+s}(\omega)$ is given by

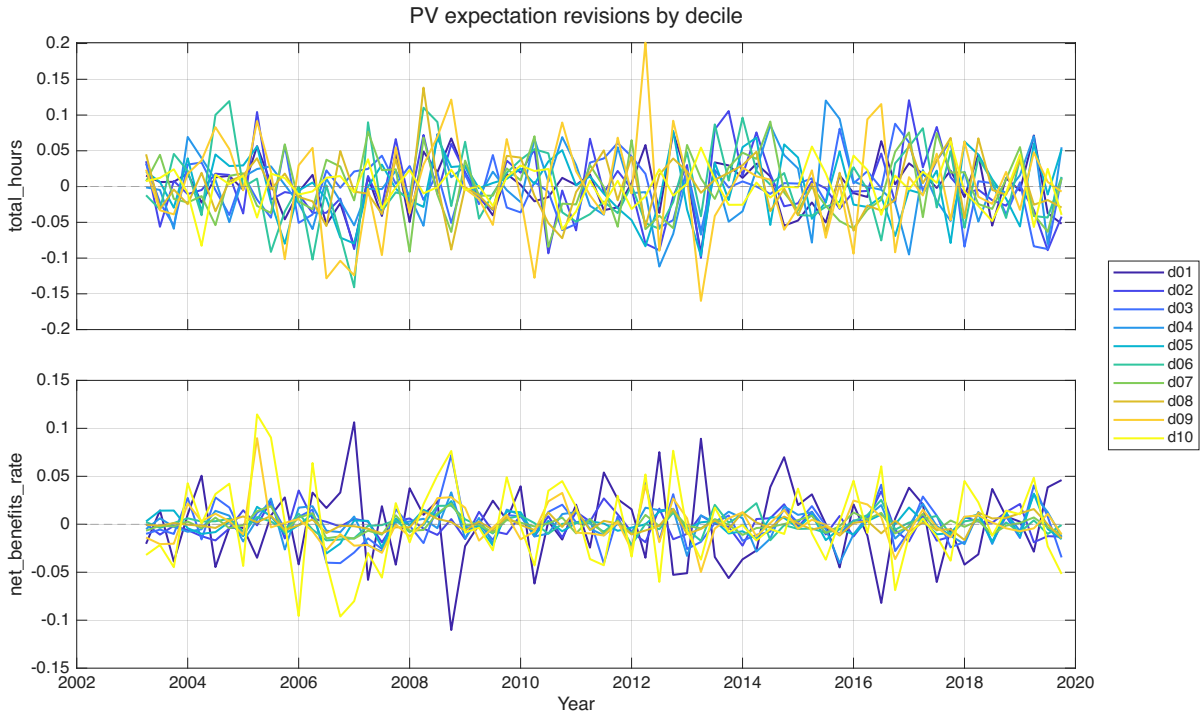
$$\begin{aligned} &\int g(i) \frac{Wn(i)}{WN} \sum_s \beta^s \left(\hat{\theta}_{t+s}(\omega) + \hat{\tau}_{t+s}(\omega) \right) \pi(i|\omega) \, di - \int \frac{Wn(i)\psi}{\sigma e(i) + Wn(i)\psi} \sum_s \beta^s \hat{\theta}_{t+s}(\omega) \pi(i|\omega) \, di \\ &\quad = \xi_t + b_R(\omega) \sum_s \beta^s \left(\hat{R}_{t+s} - \pi^Q_{CPI,t+s+1} \right) + b_Q(\omega) \hat{Q}_{CPI,t}, \end{aligned} \tag{26}$$

where $\pi(i|\omega)$ is the steady state distribution of household types (i) in a given group ω. A few remarks are in order. First, the redistribution loss captures lifetime utility changes, so the optimality condition is in present value terms. Second, even though the government might not be able to fully target household i, the distribution of types within groups matters for the optimal level of transfers. For example, the coefficient $b_R(\omega)$ captures the average group-level exposure to real interest changes, which follows from the joint distribution of debt and welfare weights in steady state. Third, these present value terms are revisions to expectations: they capture responses to unanticipated shocks rather than dynamic adjustments to past shocks. Finally, estimating an equation of this form recovers only the average welfare weight for each group. To recover individual welfare weights we exploit the expression for $g(i)$, and apply a moment matching procedure. We lay out each step of the estimation in what follows.

