

Explanatory Theories of Presupposition Projection I: Trivalent and Transparency Theories

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Presupposition Reading Group

1 The explanatory problem

Classical dynamic semantics is very powerful in that one can define multiple dynamic counterparts of a single classical operator that agree in terms of classical truth-conditions, but differ with respect to presupposition projection.

This was first pointed out independently by [Soames 1989](#) and a personal letter by Mats Rooth to Irene Heim (dated March 27, 1987; [Heim 1990](#)), contrary to remarks given in [Heim 1982, 1983](#).

1.1 Connectives in $\mathcal{L}_{\text{SAT}}$

To see this issue concretely, let us go back to our reformulation of Satisfaction Theory, $\mathcal{L}_{\text{SAT}}$. $\mathcal{L}$ is Propositional Logic and $\mathcal{A}$ is the set of atomic formulas in $\mathcal{L}$.

- (1) *Syntax of $\mathcal{L}_{\text{SAT}}$*
- a. If $\pi \in \mathcal{L}$ and $p_i \in \mathcal{A}$, then $\pi p_i \in \mathcal{L}_{\text{SAT}}$.
 - b. If $\varphi, \psi \in \mathcal{L}_{\text{SAT}}$, then $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi) \in \mathcal{L}_{\text{SAT}}$.
 - c. Nothing else is a formula of $\mathcal{L}_{\text{SAT}}$.

We defined the semantics as follows. Recall that each $v \in V$ is an $\mathcal{L}$ -valuation, a model for $\mathcal{L}$, and $\|\cdot\|$ maps each formula in $\mathcal{L}$ to a function from $\mathcal{L}$ -valuations to truth-values.

- (2)
- a. $\|\pi p_i\| = \lambda V: \text{for each } v \in V, \|\pi\|(v) = 1. \{v \in V \mid \|p_i\|(v) = 1\}$
 - b. $\|\neg\varphi\| = \lambda V: V \in \text{dom}(\|\varphi\|). V - \|\varphi\|(V)$
 - c. $\|(\varphi \wedge \psi)\| = \lambda V: V \in \text{dom}(\|\varphi\|) \text{ and } \|\varphi\|(V) \in \text{dom}(\|\psi\|). \|\psi\|(\|\varphi\|(V))$
 - d. $\|(\varphi \vee \psi)\| = \lambda V: V \in \text{dom}(\|\varphi\|) \text{ and } \|\neg\varphi\|(V) \in \text{dom}(\|\psi\|). \|\varphi\|(V) \cup \|\psi\|(\|\neg\varphi\|(V))$
 - e. $\|(\varphi \rightarrow \psi)\| = \lambda V: V \in \text{dom}(\|\varphi\|) \text{ and } \|\varphi\|(V) \in \text{dom}(\|\psi\|). V - \|\neg\psi\|(\|\varphi\|(V))$

As we remarked before, this semantics can be restated in terms of the lifted version of Propositional Logic $\mathcal{L}$, where the semantics is stated in terms of sets of valuations, according to the following general rule:

- (3) For any $\varphi \in \mathcal{L}$, $\|\varphi\|(V) = \{v \in V \mid \|\varphi\|(v) = 1\}$

This lifting operation makes no substantial change to $\mathcal{L}$, because we can go back to $\|\cdot\|$ using the following general rule:

- (4) For any $\varphi \in \mathcal{L}$, $\|\varphi\|(v) = 1$ iff $\|\varphi\|(\{v\}) = \{v\}$.

Now, using $\llbracket \cdot \rrbracket$, we can rewrite the semantics of $\mathcal{L}_{\text{SAT}}$ as (5).

- (5) a. $\llbracket \neg\varphi \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket). \llbracket \neg\varphi \rrbracket(V)$
b. $\llbracket (\varphi \wedge \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket (\varphi \wedge \psi) \rrbracket(V)$
c. $\llbracket (\varphi \vee \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \neg\varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket (\varphi \vee \psi) \rrbracket(V)$
d. $\llbracket (\varphi \rightarrow \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket (\varphi \rightarrow \psi) \rrbracket(V)$

These representations particularly clearly show that the assertive component of these operators correspond transparently to what they are in Propositional Logic $\mathcal{L}$, and crucially that their presuppositions are not directly related to the assertive component.

In fact, it is easy to come up with alternative versions of these operators that have the same assertive component but differ in terms of presuppositions, e.g. for conjunction:

- (6) a. $\llbracket (\varphi \wedge \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket (\varphi \wedge \psi) \rrbracket(V)$
b. $\llbracket (\varphi \dot{\wedge} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } V \in \text{dom}(\llbracket \psi \rrbracket). \llbracket (\varphi \wedge \psi) \rrbracket(V)$
c. $\llbracket (\varphi \ddot{\wedge} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket). \llbracket (\varphi \wedge \psi) \rrbracket(V)$
d. $\llbracket (\varphi \ddot{\wedge} \psi) \rrbracket = \lambda V: \begin{array}{l} V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket) \\ \text{or} \\ V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket) \end{array} . \llbracket (\varphi \wedge \psi) \rrbracket(V)$

Obviously, there are other possibilities too, because you can really state anything you want in the presupposition, as long as the assertion is $\llbracket (\varphi \wedge \psi) \rrbracket(V)$, but the ones in (6) are perhaps among the most ‘natural’ ones in some sense, given that the following dynamic denotations suggest them.

- (7) a. $\llbracket (\varphi \wedge \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket \psi \rrbracket(\llbracket \varphi \rrbracket(V))$
b. $\llbracket (\varphi \dot{\wedge} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } V \in \text{dom}(\llbracket \psi \rrbracket). \llbracket \varphi \rrbracket(V) \cap \llbracket \psi \rrbracket(V)$
c. $\llbracket (\varphi \ddot{\wedge} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket). \llbracket \varphi \rrbracket(\llbracket \psi \rrbracket(V))$
d. $\llbracket (\varphi \ddot{\wedge} \psi) \rrbracket = \lambda V: \begin{array}{l} V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket) \\ \text{or} \\ V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket) \end{array} . \begin{cases} \llbracket \psi \rrbracket(\llbracket \varphi \rrbracket(V)) & \text{if } V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket) \\ \llbracket \varphi \rrbracket(\llbracket \psi \rrbracket(V)) & \text{otherwise} \end{cases}$

Similarly, we can come up with alternative denotations for disjunction, e.g.:

- (8) a. $\llbracket (\varphi \vee \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \neg\varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket). \llbracket \varphi \rrbracket(V) \cup \llbracket \psi \rrbracket(\llbracket \neg\varphi \rrbracket(V))$
b. $\llbracket (\varphi \dot{\vee} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } V \in \text{dom}(\llbracket \psi \rrbracket). \llbracket \varphi \rrbracket(V) \cup \llbracket \psi \rrbracket(V)$
c. $\llbracket (\varphi \ddot{\vee} \psi) \rrbracket = \lambda V: V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \neg\psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket). \llbracket \psi \rrbracket(V) \cup \llbracket \varphi \rrbracket(\llbracket \neg\psi \rrbracket(V))$
d. $\llbracket (\varphi \ddot{\vee} \psi) \rrbracket = \lambda V: \begin{array}{l} V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \neg\varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket) \\ \text{or} \\ V \in \text{dom}(\llbracket \psi \rrbracket) \text{ and } \llbracket \neg\psi \rrbracket(V) \in \text{dom}(\llbracket \varphi \rrbracket) \end{array} . \begin{cases} \llbracket \varphi \rrbracket(V) \cup \llbracket \psi \rrbracket(\llbracket \neg\varphi \rrbracket(V)) & \text{if } V \in \text{dom}(\llbracket \varphi \rrbracket) \text{ and } \llbracket \neg\varphi \rrbracket(V) \in \text{dom}(\llbracket \psi \rrbracket) \\ \llbracket \psi \rrbracket(V) \cup \llbracket \varphi \rrbracket(\llbracket \neg\psi \rrbracket(V)) & \text{otherwise} \end{cases}$

As can be proven, the assertions of these entries all correspond to $\llbracket (\varphi \vee \psi) \rrbracket(V)$ whenever the respective presuppositions are true.

