Presupposition projection and quantification

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Presupposition Reading Group

We will present a version of Satisfaction Theory with generalized quantifiers as an extension of $\mathcal{L}^{\mathsf{Q}}_{\mathsf{SAT}}$ (Propositional Satisfaction Theory).
We will begin with a bivalent logic with classical quantifiers, $\mathcal{L}^{\mathsf{Q}}$.

1 Predicate Logic with Classical Quantifiers $\mathcal{L}^{\mathsf{Q}}$

1.1 Syntax

We introduce predicates, in addition to unanalyzable atomic propositions.

- Predicates require a specific number of arguments.
  - Predicates of arity 1 (unary predicates): $\mathtt{P}^1_1, \mathtt{P}^1_2, \ldots$
  - Predicates of arity 2 (binary predicates): $\mathtt{P}^2_1, \mathtt{P}^2_2, \ldots$
  - Predicates of arity 3 (ternary predicates): $\mathtt{P}^3_1, \mathtt{P}^3_2, \ldots$
- The atomic propositions ($\mathtt{p}_1, \mathtt{p}_2, \ldots$) of $\mathcal{L}$ (Propositional Logic) are seen as 0-place predicates.

Arguments of predicates are called terms. There are two types.

- Constants $\mathtt{c}_1, \mathtt{c}_2, \ldots$ are terms.
- Variables $\mathtt{v}_1, \mathtt{v}_2, \ldots$ are terms.
- Nothing else is a term.

We combine predicates and terms to form propositions. We denote the set of all simple propositions that contain no connectives or quantifiers by $\mathcal{S}$.

- Each atomic proposition $\mathtt{p_i}$ is a member of $\mathcal{S}$.
- If Π is an n-ary predicate and $\tau^1, \ldots, \tau^n$ are all terms, then $\Pi\,\tau^1 \ldots \tau^n \in \mathcal{S}$.
- Nothing else is in $\mathcal{S}$.

Now, we form complex sentences. Our quantifiers are generalized quantifiers. (Note that we allow vacuous quantification, for the sake of simplicity). $\mathcal{L}^{\mathsf{Q}}$ is the smallest set such that:

- $\mathcal{S} \subseteq \mathcal{L}^{\mathsf{Q}}$.
- If $\varphi, \psi \in \mathcal{L}^{\mathsf{Q}}$, then $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi) \in \mathcal{L}^{\mathsf{Q}}$.
- If ξ is a variable and $\varphi \in \mathcal{L}^{\mathsf{Q}}$, then $\exists\xi\varphi, \forall\xi\varphi \in \mathcal{L}^{\mathsf{Q}}$.

1.2 Semantics

Anticipating Satisfaction Theory, we will include possible worlds: A model $\mathcal{M}$ for $\mathcal{L}_{\mathsf{SAT}}$ is a triple $\langle \mathcal{D}, \mathcal{W}, \mathcal{I} \rangle$, where:

- $\mathcal{D}$: a non-empty set of individuals
- $\mathcal{W}$: a non-empty set of possible worlds, disjoint from $\mathcal{D}$
- $\mathcal{I}$: interpretation function such that for each $w \in \mathcal{W}$,
  - for each constant γ, $\mathcal{I}(\gamma) \in \mathcal{D}$; and
  - for each 0-ary predicate $\mathtt{p_i}$, $\mathcal{I}(w)(\mathtt{p_i}) \in \{0, 1\}$; and
  - for each n-ary predicate π (for $n \geqslant 1$), $\mathcal{I}(w)(\pi) \in \mathcal{D}^n$.

We will use *assignment functions* g to interpret variables. Assignment functions are (total) functions from the set of all variables to $\mathcal{D}$.