Data construction. We allow households with different productivity levels to be hit by (potentially) different productivity shocks. Balancing concerns for heterogeneous exposure and sample size limitations, our main specifications constructs deciles from observed data on (pre-tax) wage income. To make this assignment realistic, we also allow for income differences that are important in the UK context but cannot be accounted for in our model, like regional differences and the age profile of wage earnings. Specifically, we estimate the following regression at the household level

$$ln\left(\text{Labor Income}_{i,t}\right) = \eta_{r,t} \cdot \text{Region}_{i,t} + \beta a_{i,t} + \gamma a_{i,t}^2 + \mu_{i,t},$$

where we interpret $\mu_{i,t}$ as capturing permanent differences in earnings potential. We then assign each household to a decile by using the fitted $\hat{\mu}_{i,t}$. Given this assignment, we proceed to construct group-level averages of income, expenditure, and net benefits, where we use the sampling weights provided by the LCF to take care of under/over-sampling of households with certain



Notes: Present value revisions to total hours worked and the net benefits share for each decile over time. The hours variable is logged while the benefits are expressed as a share of total gross income.

Figure A1: Time-series of Present Value revisions

characteristics.[43] Given that the group assignment is done on labour income, only households where at least one adult works will remain in our sample. Furthermore, we exclude part-time workers by restricting the sample to households whose weekly hours exceed 30. If we do not impose this restriction, part-time workers end up populating the lowest decile of wage earners, while they are not necessarily low-skill, and might have different motives for working part-time that we cannot capture or control for.

Estimation Procedure. The household-level data come from the LCF survey, which is conducted at quarterly frequency and which we aggregate to deciles as specified in the previous section. We augment these decile-level variables with ONS data on quarterly GDP, the price level, nominal wages, and the one-year yield. We estimate a joint VAR of the group-level and aggregate variables, with the restriction that group-level variables cannot affect aggregate ones either contemporaneously or with a lag. The baseline specification allows four lags, but the results are robust to varying the number of lags. Note that we do not need to identify structural shocks, and we work directly with the reduced-form errors. In particular, given the estimated VAR coefficients, we recover the reduced-form shocks u_t and use them to construct the net present-value innovations for each decile-level variable at every point in time.

Figure A1 plots the revisions to the expected present value of hours worked and of the net transfer rate for each decile over our sample period. Two features stand out. First, the revisions to net transfers are largest for the bottom decile, as one would expect given that benefits make up a larger share of its total resources. Interestingly, they have also fluctuated strongly for the top decile, especially in the run-up to and during the Great Financial Crisis. Second, the revisions are far from perfectly correlated across deciles. For hours worked, the average correlation across all pairs of deciles is only 0.10, and it is just 0.20 for neighbouring deciles. Net transfers co-move more strongly, especially among the third to tenth deciles, where the average pairwise correlation is 0.46. The bottom decile is the exception: its revisions are negatively correlated with those of most other deciles (-0.23 on average, and -0.33 with the top decile). This heterogeneity matters for identification: a revision that moved all deciles in the same way would be absorbed by the common factor and the aggregate controls in the regression, so the decile-specific weights $\gamma(\omega)$ are identified from the variation that differs across deciles.

Our identifying equation uses revisions to net benefits and productivity at the decile level, but the latter are not directly observed. In our baseline specification, we measure productivity using revisions to hours worked, which we observe directly in the LCF. We view hours as a good measure of the group-level productivity shock. As a robustness check, however, we also consider an alternative specification that recovers productivity from the observed responses of expenditure and total wage income. This relies on the labour optimality condition for each household, under which labour supply responds to real wage changes, the productivity shock, and a wealth effect. Since the wealth effect is captured by real consumption changes, we can remove it using revisions to real expenditure. Both methods yield similar results, and the alternative specification does not change the overall pattern of how average empirical weights vary across the distribution of households.

[43] Similar to other household surveys of the same nature, like the CEX, the LCF also struggles to sample households in the rightmost tail of the income distribution. However, this is unlikely to be a concern for our exercise.