Exercise: Come up with different variants of $\rightarrow$.

1.2 The explanatory problem

It appears that the correct analysis of ‘and’ in English is $\wedge$, which evaluates the presupposition of the second conjunct against the context set updated with the content of the first conjunct, given the asymmetric projection we observe in examples like (9).

- (9) a. John is an Englishman, and Richard is also an Englishman.
b. #Richard is also an Englishman, and John is an Englishman.
- If ‘and’ means $\wedge$, then it is correctly predicted that (9a) has no presupposition overall. Also, one can analyze the infelicity of (9b) in reference to the unfiltered additive presupposition + the lack of additive marker in the second sentence (see [Bade 2016](#) for why an additive marker is required here).
 - On the other hand, if ‘and’ meant $\dot{\wedge}$, we’d expect the additive presupposition to project out in both sentences, and consequently, (9a) to be as infelicitous as (9b).
 - If ‘and’ meant $\ddot{\wedge}$, we’d expect (9b) to be felicitous and (9a) to be infelicitous, the opposite of what we actually observe.
 - If ‘and’ meant $\ddot{\vee}$, both sentences should be felicitous.

The question, then, is why ‘and’, and other conjunctions in other natural languages, seem to all mean $\wedge$, rather than the other logical options. Satisfaction Theory itself does not answer this question.

To make the matter worse, it’s actually not entirely clear what the correct analysis for ‘or’ is. The best analysis among the options given above might actually be $\ddot{\vee}$, given that filtering seems to be possible from either direction.

- (10) a. Either Benjamin is not from Russia, or he has to go back to Russia soon.
b. Either Benjamin has to go back to Russia soon, or he is not from Russia.

For now, we assume:

- Conjunction is asymmetric in that the content of the first conjunct can be used to filter out the presupposition of the second conjunct, but not vice versa.
- Disjunction is symmetric and the content of either disjunct can be used to filter out the presupposition of the other disjunct.

We’ll discuss more data later.

Exercise: Check the predictions of the variants of conditional you came up with above against English examples (adopting the arguably wrong assumption that English *if*-conditionals are to be analyzed as material implication). Note that English allows *if*-conditionals to precede or follow the consequent.

1.3 Stalnaker’s conjecture

[Stalnaker 1973, 1974](#) provided the following idea for deriving the presupposition of ‘and’, and conditionals.

The pragmatic account of presupposition gives a natural intuitive explanation for a rule which, on the semantic account separate the semantic question of the truth value of a conjunction from the pragmatic question of the presuppositions it requires. Because we have made this separation, we can reconcile the semantical symmetry of the conjunction operation with the asymmetry of the conjunctive assertions with respect to the presuppositions they require. ‘*A and B*’ says exactly the same thing as ‘*B and A*’, but the first way of saying it may require different presuppositions than the second. The analogous problem for disjunctive and conditional statements is not quite so straightforward, but I expect that a reasonably natural explanation for the facts can be given using plausible assumptions about the way background assumptions change in the course of a conversation. (Stalnaker 1973: p. 455f)

[...] if we regard Karttunen’s generalization as a generalization about *pragmatic* presuppositions, then we can reconcile it with the standard truth-functional account of *and*, and we can explain the generalization without postulating any *ad hoc* semantics or pragmatic rules. The explanation goes like this: first, once a proposition has been asserted in a conversation, then (unless or until it is challenged) the speaker can reasonably take it for granted for the rest of the conversation. In particular, when a speaker says something of the form *A and B*, he may take it for granted that *A* (or at least that his audience recognizes that *he* accepts that *A*) after he has said it. The proposition that *A* will be added to the background of common assumptions before the speaker asserts that *B*. Now suppose that *B* expresses a proposition that would, for some reason, be inappropriate to assert except in a context where *A*, or something entailed by *A*, is presupposed. Even if *A* is *not* presupposed initially, one may still assert *A and B* since by the time one gets to saying that *B*, the context has shifted, and it is by then presupposed that *A*.

As with the explanation sketched in the earlier discussion, this explanation rests on just two things: first, a simple pragmatic assumption about the way presuppositions shift in the course of a conversation—an assumption that says, roughly, that a speaker may build on what has already been said; second, an uncontroversial assumption about the semantic properties of the word *and*—in particular, that when one asserts a conjunction, he asserts both conjuncts. If we interpreted presupposition to mean *pragmatic* presupposition, then we can deduce Karttunen’s generalization from these two almost trivial assumptions.

The analogous generalization about conditional statements is explainable on equally simple assumptions. Here we need first the assumption that what is explicitly *supposed* becomes (temporarily) a part of the background of common assumptions in subsequent conversation, and second that an *if* clause is an explicit supposition. Again, Karttunen’s generalization can be derived from these obvious assumptions. (Stalnaker 1974; reprinted in Stalnaker 1998: p. 60)

However, Schlenker 2008a contends that this idea is not general enough:

This account has the ring and the simplicity of truth. It has proven immensely influential; but on closer inspection it raises three difficulties: it conflates the belief that *p* has been asserted with the belief that *p* is true; it fails to generalize to conjunction that are embedded under other operators; and it fails to apply to other connectives, even when they are not embedded [...].

(Schlenker 2008a: p. 162)

I find the first two issues not so significant.

- The point about the difference between assertion and belief is anticipated by Stalnaker in the 1974 quote above: “when a speaker says something of the form *A and B*, he may take it for granted that *A* (**or at least that his audience recognizes that he accepts that *A***) after he has said it” (emphasis added).
 - Certainly, this on its own doesn’t mean that the truth of *A* becomes common ground, but we could generalize Stalnaker’s idea of temporary common assumptions of explicit suppositions to all cases, and maintain that the content of *A* becomes a tentative/hypothetical update of the common ground during the processing, and only becomes part of the actual common ground after it’s clear that you, as the hearer, (and everyone else in the conversation) accept the overall utterance.
 - I’d say this is more realistic given that an utterance of *A and ...* does not guarantee that *A* is asserted. For instance, the whole utterance might be *A and B, if C*.
 - [Farkas & Bruce 2010](#) put forward a similar view of assertion (and other speech acts): An assertion of *A* is merely a proposal to add the truth of *A* to the common ground. If everyone accepts the proposal, the truth of *A* becomes part of the common ground. This sounds reasonable, and if it’s true, then conversational participants must process the content of an utterance without actual belief updates, but perhaps something like hypothetical updates.
- The second point about conjunction under other operators is probably not a huge issue, once we have the idea of temporary, hypothetical updates. Schlenker talks about the example in (11).

(11) I have never met an academic who was an idiot and who knew it.
([Schlenker 2008a](#): p. 163)

It’s true that this is “not asserting of anybody that he is an idiot” (p. 163), but nonetheless, one could say that there is a hypothetical update of the context that there is and the second conjunct is interpreted against that, as a dynamic semantic analysis of this sentence would have it.

As for the third point, Schlenker presents the following disjunctive example.

(12) John isn’t incompetent, or he knows that he is. ([Schlenker 2008a](#): p. 163)

In order to account for the filtering effect, Stalnaker would have to presume that a disjunctive sentence *A or B* involves a hypothetical update of the form, *A or [not A and B]*, or maybe *A or [if not A, then B]*. This is not impossible, but it is far from obvious why this needs to be the case.

To complicate the matter, filtering in the opposite direction is also possible.

(13) John knows that he is incompetent, or he isn’t incompetent.

[Schlenker 2008a](#) does not deny the possibility that those disjunctive cases involve local accommodation; if the factive presupposition of (13) projected out, its satisfaction would render the second disjunct redundant and the whole disjunction, infelicitous.

(14) #John is incompetent. He know that he is, or he isn’t incompetent.

He raises examples where this confound is removed: the presuppositional content is weaker than the negation of the other disjunct.