We will see denotations as functions from pairs $\langle w, g \rangle$ consisting of a possible world and an assignment function. It'd be cumbersome to always write pairs $\langle w, g \rangle$, so let us introduce a designated world variable $\mathtt{v}_0$, and represent each $\langle w, g \rangle$ by an *extended assigment* a such that for each variable ξ,

$$a(\xi) = \begin{cases} w & \text{if } \xi = \mathtt{v}_0 \\ g(\xi) & \text{otherwise} \end{cases}$$

We denote the set of all extended assignments by $\mathcal{A}$.

Assignment modification: For any extended assignment function a, any variable ξ distinct from $\mathtt{v}_0$, and any $d \in D$, $a[\xi \mapsto d]$ is that extended assignment function such that for each variable ξ':

$$a[\xi \mapsto d](\xi') = \begin{cases} d & \text{if } \xi \equiv \xi' \\ a(\xi) & \text{otherwise} \end{cases}$$

- Terms
  - For each constant γ, $\|\gamma\|(a) = \mathcal{I}(\gamma)$.
  - For each variable ξ distinct from $\mathtt{v}_0$, $\|\xi\|(a) = a(\xi)$.
- Formulas
  - $\|\mathtt{p_i}\|(a) = \mathcal{I}(a(\mathtt{v}_0))(\mathtt{p_i})$.
  - $\|\Pi\tau^1 \ldots \tau^n\|(a) = \begin{cases} 1 & \text{if } \langle \|\tau^1\|(a), \ldots, \|\tau^n\|(a) \rangle \in \mathcal{I}(a(\mathtt{v}_0))(\Pi) \\ 0 & \text{otherwise} \end{cases}$
  - $\|\neg\varphi\|(a) = \begin{cases} 1 & \text{if } \|\varphi\|(a) = 0 \\ 0 & \text{otherwise} \end{cases}$
  - $\|(\varphi \wedge \psi)\|(a) = \begin{cases} 1 & \text{if } \|\varphi\|(a) = \|\psi\|(a) = 1 \\ 0 & \text{otherwise} \end{cases}$
  - $\|(\varphi \vee \psi)\|(a) = \begin{cases} 0 & \text{if } \|\varphi\|(a) = \|\psi\|(a) = 0 \\ 1 & \text{otherwise} \end{cases}$

– $\|(\varphi \to \psi)\|(a) = \begin{cases} 0 & \text{if } \|\varphi\|(a) = 1 \text{ and } \|\psi\|(a) = 0 \\ 1 & \text{otherwise} \end{cases}$

– $\|\exists\xi\varphi\|(a) = \begin{cases} 1 & \text{for some } x \in \mathcal{D},\ \|\varphi\|(a[\xi \mapsto x]) = 1 \\ 0 & \text{otherwise} \end{cases}$

– $\|\forall\xi\varphi\|(a) = \begin{cases} 0 & \text{for some } x \in \mathcal{D},\ \|\varphi\|(a[\xi \mapsto x]) = 0 \\ 1 & \text{otherwise} \end{cases}$

2 Dynamic Predicate Logic $\mathcal{L}^{\mathsf{Q}}_{\mathsf{D}}$

Before Satisfaction Theory, we should review Dynamic Predicate Logic (DPL; Groenendijk & Stokhof 1991). DPL is properly dynamic in its interpretation of quantifiers.
$\mathcal{L}^{\mathsf{Q}}_{\mathsf{D}}$ has the same syntax as $\mathcal{L}^{\mathsf{Q}}$ (so $\mathcal{L}^{\mathsf{Q}} = \mathcal{L}^{\mathsf{Q}}_{\mathsf{D}}$), but $\mathcal{L}^{\mathsf{Q}}_{\mathsf{D}}$ formulas are interpreted as relations between (extended) assignments.

2.1 Semantics

The existential quantifier has a special function called *random assignment*: Let ξ be a variable distinct from v_0. Then a and a' are said to differ at most with respect to ξ (written $a \approx_\xi a'$) iff for each variable ζ, if $\zeta \not\equiv \xi$, then $a(\zeta) = a'(\zeta)$.