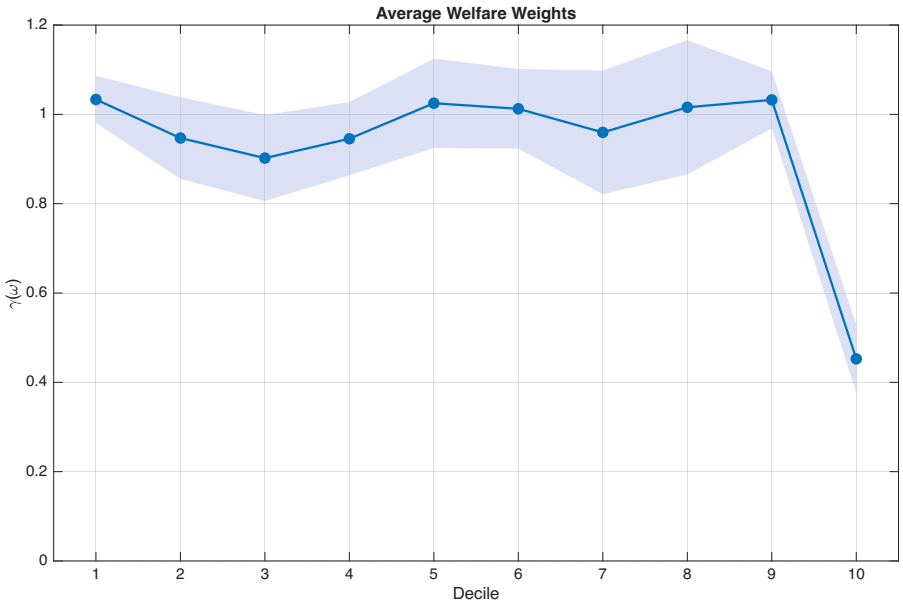


Figure A2: Average Welfare Weights by Decile

Welfare weight regression. We now have all the ingredients required to recover the empirical welfare weights, and we proceed in two steps. First, using expectation revisions for productivity, benefits, real interest rates and the price level, we estimate average weights for different groups by regressing

$$\sum_s \beta^s \hat{\theta}_{t+s}(\omega) = \gamma(\omega) \sum_s \beta^s \left(\hat{\theta}_{t+s}(\omega) + \hat{\tau}_{t+s}(\omega)\right) + b(\omega)\xi_t + b_R(\omega) \sum_s \beta^s \left(\hat{R}_{t+s} - \pi^Q_{CPI,t+s+1}\right) + b_Q(\omega)\hat{Q}_{CPI,t},$$

where we deal with the term $b(\omega)\xi_t$ by using Bai (2009). This expression follows from a simple rearrangement of the optimality condition given in 26.

Figure A2 plots the estimated $\gamma(\omega)$, together with the confidence intervals. The coefficients are close to one for the bottom nine deciles, and fall sharply to about 0.45 for the top decile of households. Note that the coefficient

$$\gamma(\omega) = \int \frac{g(i)\frac{Wn(i)}{WN}}{\int \frac{Wn(i)\psi}{\sigma e(i)+Wn(i)\psi}\pi(i|\omega)\,di}\pi(i|\omega)\,di,$$

collapses to one when the social welfare function is linear $\left(g(i) = \frac{1}{\sigma e(i)+Wn(i)\psi}\right)$, which allows for a clear interpretation of the results. Specifically, our estimates suggest that on average social welfare weights agree with the individual's utility cost of fluctuations for the bottom 90% of the distribution, but underweigh them for the top 10% of households.

Individual weights. The final step is to recover household-level welfare weights. To do so we use the expression for weights given by

$$g(e, Wn) = \frac{1}{1-R^{-1}}e^{-1}\chi(V(e, Wn)) + \frac{1}{e+Wn\psi},$$

and the estimated averages $\gamma(\omega)$, to approximate the unknown function $\chi(V(e, Wn))$. Note that $\chi(V) = -\frac{1}{1-R^{-1}}G''(V)/G(V)$ captures the curvature of the social welfare function, with $\chi(\cdot) = 0$ being the linear case. Because we only have ten moment conditions (one for each group), we can approximate $\chi(\cdot)$ using only a few terms. In the baseline we use a quadratic expansion which we estimate with a simple OLS. In particular, the decile-level regressors $\{Z_{1,\omega}, Z_{2,\omega}\}$ are given by

$$Z_{1,\omega} = \int \frac{Wn(i)}{e(i)}V(i)\pi(i|\omega)\,di,$$
$$Z_{2,\omega} = \int \frac{Wn(i)}{e(i)}V(i)^2\pi(i|\omega)\,di,$$

where the households steady state lifetime utility is $V(i) = \frac{1}{1-R^{-1}}\left(ln(e(i)) - \frac{Wn(i)}{(1+\psi^{-1})e(i)}\right)$. Intuitively, households that have higher expenditure and more assets (so that they have to work less to maintain a given level of consumption) have higher lifetime utility. Note that the weight $\frac{Wn(i)}{e(i)}$ in the regressors comes from the fact that our estimated coefficients are weighted by the labour share. When constructing the decile-level regressors we use the distribution of households for 2019, and which is also what we feed in the model for our quantitative exercise. Finally, we redefine the outcome variable to correct for the denominator in $\gamma(\omega)$, and to