- (15) a. Mary doesn't have a violin, or else her instrument is well hidden.
b. Mary's instrument is well hidden, or else she doesn't have a violin.
(Schlenker 2008a: p. 187)
- (16) a. Mary doesn't have cancer, or else her doctor will realize that she is sick.
b. Mary's doctor will realize that she is sick, or else she doesn't have cancer.
(Schlenker 2008a: p. 187)

Schlenker admits that both orders allow for readings without presuppositions. We'll come back to these examples later.

2 Trivalent Theories

Trivalent theories offer a general recipe, and thereby address the explanatory problem. However, none of them seem to predict the difference in (a)symmetry between conjunction and disjunction (but see a recent attempt by Elliott & Kalomoiros 2026 to address this issue in the trivalent approach).

We'll work on the same set of formulas as $\mathcal{L}_{\text{SAT}}$ (see the handout on Trivalent Theories for details).

2.1 Weak Kleene

The general recipe for Weak Kleene is that the third-value, 2, projects out.

- Start from the bivalent version of a n -ary operator.
- Cases where all n arguments are classical truth-values, 0 or 1 are taken care of by the bivalent meaning.
- So all new cases involve at least one instance of 2 among the arguments. All such cases denote #.

$$(17) \quad \text{a.} \quad \|\neg\varphi\|(v) = \begin{cases} 1 & \text{if } \|\varphi\|(v) = 0 \\ 0 & \text{if } \|\varphi\|(v) = 1 \end{cases}$$

$$\text{b.} \quad \llbracket \neg\varphi \rrbracket_{\text{WK}}(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 0 \\ 0 & \text{if } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 1 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 2 \end{cases}$$

$$(18) \quad \text{a.} \quad \|(\varphi \wedge \psi)\|(v) = \begin{cases} 1 & \text{if } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 1 \\ 0 & \text{if either } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 0 \\ & \text{or } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 1 \\ & \text{or } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 0 \end{cases}$$

$$\text{b. } \llbracket (\varphi \wedge \psi) \rrbracket_{\text{WK}}(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 1 \\ 0 & \text{if either } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 1 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 0 \\ 2 & \text{if either } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 1 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 2 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 2 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{WK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{WK}}(v) = 2 \end{cases}$$

Similarly for the other operators.

As we discussed before, Week Kleene doesn't account for presupposition filtering at all, so is inadequate as a theory of presupposition projection in natural language.

2.2 Strong Kleene

Strong Kleene sees 2 as uncertainty between 0 and 1. The general receipt therefore is:

- As before, all new cases to consider for an n -ary operator involve at least one of the n arguments denoting 2.
- If replacing 2 with 0 and replacing it with 1 yield the same overall truth-value, that truth-value is the denotation of the whole formula; if the two ways of replacing 2 do not converge, the whole formula denotes 2.

$$(19) \quad \text{a. } \llbracket \neg \varphi \rrbracket(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket(v) = 0 \\ 0 & \text{if } \llbracket \varphi \rrbracket(v) = 1 \end{cases}$$

$$\text{b. } \llbracket \neg \varphi \rrbracket_{\text{SK}}(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 0 \\ 0 & \text{if } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 1 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 2 \end{cases}$$

$$(20) \quad \text{a. } \llbracket (\varphi \wedge \psi) \rrbracket(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket(v) = 1 \text{ and } \llbracket \psi \rrbracket(v) = 1 \\ 0 & \text{if either } \llbracket \varphi \rrbracket(v) = 0 \text{ and } \llbracket \psi \rrbracket(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket(v) = 0 \text{ and } \llbracket \psi \rrbracket(v) = 1 \\ & \text{or } \llbracket \varphi \rrbracket(v) = 1 \text{ and } \llbracket \psi \rrbracket(v) = 0 \end{cases}$$

$$\text{b. } \llbracket (\varphi \wedge \psi) \rrbracket_{\text{SK}}(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 1 \\ 0 & \text{if either } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 1 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 0 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 2 \\ 2 & \text{if either } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 1 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 2 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 2 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{SK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{SK}}(v) = 2 \end{cases}$$

Similarly for other connectives.

A problem of this analysis of conjunction is that it predicts symmetric filtering, as we discussed before.

- The pragmatic presupposition of an assertion of a sentence denoting φ is that with respect to every world v in the context set c , φ denotes a classical truth-value, 0 or 1. (Stalnaker's rationale: the hearer wouldn't know what to do with worlds where φ denotes 2)
- If φ denotes 0 with respect to an arbitrary world v , φ is allowed to denote 2 with respect to v , whereas if φ denotes 1 with respect to v , we have to make sure that φ denotes either 0 or 1 with respect to v , in order to avoid pragmatic presupposition failure. This is a conditional presupposition that if φ denotes 1, then ψ denotes either 0 or 1, i.e., the presupposition of ψ is true.
- Not only that, parallel reasoning with the arguments switched gives us a conditional presupposition that if ψ denotes 1 with respect to v , then the presupposition of φ needs to be true with respect to v , in order to avoid pragmatic presupposition failure.
- If both of these are the case, the pragmatic presupposition of $(\varphi \wedge \psi)$ will be satisfied.

As we remarked above, it is generally considered that conjunction gives rise to asymmetric filtering, so the symmetric prediction of Strong Kleene is thought to be a problem.

But it might be a good prediction for disjunction. More on this below.

2.3 Middle Kleene

Middle Kleene (as [Krahmer 1998](#) calls it) introduces an asymmetry between the two conjuncts in a conjunctive formula, by taking into consideration linear order.

Middle Kleene was originally proposed by [Peters 1979](#) as a trivalent re-formulation of the projection rules of [Karttunen 1974](#), and Peters is largely implicit about the general recipe.

[George 2008](#) and [Fox 2008](#) clearly state the general recipe, and the role of linear order in it.

- The idea is to do Strong Kleene reasoning from left-to-right.
- For a binary operator $\square$ in $(\varphi \square \psi)$, assume that we haven't seen ψ yet. That is to say that we temporarily fix the denotation of ψ to 2, as 2 represents uncertainty in Strong Kleene, and do Strong Kleene reasoning on different values of φ . While keeping the results of the first step, we run Strong Kleene reasoning about different values of φ and ψ at the same time.
- Generalizing this to an n -ary operator, $O(\varphi_1, \dots, \varphi_n)$:
 - First, run Strong Kleene reasoning on φ_1 in $O(\varphi_1, \dots, \varphi_n)$ on the assumption that $\varphi_2, \dots, \varphi_n$ all denote 2.
 - Second, while keeping the results of the first step, run Strong Kleene reasoning about φ_1 and φ_2 simultaneously in $O(\varphi_1, \dots, \varphi_n)$ with the assumption that $\varphi_3, \dots, \varphi_n$ all denote 2.
 - Third, while keeping the results of the first two steps, run Strong Kleene reasoning about $\varphi_1 \dots \varphi_3$ in $O(\varphi_1, \dots, \varphi_n)$ with the assumption that $\varphi_4, \dots, \varphi_n$ all denote 2.
 - $\vdots$

- Finally, while keeping the results of all the previous steps, run Strong Kleene reasoning about $\varphi_1, \dots, \varphi_n$ in $O(\varphi_1, \dots, \varphi_n)$.

$$(21) \quad \text{a.} \quad \|\neg\varphi\|(v) = \begin{cases} 1 & \text{if } \|\varphi\|(v) = 0 \\ 0 & \text{if } \|\varphi\|(v) = 1 \end{cases}$$

$$\text{b.} \quad \llbracket \neg\varphi \rrbracket_{\text{MK}}(v) = \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 0 \\ 0 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 2 \end{cases}$$

$$(22) \quad \text{a.} \quad \|(\varphi \wedge \psi)\|(v) = \begin{cases} 1 & \text{if } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 1 \\ 0 & \text{if either } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 0 \\ & \text{or } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 1 \\ & \text{or } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 0 \end{cases}$$

b. Assume that $\llbracket \psi \rrbracket_{\text{MK}}(v) = 2$.

$$\llbracket (\varphi \wedge \psi) \rrbracket_{\text{MK}}(v) = \begin{cases} 0 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 0 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 2 \end{cases}$$

c. While keeping the results above, we now reason about different values of φ and ψ that have not been covered yet. Those are different cases of $\llbracket \varphi \rrbracket_{\text{MK}} = 1$.