- For any $\varphi \in \mathcal{S}$, $[\![\varphi]\!] = \{ \langle a, a' \rangle \mid a = a' \text{ and } \|\varphi\|(a) = 1 \}$
- $[\![\neg\varphi]\!] = \{ \langle a, a' \rangle \mid a = a' \text{ and for no } b \in \mathcal{A}, \langle a, b \rangle \in [\![\varphi]\!] \}$
- $[\![(\varphi \wedge \psi)]\!] = \{ \langle a, a' \rangle \mid \text{for some } b \in \mathcal{A},\ \langle a, b \rangle \in [\![\varphi]\!] \text{ and } \langle b, a' \rangle \in [\![\psi]\!] \}$
- $[\![(\varphi \vee \psi)]\!] = \left\{ \langle a, a' \rangle \,\middle|\, \begin{array}{l} a = a' \text{and for some } b \in \mathcal{A}, \text{ either } \langle a, b \rangle \in [\![\varphi]\!], \\ \text{or else } \langle a, b \rangle \in [\![(\neg\varphi \wedge \psi)]\!] \end{array} \right\}$
- $[\![(\varphi \to \psi)]\!] = [\![\neg(\varphi \wedge \neg\psi)]\!] = \left\{ \langle a, a' \rangle \,\middle|\, \begin{array}{l} a = a' \text{ and for every } b \in \mathcal{A} \text{ such that } \langle a, b \rangle \in [\![\varphi]\!] \\ \text{there is no } b' \in \mathcal{A} \text{ such that } \langle b, b' \rangle \in [\![\psi]\!] \end{array} \right\}$
- $[\![\exists\xi\varphi]\!] = \{ \langle a, a' \rangle \mid \text{for some } b \in \mathcal{A},\ a \approx_\xi b \text{ and } \langle b, a' \rangle \in [\![\varphi]\!] \}$
- $[\![\forall\xi\varphi]\!] = [\![\neg\exists\xi\neg\varphi]\!] = \left\{ \langle a, a' \rangle \,\middle|\, \begin{array}{l} a = a' \text{ and for every } b \in \mathcal{A} \text{ such that } a \approx_\xi b, \\ \text{there is some } b' \in \mathcal{A} \text{ such that } \langle b, b' \rangle \in [\![\varphi]\!] \end{array} \right\}$

Note that the existential quantifier $\exists\xi$ could be seen as a formula on its own. If it were, it would receive the following interpretation.

- $[\![\exists\xi]\!] = \{ \langle a, a' \rangle \mid a \approx_\xi a' \}$
- $[\![\exists\xi\varphi]\!] = [\![(\exists\xi \wedge \varphi)]\!]$

2.2 Dynamic Predicate Logic with Unselective Generalized Quantifiers $\mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$

$\mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$ is the smallest set such that:

- $\mathcal{L}_{\mathsf{D}}^{\mathsf{Q}} \subseteq \mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$.
- If $\varphi, \psi \in \mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$ and ξ is a variable, then $\mathsf{SOME}^{\xi}(\varphi)(\psi), \mathsf{EVERY}^{\xi}(\varphi)(\psi), \mathsf{NO}^{\xi}(\varphi)(\psi) \in \mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$.

These quantifiers are *unselective* in that they quantify over assignments, rather than over individuals.