Decile	Y actual	Y fit	Residual
1	0.0116	0.0782	-0.0666
2	-0.0245	0.0269	-0.0514
3	-0.0511	-0.0064	-0.0447
4	-0.0311	-0.0294	-0.0017
5	0.0146	-0.0361	0.0506
6	0.0076	-0.0303	0.0378
7	-0.0251	0.0246	-0.0497
8	0.0101	-0.0051	0.0151
9	0.0211	-0.1008	0.1219
10	-0.3694	-0.2910	-0.0784
$R^2 = 0.7014$			

Table 2: Decile fit, quadratic specification

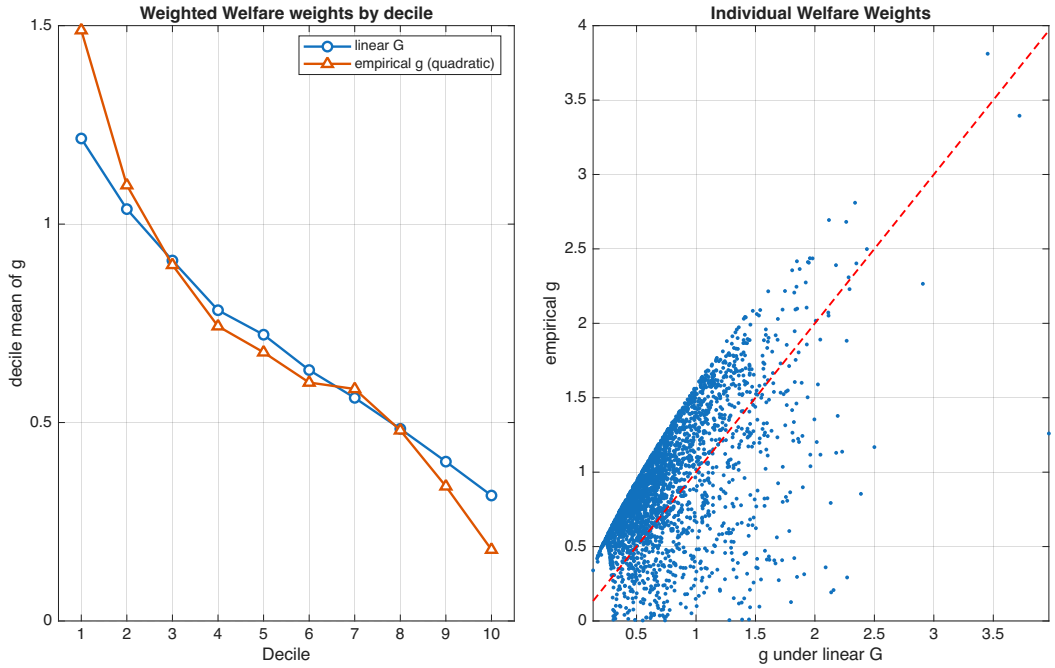
remove the linear part from the welfare weights. Specifically, we define the dependent variable as

$$\hat{y}_w \equiv \left(\int \frac{Wn(i)\psi}{\sigma e(i) + Wn(i)\psi} \pi(i|\omega)\, di \right) \hat{\gamma}(\omega) - \int \frac{Wn(i)}{e(i) + Wn(i)\psi} \pi(i|\omega)\, di.$$

We regress $\hat{y}_w$ on $\{Z_{1,\omega}, Z_{2,\omega}\}$ and a constant. Table 2 shows that the quadratic approximation provides a reasonable fit ($R^2 \approx 0.70$) of the moment conditions. Given the estimated coefficients, we can now construct individual welfare weights for every household.[44]

Figure A3 shows how empirical and linear welfare weights compare. The left panel shows that for either measure, the average welfare weight decreases with the income decile. This is to be expected since higher deciles tend to earn more income and consume more. Empirical weights are above the linear ones for the bottom decile, and below the linear case for the top two deciles, reflecting the quadratic fit, since $\hat{\gamma}(\omega)$ drops only for the top decile. They otherwise coincide for the rest of the distribution. Beyond these averages, the right panel shows that going from linear to (total) social welfare weights is not a one-for-one mapping as there is quite a lot of variation that is introduced by the individual dispersion in income and expenditure. Finally, note that for most households (around 80%), the social welfare function assigns a higher weight than their own preferences imply.

[44]Because we do not impose any other restrictions in the estimation of the social welfare curvature $\chi(\cdot)$, it is the case that for a small number of households ($\approx X\%$) we recover empirical weights that are negative. These cases are theoretically problematic (though they are not quantitatively relevant), so we impose positive welfare weights when quantifying the welfare losses.



Notes: The decile-level average welfare weights are weighted by the within-decile labour share. The right panel excludes the few observations in which we recover a negative weight.

Figure A3: Empirical vs Linear Welfare Weights