$$\begin{aligned} \llbracket (\varphi \wedge \psi) \rrbracket_{\text{MK}}(v) &= \begin{cases} 0 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 0 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 2 \\ 1 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 1 \\ 0 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 0 \\ 2 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 2 \end{cases} \\ &= \begin{cases} 1 & \text{if } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 1 \\ 0 & \text{if either } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 0 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 0 \\ 2 & \text{if either } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 2 \\ & \text{or } \llbracket \varphi \rrbracket_{\text{MK}}(v) = 1 \text{ and } \llbracket \psi \rrbracket_{\text{MK}}(v) = 2 \end{cases} \end{aligned}$$

Looking at the condition for $\llbracket (\varphi \wedge \psi) \rrbracket_{\text{MK}}(v) = 2$, in order to make sure that's not the case, it must be the case that $\llbracket \varphi \rrbracket_{\text{MK}}(v) = 1$ or $\llbracket \varphi \rrbracket_{\text{MK}}(v) = 0$ —which is to say that the presupposition of φ is true with respect to v —and also that in case $\llbracket \varphi \rrbracket_{\text{MK}}(v) = 1$, $\llbracket \psi \rrbracket_{\text{MK}} = 1$ or $\llbracket \psi \rrbracket_{\text{MK}} = 0$ —which is a conditional presupposition that if φ is true with respect to v , the presupposition of ψ must be true with respect to v .

We have therefore an asymmetric conjunction.

2.4 Disjunction

Strong Kleene generally makes symmetric predictions and Middle Kleene generally makes asymmetric predictions.

- For conjunction, Middle Kleene is arguably better.
- But for disjunction, a symmetric prediction seems more desirable.

$$(23) \quad \|(\varphi \vee \psi)\|(v) = \begin{cases} 1 & \text{if either } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 1 \\ & \text{or } \|\varphi\|(v) = 1 \text{ and } \|\psi\|(v) = 0 \\ & \text{or } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 1 \\ 0 & \text{if either } \|\varphi\|(v) = 0 \text{ and } \|\psi\|(v) = 0 \end{cases}$$

2.4.1 Strong Kleene disjunction

$$(24) \quad \llbracket(\varphi \vee \psi)\rrbracket_{SK}(v) = \begin{cases} 1 & \text{if } \llbracket\varphi\rrbracket_{SK}(v) = 1 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 1 \\ & \text{or } \llbracket\varphi\rrbracket_{SK}(v) = 1 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 2 \\ & \text{or } \llbracket\varphi\rrbracket_{SK}(v) = 2 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 1 \\ 0 & \text{if either } \llbracket\varphi\rrbracket_{SK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 0 \\ 2 & \text{if either } \llbracket\varphi\rrbracket_{SK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 2 \\ & \text{or } \llbracket\varphi\rrbracket_{SK}(v) = 2 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 0 \\ & \text{or } \llbracket\varphi\rrbracket_{SK}(v) = 2 \text{ and } \llbracket\psi\rrbracket_{SK}(v) = 2 \end{cases}$$

In order for $\llbracket(\varphi \vee \psi)\rrbracket_{SK}(v) \neq 2$, we have to make sure three things:

- It's not the case that $\llbracket\varphi\rrbracket_{SK}(v) = 0$ and $\llbracket\psi\rrbracket_{SK}(v) = 2$.
- It's not the case that $\llbracket\varphi\rrbracket_{SK}(v) = 2$ and $\llbracket\psi\rrbracket_{SK}(v) = 0$.
- It's not the case that $\llbracket\varphi\rrbracket_{SK}(v) = 2$ and $\llbracket\psi\rrbracket_{SK}(v) = 2$.

So the predicted presupposition is the conjunction of the following three conditions.

- If φ is false with respect to v , the presupposition of ψ is true with respect to v ; and
- if ψ is false with respect to v , the presupposition of φ is true with respect to v ; and
- The presupposition of at least one of φ and ψ is true.

2.4.2 Middle Kleene disjunction

(25) a. Assume that $\llbracket\psi\rrbracket_{MK}(v) = 2$.

$$\llbracket(\varphi \vee \psi)\rrbracket_{MK}(v) = \begin{cases} 1 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 1 \\ 2 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 2 \end{cases}$$

b. Now, while keeping the results of the first step, reason about both φ and ψ .

$$\begin{aligned} \llbracket(\varphi \vee \psi)\rrbracket_{MK}(v) &= \begin{cases} 1 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 1 \\ 2 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 2 \\ 1 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 1 \\ 0 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 0 \\ 2 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 2 \end{cases} \\ &= \begin{cases} 1 & \text{if either } \llbracket\varphi\rrbracket_{MK}(v) = 1 \\ & \text{or } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 1 \\ 0 & \text{if } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 0 \\ 2 & \text{if either } \llbracket\varphi\rrbracket_{MK}(v) = 2 \\ & \text{or } \llbracket\varphi\rrbracket_{MK}(v) = 0 \text{ and } \llbracket\psi\rrbracket_{MK}(v) = 2 \end{cases} \end{aligned}$$

In order for $\llbracket (\varphi \vee \psi) \rrbracket_{\text{MK}}(v) \neq 2$, we have to make the following two things to be the case:

- $\llbracket \varphi \rrbracket_{\text{MK}}(v) \neq 2$, i.e., the presupposition of φ is true with respect to v ; and
- If φ is false with respect to v , the presupposition of ψ is true with respect to v .

So, assuming that conjunction is asymmetric, while disjunction is symmetric, it seems that Middle Kleene is a good system for the former, but not the latter, while Strong Kleene is a good system for the latter, but not for the former.

See the handout on Trivalent Theories for versions of these theories augmented with quantifiers, which pose further issues, as well as for refinements proposed by [George 2008](#) and [Fox 2012](#).

3 Incremental Transparency

[Schlenker 2008a](#) proposes a different explanatory theory that is built on a bivalent semantics and two pragmatic principles.

3.1 Bivalent Semantics

We continue to work on the same language as $\mathcal{L}_{\text{SAT}}$, but with a completely bivalent semantics. It's basically like Classical Propositional Logic $\mathcal{L}$ and its semantics $\|\cdot\|$ except for the first clause.

- (26)
- $\llbracket \pi p_i \rrbracket_{\text{TR}}(v) = 1$ iff $\|\pi\|(v) = \|p_i\|(v) = 1$
 - $\llbracket \neg \varphi \rrbracket_{\text{TR}}(v) = 1$ iff $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 0$
 - $\llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = 1$ iff $\llbracket \varphi \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v) = 1$
 - $\llbracket (\varphi \vee \psi) \rrbracket_{\text{TR}}(v) = 0$ iff $\llbracket \varphi \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v) = 0$
 - $\llbracket (\varphi \rightarrow \psi) \rrbracket_{\text{TR}}(v) = 0$ iff $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$ and $\llbracket \psi \rrbracket_{\text{TR}}(v) = 0$

Although we have the subscripted formula π on atomic formulas, as in $\mathcal{L}_{\text{SAT}}$, the semantics is completely bivalent and has no notion of presuppositions.

We have been following [Schlenker 2008a](#) all along in separating the Projection Problem and Triggering Problem.

We assume, then, that the ‘presupposition’ of an elementary clause [i.e., π of πp_i] is just a distinguished part of its bivalent meaning, which is conceptualized as a ‘pre-condition’ of the whole. For present purposes we take this notion as primitive, and stipulate in the syntax of our logical forms that certain elements are ‘pre-conditions’ of others. Eventually we would want to *explain* why an entailment e of a meaning m is sometimes conceptualized as a pre-condition of m , but this is really a different question, which doesn’t pertain to the Projection Problem, but rather to the Triggering Problem – i.e. to the problem of determining why presuppositions of elementary clauses are generated to begin with. ([Schlenker 2008a](#): p. 170f)

3.2 Pragmatics

In the theory of [Schlenker 2008a](#), two pragmatic principles give rise to presupposition projection.