- $[\![\mathsf{SOME}^{\xi}(\varphi)(\psi)]\!] = [\![\exists\xi(\varphi \wedge \psi)]\!]$
  $= \{ \langle a, a' \rangle \mid \text{for some } b, b' \in \mathcal{A},\ a \approx_{\xi} b \text{ and } \langle b, b' \rangle \in [\![(\varphi \wedge \psi)]\!] \ \}$
- $[\![\mathsf{EVERY}^{\xi}(\varphi)(\psi)]\!]$
  $$= \left\{ \langle a, a' \rangle \;\middle|\; \begin{array}{l} a = a' \text{ and} \\ \text{for every } b \in \mathcal{A} \text{ such that } a \approx_{\xi} b \text{ and for some } b' \in \mathcal{A}\ \langle b, b' \rangle \in [\![\varphi]\!], \\ \text{there is some } b'' \in \mathcal{A} \text{ such that } \langle b, b'' \rangle \in [\![(\varphi \wedge \psi)]\!] \end{array} \right\}$$
  $= [\![\neg\mathsf{SOME}^{\xi}(\varphi)(\neg\psi)]\!]$
- $[\![\mathsf{NO}^{\xi}(\varphi)(\psi)]\!]$
  $$= \left\{ \langle a, a' \rangle \;\middle|\; \begin{array}{l} a = a' \text{ and for no } b \in \mathcal{A}, \\ a \approx_{\xi} b \text{ and there is } b' \in \mathcal{A} \text{ such that } \langle b, b' \rangle \in [\![(\varphi \wedge \psi)]\!] \end{array} \right\}$$
  $= [\![\neg\mathsf{SOME}^{\xi}(\varphi)(\psi)]\!]$

These are all definable in terms of $\exists$, so they are not extending the language in any substantial way.

Unselective quantifiers have been shown to be empirically problematic for donkey anaphora, which led to the development of dynamic *selective* quantifiers in the 1990s (Van Eijck & De Vries 1992, Kanazawa 1993, 1994, Van den Berg 1991, Van den Berg 1996, Chierchia 1992, 1995).

But as far as I know not much has been done about the projective properties of dynamic selective quantifiers.

2.3 Lifting

The relational semantics of DPL looks non-deterministic. But it can be lifted to functions over sets of extended assignments.

- For any $\varphi \in \mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$, $[\![\![\varphi]\!]\!](A) = \{ a' \mid \text{for some } a \in A,\ \langle a, a' \rangle \in [\![\varphi]\!] \}$

Note that this is not changing anything since $[\![\cdot]\!]$ can be redefined in terms of $[\![\![\cdot]\!]\!]$:

- For any $\varphi \in \mathcal{L}_{\mathsf{D}}^{\mathsf{UGQ}}$, $[\![\varphi]\!] = \{ \langle a, a' \rangle \mid a' \in [\![\![\varphi]\!]\!](\{ a \}) \}$

3 Satisfaction Theory with Unselective Generalized Quantifiers $\mathcal{L}_{\mathsf{SAT}}^{\mathsf{UGQ}}$

3.1 Syntax

As in the case the propositional fragment of Satisfaction Theory, $\mathcal{L}_{\mathsf{SAT}}$, we will restrict our attention to cases where presuppositions are triggered within simple propositions.

- Ultimately we want presuppositions to come from atomic expressions—i.e., predicates and terms—but we will not deal with presupposition projection in sub-propositional compositionality here in order to keep our discussion simple.
- So we continue to assume that presuppositions are simply attached to simple propositions.

The syntax of $\mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$ is parallel to that of $\mathcal{L}_{\mathsf{SAT}}$ except that we have a clause for quantifiers.

- If $\pi \in \mathcal{L}^{\mathsf{Q}}$ and $\varphi \in \mathcal{S}$, then ${}_{\pi}\varphi \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$.
- If $\varphi, \psi \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$, then $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \to \psi) \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$.
- If ξ is a variable and $\varphi \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$, then $\exists\xi\varphi, \forall\xi\varphi, \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$.
- If ξ is a variable and $\varphi, \psi \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$, then $\mathsf{SOME}^{\xi}(\varphi)(\psi), \mathsf{EVERY}^{\xi}(\varphi)(\psi), \mathsf{NO}^{\xi}(\varphi)(\psi) \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$.
- Nothing else is a formula of $\mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$.

3.2 Semantics

Formulas of $\mathcal{L}^{\mathsf{Q}}_{\mathsf{SAT}}$ are interpreted over functions over sets of extended assignments.