Ultimately, the pragmatic principles should be stated for speech acts involving expressions of English, or whichever object natural language one assumes. But let's for now pretend that they are about speech acts involving expressions of $\mathcal{L}_{\text{TR}}$.

(27) **Be Articulate**

In any syntactic environment, make sure πp_i occurs as the second conjunct of a conjunction $(\pi \wedge \pi p_i)$
(unless *Be Brief* rules out a felicitous use of $(\pi \wedge \pi p_i)$)

This could be reduced to a more general pragmatic principle, but we won't discuss that.

Be Articulate should ultimately be derived from Grice's 'Maxim of Manner', and specially from the requirement that one be 'orderly'. For present purposes, however, we take it as primitive. (Schlenker 2008a: p. 171)

Schlenker 2008a discusses two ways of stating the other principle, *Be Brief*. The first one we consider is based on the notion of *incremental transparency*:

(28) φ in a string ' $\alpha(\varphi \wedge \dots$ ' is **incrementally transparent** with respect to context set c if for any ψ and for any good final β and for any $v \in c$, $\llbracket \alpha(\varphi \wedge \psi)\beta \rrbracket_{\text{TR}}(v) = \llbracket \alpha\beta \rrbracket_{\text{TR}}(v)$.

(29) **Be Brief**

Asserting an expression Γ with respect to context set c is infelicitous if $\Gamma = \alpha(\varphi \wedge \dots$ and φ is incrementally transparent with respect to c .

This, too, could be reduced to a more general principle (as Schlenker 2008a: p. 174 remarks), but it will be crucial how exactly it is stated, so we'll stick to this definition.

3.3 Examples

3.3.1 No embedding: πp_i

- (30) a. πp_i
b. $(\pi \wedge \pi p_i)$

Suppose that c_0 doesn't entail π but is compatible with it.

- This means: for some $v \in c_0$, $\|\pi\|(v) = 1$ and for some $v \in c_0$, $\|\pi\|(v) = 0$.
- Then π in ' $\varepsilon(\pi \wedge \dots$ ' (where ε is an empty string) is *not* incrementally transparent with respect to c_0 , because, e.g., for the worlds $v \in c_0$ such that $\|\pi\|(v) = 0$,
 - $\llbracket \varepsilon(\pi \wedge \top)\varepsilon \rrbracket_{\text{TR}}(v) = 0$; while
 - $\llbracket \varepsilon\top\varepsilon \rrbracket_{\text{TR}}(v) = 1$

where $\top$ is a tautological proposition (e.g., $(\varphi \vee \neg\varphi)$).

- Consequently, *Be Articulate* prohibits with respect to c_0 a use of πp_i alone, in favor of $(\pi \wedge \pi p_i)$.

Suppose c_1 entails that π is false. Then similarly to the reasoning above, for any world $v \in c_1$, we have $\llbracket \varepsilon(\pi \wedge \top)\varepsilon \rrbracket_{\text{TR}}(v) = 0$ and $\llbracket \varepsilon\top\varepsilon \rrbracket_{\text{TR}}(v) = 1$, so π in ' $\varepsilon(\pi \wedge \dots$ ' is *not* incrementally transparent with respect to c_1 . Therefore, πp_i cannot be used on its own.

Suppose now that c_2 entails π .

- This means that for each $v \in c_2$, $\|\pi\|(v) = 1$.
- Then π in $\ulcorner \varepsilon(\pi \wedge \dots) \urcorner$ is incrementally transparent with respect to c_2 , because:
 - No matter what ψ is, all good finals β of $\varepsilon(\pi \wedge \psi) \dots$ is ε .
 - No matter what ψ is, for each world $v \in c_2$, we have: $\llbracket \varepsilon(\pi \wedge \psi) \varepsilon \rrbracket_{\text{TR}}(v) = \llbracket \varepsilon \psi \varepsilon \rrbracket_{\text{TR}}(v)$.
- Then *Be Brief* renders $(\pi \wedge \pi p_i)$ in felicitous with respect to c_2 , so πp_i can be used on its own with respect to c_2 .

Consequently, the pragmatic presupposition of πp_i amounts to that π be entailed by the context.

3.3.2 Negation: $\neg \pi p_i$

- (31) a. $\neg \pi p_i$
 b. $\neg(\pi \wedge \pi p_i)$

Suppose that c_0 doesn't entail π but is compatible with it.

- Then π in $\ulcorner \neg(\pi \wedge \dots) \urcorner$ is *not* incrementally transparent with respect to c_0 , because, e.g., for the worlds $v \in c_0$ such that $\|\pi\|(v) = 0$,
 - $\llbracket \neg(\pi \wedge \top) \varepsilon \rrbracket_{\text{TR}}(v) = 1$; while
 - $\llbracket \neg \top \varepsilon \rrbracket_{\text{TR}}(v) = 0$
- Consequently, $\neg \pi p_i$ is infelicitous with respect to c_0 .

By the same reasoning, $\neg \pi p_i$ is infelicitous with respect to c_1 that entails that π is false.

For c_2 , which entails that π true,

- π in $\ulcorner \varepsilon(\pi \wedge \dots) \urcorner$ is incrementally transparent with respect to c_2 , because:
 - No matter what ψ is, all good finals β of $\neg(\pi \wedge \psi) \dots$ is ε .
 - No matter what ψ is, for each world $v \in c_2$, we have: $\llbracket \varepsilon(\pi \wedge \psi) \varepsilon \rrbracket_{\text{TR}}(v) = \llbracket \varepsilon \psi \varepsilon \rrbracket_{\text{TR}}(v)$.
- Then *Be Brief* renders $(\pi \wedge \pi p_i)$ in felicitous with respect to c_2 , so πp_i can be used on its own with respect to c_2 .

Consequently, $\neg \pi p_i$ has the same pragmatic presupposition as πp_i , that π be entailed by the context set.

3.3.3 First conjunct: $(\pi p_i \wedge \varphi)$

- (32) a. $(\pi p_i \wedge \varphi)$
 b. $((\pi \wedge \pi p_i) \wedge \varphi)$

For c_0 , which is compatible with π but does not entail it to be false, π in $\ulcorner ((\pi \wedge \dots) \urcorner$ is *not* incrementally transparent, because for any world $v \in c_0$ such that $\|\pi\|(v) = 0$, we have $\llbracket ((\pi \wedge \top) \wedge \top) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\top \wedge \top) \rrbracket_{\text{TR}} = 1$.

By the same token, for c_1 , which entails that π is false, for any world $v \in c_1$, we have $\llbracket ((\pi \wedge \top) \wedge \top) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\top \wedge \top) \rrbracket_{\text{TR}} = 1$, so π in $\ulcorner ((\pi \wedge \dots) \urcorner$ is *not* incrementally transparent.

For c_2 , which entails that π is true, π in $\ulcorner ((\pi \wedge \dots) \urcorner$ is incrementally transparent, because:

- For any ψ , any good final of $\ulcorner (\pi \wedge \psi) \dots \urcorner$ involves one and only one instance of a binary connective. So we only need to reason about the following three possibilities (for some formula χ):
 - $((\pi \wedge \psi) \wedge \chi)$
 - $((\pi \wedge \psi) \vee \chi)$
 - $((\pi \wedge \psi) \rightarrow \chi)$
- Notice, incidentally, that for any ψ for any $v \in c_2$, $\llbracket (\pi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$.
- Given the compositionality of the language, for any formulas ψ and χ and for any $v \in c_2$, we have:
 - $\llbracket ((\pi \wedge \psi) \wedge \chi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \chi) \rrbracket_{\text{TR}}(v)$
 - $\llbracket ((\pi \wedge \psi) \vee \chi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \vee \chi) \rrbracket_{\text{TR}}(v)$
 - $\llbracket ((\pi \wedge \psi) \rightarrow \chi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \rightarrow \chi) \rrbracket_{\text{TR}}(v)$

This means that $(\pi p_i \wedge \varphi)$ has the same pragmatic presupposition as πp_i alone (and possibly more if φ also triggers pragmatic presuppositions), which is to say that the presupposition of the first conjunct projects out.

3.3.4 First disjunct: $(\pi p_i \vee \varphi)$

Notice that the above reasoning about the first conjunct does not involve reference to $\wedge$, but only to the fact that there is a binary operator following πp_i (which is indicated by the extra opening parenthesis).

So the same reasoning applies to the first component of any binary operator.

Consequently, the presupposition of πp_i is predicted to project out in $(\pi p_i \vee \varphi)$.

3.3.5 Conditional antecedent: $(\pi p_i \rightarrow \varphi)$

By the same reasoning, the presupposition of πp_i is predicted to project out in $(\pi p_i \rightarrow \varphi)$.

3.3.6 Second conjunct: $(\varphi \wedge \pi p_i)$

- (33) a. $(\varphi \wedge \pi p_i)$
 b. $(\varphi \wedge (\pi \wedge \pi p_i))$

Consider c_0 , which entails that φ is false. Then π in $\ulcorner (\varphi \wedge (\pi \wedge \dots) \urcorner$ will be incrementally transparent with respect to c_0 , because

- For any ψ , the only good final of $\ulcorner (\varphi \wedge (\pi \wedge \psi) \dots \urcorner$ is a closing parenthesis).
- For any ψ and for any world in $v \in c_0$, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = 0$.

Consider c_1 , which entails that φ is true. Among such contexts:

- We have c_1^0 that entails that π is false. With respect to c_1^0 , π in $\ulcorner (\varphi \wedge (\pi \wedge \dots) \urcorner$ will *not* be incrementally transparent, because for any world $v \in c_1^0$, we have $\llbracket (\varphi \wedge (\pi \wedge \top)) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\varphi \wedge \top) \rrbracket_{\text{TR}}(v) = 1$.

- We also have c_1^1 that is compatible with π , but does not entail it. With respect to c_1^1 , π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will *not* be incrementally transparent, either, because for any world $v \in c_1^1$ such that $\|\pi\|(v) = 0$, we have $\llbracket (\varphi \wedge (\pi \wedge \top)) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\varphi \wedge \top) \rrbracket_{\text{TR}}(v) = 1$.
- Now consider c_1^2 that entails that π is true. Then π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will be incrementally transparent with respect to c_1^2 , because:
 - The cases we have to consider are all of the form $(\varphi \wedge (\pi \wedge \psi))$ for different ψ .
 - It is clear that for any ψ and for any $v \in c_1^2$, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$.

Finally, consider c_2 , which is compatible with φ but does not entail it to be true. Among such contexts:

- We have c_2^0 that entails that π is false. Then π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will *not* be incrementally transparent with respect to c_2^0 , because for any world $v \in c_2^0$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$, $\llbracket (\varphi \wedge (\pi \wedge \top)) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\varphi \wedge \top) \rrbracket_{\text{TR}}(v) = 1$.
- For any c_2^1 that entails that π is true, π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will be incrementally transparent with respect to c_2^1 , because:
 - For any ψ , the only good final of $\ulcorner \varphi \wedge (\pi \wedge \psi) \dots \urcorner$ is a closing parenthesis).
 - For any ψ and for any $v \in c_2^1$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 0$, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = 0$.
 - For any ψ and for any $v \in c_2^1$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$.
- We also have c_2^2 that is compatible with π but does not entail it to be true. Among these, there are two cases:
 - c_2^{2a} that is simultaneously compatible with the truth of φ and the falsity of π . Here, π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will *not* be incrementally transparent with respect to c_2^{2a} , because for any world $v \in c_2^{2a}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$ and $\|\pi\|(v) = 0$, $\llbracket (\varphi \wedge (\pi \wedge \top)) \rrbracket_{\text{TR}}(v) = 0$ but $\llbracket (\varphi \wedge \top) \rrbracket_{\text{TR}}(v) = 1$.
 - c_2^{2b} that is not compatible with the truth of φ and the falsity of π at the same time, which is to say that φ entails π . Then, π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ will be incrementally transparent with respect to c_2^{2b} , because:
 - For any $v \in c_2^{2b}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 0$, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v) = 0$.
 - For any $v \in c_2^{2b}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$, it is guaranteed that $\|\pi\|(v) = 1$, and then since for any $v \in c_2^{2b}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = \|\pi\| = 1$ and for any ψ , $\llbracket (\pi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$, we also have, $\llbracket (\varphi \wedge (\pi \wedge \psi)) \rrbracket_{\text{TR}}(v) = \llbracket (\varphi \wedge \psi) \rrbracket_{\text{TR}}(v)$.

To sum up, the contexts that render π in $\ulcorner \varphi \wedge (\pi \wedge \dots \urcorner$ incrementally transparent are the following three types:

- contexts c_0 that entail that φ is false.
- contexts c_1^2 that entail that φ is true and entails that π is true.
- contexts c_2^1 that are compatible with φ but do not entail it, and entails that π is true.

- contexts c_2^{2b} that are compatible with φ but do not entail it, are compatible with π and do not entail it, but φ contextually entails π .

The latter two cases can be put into one category: contexts that are compatible with φ but do not entail it, and are compatible with π and license contextual entailment from φ to π . (NB: contextual entailment holds when π is just true everywhere in the context)

This in turn can be put together with the second one: contexts that are compatible with both φ and π and license contextual entailment from φ to π .

So, the pragmatic presupposition that π gives rise to in $(\varphi \wedge (\pi p_i))$ is: Either the context entails φ to be false, or else it is compatible with both φ and π and licenses contextual entailment from φ to π .

We can ignore the contexts that entail the falsity of φ , as well as contexts that entail the falsity of π , because in such a context the conjunction is unassertable anyway for independent reason, so we are left with the conditional presupposition that if φ is the case, π is the case.

3.3.7 *Second disjunct*: $(\varphi \vee \pi p_i)$

The reasoning would be parallel to the case of second conjunct, except that the roles of the truth and falsity of φ will flip: When it is true, we'll know that the whole disjunction will be true, regardless of π .

Consequently, we will drive the conditional presupposition: if φ is false, π is true.

Exercise: Spell out the reasoning in full.

3.3.8 *Conditional consequent*: $(\varphi \vee \pi p_i)$

Here, the reasoning will be similar to the case of projection from the second conjunct. When φ is false, π won't matter.

Exercise: Spell out the reasoning in full.

3.4 Notes on presupposition accommodation

As [Schlenker 2008a](#): §2.5.1 discusses, Lewisian global accommodation adopted as is under Transparent Theory, in terms of adjusting the conversational common ground.

He also suggests a way to model local accommodation in Transparent Theory: “[a]ll we need to say is that, under duress (e.g. to avoid a very bad conversational outcome, such as the utterance of a contradiction or a triviality), one may assume that the speaker did not obey *Be Articulate*. This leaves us with an unadorned bivalent meaning [...]” (p. 181).

3.5 Object language

We reasoned about our intermediate formal language, $\mathcal{L}_{\text{SAT}}$, which contains parentheses, which unambiguously indicate the structure of a given formula. In particular, it is important in reasoning about good finales that each opening parenthesis is matched with one and only one closing parenthesis.

[Schlenker 2008a](#) remarks on this point:

It is worth noting that in this statement α , β , and γ [in the definition of *Be Brief* in (18) on p. 174] range over *strings of symbols* of a language that includes parentheses to disambiguate structure (equivalently, one could think of α , β , and γ as

-ranging over *parts of a syntactic tree*, though this would require further elaborations). (Schlenker 2008a: p. 174)

3.5.1 Summary

- A presupposition triggered in the left argument of a binary connective projects out.
- a presupposition triggered in the right argument of a binary connective gives rise to a conditional presupposition.

For the propositional language $\mathcal{L}_{\text{SAT}}$, the predictions of the theory are identical to Satisfaction Theory (see Schlenker 2007 for a formal proof).

For non-canonical binary connectives, the predictions depend on their (bivalent) semantics. There are $2^4 = 16$ possible binary connectives in Classical Propositional Logic. In addition to the standard three ($\wedge$, $\vee$, $\rightarrow$), we have:

φ	ψ	$(\varphi T \psi)$	$(\varphi F \psi)$	$(\varphi L \psi)$	$(\varphi R \psi)$	$(\varphi \bar{L} \psi)$	$(\varphi \bar{R} \psi)$
1	1	1	0	1	1	0	0
1	0	1	0	1	0	0	1
0	1	1	0	0	1	1	0
0	0	1	0	0	0	1	1

φ	ψ	$(\varphi \leftarrow \psi)$	$(\varphi \leftrightarrow \psi)$	$(\varphi \perp \psi)$	$(\varphi \uparrow \psi)$	$(\varphi \downarrow \psi)$	$(\varphi \rightarrow \psi)$	$(\varphi \leftarrow \psi)$
1	1	1	1	0	0	0	0	0
1	0	1	0	1	1	0	1	0
0	1	0	0	1	1	0	0	1
0	0	1	1	0	1	1	0	0

- Those that ignore one or both of the arguments (T , F , L , R , $\bar{L}$, $\bar{R}$) will of course ignore presuppositions triggered there, and presuppositions triggered in the argument that is not ignored will simply project out.
- For the rest, which have more content, ($\leftarrow$, $\leftrightarrow$, $\perp$, $\uparrow$, $\downarrow$, $\rightarrow$, $\leftarrow$), we derive the same asymmetric pattern as above.

Exercise: Spell out Transparency reasoning with these non-canonical connectives.

For natural language applications, it is important that linear order is preserved in the translation, e.g., We want to interpret ‘ ϕ and ψ ’ as $(\varphi \wedge \psi)$, rather than $(\psi \wedge \varphi)$.

But conditionals pose an issue. *If*-clauses can appear after the consequent, but the interpretation, including the projection pattern, does not seem to change.

- (34) a. If John is from Russia, he will go back there soon.
b. John will go back to Russia, if he is from there.
- (35) a. If Mary never smoked before, she will start smoking soon.
b. Mary will start smoking soon, if she has never smoked before.

So here, no matter what the surface order is, we want to interpret both in terms of something like $\rightarrow$, and never use $\leftarrow$, because otherwise we’d make the wrong prediction that the presupposition of the consequent should project out in the b-examples.

Two caveats:

- It's (almost) a consensus that the semantics of *if*-clauses is not accounted for in terms of material implication anyway.
- It's possible that the LF representations of pre-posed and post-posed conditions are similar, and maybe *if*-clauses always 'precede' the consequent at LF (although it's strange to talk about linear order at LF).

Regarding the second point, [Schlenker 2008a](#): Appendix F remarks:

In the incremental version of Transparency, we have taken linear order to be crucial. But it would be reasonable to explore versions of the theory in which syntactic structures are ordered by other principles. In fact, it would make good conceptual sense to take *order of processing* to be the primitive notion, without being committed to the view that the order of processing is just linear order. Other alternatives could be considered as well, for instance ordering by the syntactician's notion of 'c-command'. Different predictions would not doubt be obtained.

In any case, the analogy to processing is perhaps a bit bizarre given that Transparency Theory only works on disambiguated representations, while in reality, natural languages are full of ambiguities (lexical, structural, and scopal ambiguity).

It's open at this point what exactly would happen with ambiguous languages that are more similar to natural languages.

3.6 Problem of ungrammatical conjunctions

[Beaver 2008](#) points out an important issue of this theory (see also [Schlenker 2008b](#): fn. 21).

(36) Mary is thinner than the King of France is fat. ([Beaver 2008](#): p. 226)

The definite description *the King of France* has a presupposition, and the Transparency Theory asks us to consider the version with an asserted conjunction, (37), but it's ungrammatical (although [Beaver 2008](#) only gives one question mark to it).

(37) ?Mary is thinner than there is a King of France and the King of France is fat. ([Beaver 2008](#): p. 226)

This issue arises with all sorts of cases that would involve violations of the Coordinate Structure Constraint.

- (38) a. When do you think that Mary is immigrating to Israel __?
 b. *When do you think that Mary is originally not from Israel and Mary is immigrating to Israel __?
- (39) a. Mary brought something that both of John's sisters like __.
 b. *Mary brought something that John has exactly two sisters and both of John's sisters like __.

But of course we want to derive pragmatic presuppositions for these sentences. This seems to me to be a pretty difficult problem.

4 Symmetric Transparency

Recall that Incremental Transparency makes asymmetric predictions for the canonical binary connectives $\wedge$, $\vee$, and $\rightarrow$.

- The presupposition triggered in the first argument will project out.
- The presupposition triggered in the second argument give rise to conditional presuppositions: $\varphi \rightarrow \pi$ in the case of $\wedge$ and $\rightarrow$, and $\neg\varphi \rightarrow \pi$ in the case of $\vee$.

This is a good result for conjunction but questionable for disjunction, which seems to be symmetric in natural language.

4.1 Symmetric Transparency

[Schlenker 2008a](#): §3.2 presents a symmetric version of the theory, where presuppositions triggered in the first argument also give rise to conditional presuppositions. This is done by making *Be Brief* insensitive to linear order:

- (40) φ occurring in $\ulcorner \alpha(\varphi \wedge \psi)\beta \urcorner$ is **symmetrically transparent** with respect to context set c if for any χ and for any $v \in c$, $\llbracket \alpha(\varphi \wedge \chi)\beta \rrbracket_{\text{TR}}(v) = \llbracket \alpha\chi\beta \rrbracket_{\text{TR}}(v)$.
- (41) **Be Brief** (symmetric)
Asserting an expression Γ with respect to context set c is infelicitous if $\Gamma = \alpha(\varphi \wedge \dots$ and φ is symmetrically transparent with respect to c .

4.2 Example: First conjunct

To illustrate how Symmetric Transparency work, let's consider the case of a presupposition triggered in the first conjunct.

- (42) a. $(\pi p_i \wedge \varphi)$
b. $((\pi \wedge \pi p_i) \wedge \varphi)$

Suppose that c_0 entails that φ is false. With respect to context, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ is symmetrically transparent, because for any ψ and for any $v \in c_0$, $\llbracket ((\pi \wedge \psi) \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$.

Suppose that c_1 entails that φ is true. Among such contexts, there are three types to consider:

- With respect to a context c_1^0 that entails that π is false, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will not be symmetrically transparent, because for any $v \in c_1^0$, $\llbracket ((\pi \wedge \top) \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$ whereas $\llbracket (\top \wedge \varphi) \rrbracket_{\text{TR}}(v) = 1$.
- With respect to a context c_1^1 that entails that π is true, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will be symmetrically transparent, because for any ψ and for any $v \in c_1^1$, $\llbracket (\pi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$, and therefore for any ψ and for any $v \in c_1^1$ $\llbracket ((\pi \wedge \psi) \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \varphi) \rrbracket_{\text{TR}}(v)$.
- With respect to a context c_1^2 that is compatible with π but does not entail it, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will not be symmetrically transparent, because for any $v \in c_1^2$ such that $\llbracket \pi \rrbracket(v) = 0$, $\llbracket ((\pi \wedge \top) \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$ whereas $\llbracket (\top \wedge \varphi) \rrbracket_{\text{TR}}(v) = 1$.

Suppose that c_2 is compatible with φ but does not entail it. Among such contexts, there are three types to consider:

- With respect to c_2^0 that entails that π is false, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will not be symmetrically transparent, because for any $v \in c_2^0$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$, $\llbracket ((\pi \wedge \top) \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$ whereas $\llbracket (\top \wedge \varphi) \rrbracket_{\text{TR}}(v) = 1$.
- With respect to c_2^1 that entails that π is true, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will be symmetrically transparent, because:
 - for any $v \in c_2^1$ and for any ψ , $\llbracket (\pi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$; and
 - therefore, for any $v \in c_2^1$ and for any ψ , $\llbracket ((\pi \wedge \psi) \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \varphi) \rrbracket_{\text{TR}}(v)$.
- With respect to c_2^2 that is compatible with π but does not entail it to be true, there are two cases to consider:
 - With respect to c_2^{2a} that are simultaneously compatible with the truth of φ and the falsity of π , π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will not be symmetrically transparent, because for any $v \in c_2^{2a}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$ and $\llbracket \pi \rrbracket_{\text{TR}}(v) = 0$, $\llbracket ((\pi \wedge \top) \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$ whereas $\llbracket (\top \wedge \varphi) \rrbracket_{\text{TR}}(v) = 1$.
 - With respect to c_2^{2b} where φ contextually entails π , π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will be symmetrically transparent, because:
 - For each world $v \in c_2^{2b}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 0$, we have $\llbracket ((\pi \wedge \psi \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \varphi) \rrbracket_{\text{TR}}(v) = 0$.
 - For each world $v \in c_2^{2b}$ such that $\llbracket \varphi \rrbracket_{\text{TR}}(v) = 1$, it is guaranteed that $\llbracket \pi \rrbracket_{\text{TR}}(v) = 1$, and for each such world v , for each ψ , $\llbracket (\pi \wedge \psi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$, and therefore $\llbracket ((\pi \wedge \psi) \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket (\psi \wedge \varphi) \rrbracket_{\text{TR}}(v) = \llbracket \psi \rrbracket_{\text{TR}}(v)$.

To summarize, π in $((\pi \wedge \pi p_i) \wedge \varphi)$ will be symmetrically transparent in four types of contexts:

- contexts c_0 that entail φ to be false.
- contexts c_1^1 that entail φ to be true and π to be true.
- contexts c_2^1 that are compatible with φ but do not entail it to be true, and entail π to be true.
- contexts c_2^{2b} that are compatible with φ but do not entail it to be true and are compatible with π but do not entail it to be true, and license contextual entailment from φ to π .

As we saw before, this amounts to the pragmatic presupposition that either the context entails φ to be false, or it is compatible with both φ and π and φ contextually entails π .

Exercise: Spell out the reasoning for presuppositions triggered in the first disjunct and the conditional antecedent.

For presuppositions triggered in the second conjunct, the reasoning will be completely identical to the reasoning under Incremental Transparency, as the good finals were anyway trivial.

4.3 Conceptual issue

About the symmetric version of *Be Brief*, Schlenker remarks:

[...] the symmetric version of *Be Brief* is admittedly harder to motivate in terms of a processing metaphor than its incremental counterpart: if one has heard the end of the sentence β , *a fortiori* one has heard the second conjunct d' [ψ in our formulation], and thus it is a bit mysterious why the latter should not be taken into account in the new version of *Be Brief*.

(Schlenker 2008a: p. 190)

As evidence for the symmetric version of *Be Brief*, Schlenker presents the following examples:

- (43) a. ??Mary is pregnant and (she is) happy, if she is pregnant.
b. ??Mary is pregnant and (she is) happy, if she is expecting a child.
c. ??Mary is pregnant and (she is) happy, if she is expecting a boy.
(Schlenker 2008a: p. 190)
- (44) a. ?If Mary is not (both) pregnant and happy, she is not pregnant.
b. ?If Mary is not (both) pregnant and happy, she is not expecting a child.
c. ?If Mary is not (both) pregnant and happy, she is not expecting a boy.
(Schlenker 2008a: p. 191)
- (45) a. ?Unless Mary is pregnant and (she is) happy, she is not pregnant.
b. ?Unless Mary is pregnant and (she is) happy, she is not expecting a child.
c. ?Unless Mary is pregnant and (she is) happy, she is not expecting a boy.
(Schlenker 2008a: p. 191)

According to his judgments, the latter two sets of examples are “slightly more acceptable than” (p. 191) the first, remarking that it might be because “a somewhat more complex reasoning is necessary to determine that the first conjunct is semantically eliminable once the entire sentence has been heard” (p. 191).

4.4 English data

The real question is what the facts are.

4.4.1 Symmetric conjunctions?

- (46) a. John knows that he is sick and he has cancer!
b. John doesn't know that he sick[sic.] and he has had pancreas cancer for five years.
c. I really doubt that John doesn't know he is sick and that he has had pancreas cancer for five years – one doesn't survive for that long with such a disease.
d. Is it true that John knows he is sick and that he has cancer?
e. Is it true that John doesn't know he is sick and that he has cancer?
(Schlenker 2008a: p. 193)
- (47) a. John has stopped smoking and he used to smoke five packs a day!
b. John hasn't stopped smoking and he used to smoke five packs a day!
c. I really doubt that John has stopped smoking and (that he) used to smoke five packs a day – one doesn't get off easily when one has been that addicted.
d. Is it true that John has stopped smoking and (that he) used to smoke five packs a day?
e. Is it true that John hasn't stopped smoking and (that he) used to smoke five

packs a day?

(Schlenker 2008a: p. 193)

With respect to all of these cases, it seems to me that it is *to some extent* possible to understand the sentences with no presupposition at all, but that such a reading is very difficult; if correct, both sides of the observation should be derived.

(Schlenker 2008a: p. 194)

4.4.2 *Symmetric conditionals?*

- (48) a. If this house has a bathroom, the bathroom is well hidden.
b. If the bathroom is not hidden, this house has no bathroom.

(Schlenker 2008a: p. 186)

Schlenker 2008a also points out that for conditionals, the order does not seem to matter.

- (49) a. The bathroom is well hidden, if this house has a bathroom.
b. Mary's doctor knows that she is expecting a child, if she is pregnant.

(Schlenker 2008a: p. 186)

Schlenker says about these examples and the conjunction data above:

In all cases, then, the facts are roughly the same; each sentence can be read without presupposition, though this is easier in the 'canonical' than in the 'reserved' order (with the possible exception of post-posed[sic.] *if*-clauses, which behave very much like pre-posed ones). It would be misguided to seek a lexical solution to this problem since it would have to make a stipulation about disjunction, and another one about conditionals, etc. A general treatment is called for.

(Schlenker 2008a: p. 186)

- (50) a. If Mary has a violin, her instrument is well hidden.
b. If Mary's instrument is not well hidden, she doesn't have a violin.

(Schlenker 2008a: p. 188)

- (51) a. If Mary had cancer, her doctor knows that she was sick.
b. If Mary's doctor didn't know that she is sick, she didn't have cancer.

(Schlenker 2008a: p. 188)

4.4.3 *Asymmetric disjunction?*

- (52) a. Mary doesn't have a violin, or else her instrument is well hidden.
b. Mary's instrument is well hidden, or else she doesn't have a violin.

(Schlenker 2008a: p. 187)

- (53) a. Mary doesn't have cancer, or else her doctor will realize that she is sick.
b. Mary's doctor will realize that she is sick, or else she doesn't have cancer.

(Schlenker 2008a: p. 187)

He remarks that the b-examples here are more easily read with projected presuppositions than the a-examples (or the examples above with post-posed conditionals, where the presupposition trigger linearly precedes the clause that entails it), but also says "the data are not as sharp as a fully asymmetric account would lead one to expect" (p. 188), and that "I believe that the data partly hinge on intonation" (fn. 23, p. 188).

4.5 Three options

After discussing the possibility to account for symmetric cases in terms of local accommodation, [Schlenker 2008a](#) concludes:

Pending further investigation, I conclude tht (i) as a first approximation, the connectives display the asymmetric behavior predicted by Heim 1983, Beaver 2001 and our existing version of Transparency; however (ii) with some difficulty, a symmetric behavior can be obtained, and it is not wholly reducible to local accommodation. ([Schlenker 2008a](#): p. 189)

But I feel that this is a bit biased, especially for the disjunction data.

[Schlenker 2008a](#): §3.4 suggests three options:

- Assume only Incremental Transparency, and account for symmetric readings by resorting to something else (e.g., local accommodation).
- Assume only Symmetric Transparency, but this doesn't seem to be able to account for the overwhelming preference for asymmetric readings of conjunctions.
- Assume both Incremental and Symmetric Transparency, but with different strengths.
 - A (redundant) conjunct is less infelicitous to be symmetrically transparent, than to both symmetrically and incrementally transparent.
 - The acceptability of a sentence with a presupposition trigger is inversely correlated with the version with an asserted conjunct expressing the content of the presupposition.

5 Quantification and attitude reports

[maybe next time]

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