- $[\![{}_{\pi}\varphi]\!] = \lambda A\!: \text{ for each } a \in A,\ \|\pi\|(a) = 1.\ \{\, a \in A \mid \|\varphi\|(a) = 1 \,\}$
- $[\![\neg\varphi]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]).\ \{\, a \in A \mid \text{for no } a' \in [\![\varphi]\!](A),\ a(\mathrm{v}_0) = a'(\mathrm{v}_0) \,\}$
- $[\![(\varphi \wedge \psi)]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\varphi]\!](A) \in \mathrm{dom}([\![\psi]\!]).\ [\![\psi]\!]([\![\varphi]\!](A))$
- $[\![(\varphi \vee \psi)]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\neg\varphi]\!](A) \in \mathrm{dom}([\![\psi]\!]).\ [\![\varphi]\!](A) \cup [\![\psi]\!]([\![\neg\varphi]\!](A))$
- $[\![(\varphi \to \psi)]\!] = [\![\neg(\varphi \wedge \neg\psi)]\!]$
  $= \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\varphi]\!](A) \in \mathrm{dom}([\![\neg\psi]\!]).\ \left\{ a \in A \;\middle|\; \begin{array}{l} \text{for no } a' \in [\![\neg\psi]\!]([\![\varphi]\!](A)), \\ a(\mathrm{v}_0) = a'(\mathrm{v}_0) \end{array} \right\}$
- $[\![\exists\xi\varphi]\!] = \lambda A\!:\ \{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\} \in \mathrm{dom}([\![\varphi]\!]).\ [\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\})$
- $[\![\forall\xi\varphi]\!] = [\![\neg\exists\xi\neg\varphi]\!]$
- $[\![\mathsf{SOME}^{\xi}(\varphi)(\psi)]\!] = [\![\exists\xi(\varphi \wedge \psi)]\!]$
- $[\![\mathsf{EVERY}^{\xi}(\varphi)(\psi)]\!] = [\![\neg\exists\xi(\varphi \wedge \neg\psi)]\!]$
- $[\![\mathsf{NO}^{\xi}(\varphi)(\psi)]\!] = [\![\neg\exists\xi(\varphi \wedge \psi)]\!]$

The outputs can be restated in terms of $[\![\![\cdot]\!]\!]$:

- $[\![{}_{\pi}\varphi]\!] = \lambda A\!: \text{ for each } a \in A,\ \|\pi\|(a) = 1.\ [\![\![\varphi]\!]\!](A)$
- $[\![\neg\varphi]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]).\ [\![\![\neg\varphi]\!]\!](A)$
- $[\![(\varphi \wedge \psi)]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\varphi]\!](A) \in \mathrm{dom}([\![\psi]\!]).\ [\![\![(\varphi \wedge \psi)]\!]\!]$
- $[\![(\varphi \vee \psi)]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\neg\varphi]\!](A) \in \mathrm{dom}([\![\psi]\!]).\ [\![\![(\varphi \vee \psi)]\!]\!]$
- $[\![(\varphi \to \psi)]\!] = \lambda A\!:\ A \in \mathrm{dom}([\![\varphi]\!]) \text{ and } [\![\varphi]\!](A) \in \mathrm{dom}([\![\neg\psi]\!]).\ [\![\![(\varphi \to \psi)]\!]\!]$
- $[\![\exists\xi\varphi]\!] = \lambda A\!:\ \{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\} \in \mathrm{dom}([\![\varphi]\!]).\ [\![\![\exists\xi\varphi]\!]\!]$

This reformulation makes it clear that the presuppositions are essentially stipulated, rather than derived in terms of a general principle.

3.3 Universal projection

A key prediction of the satisfaction theory is that all quantifiers should give rise to *universal presuppositions*.
For instance, with the translation:

(1) If S_1 and S_2 such that $\mathsf{S}_1 \rightsquigarrow \varphi$ and $\mathsf{S}_2 \rightsquigarrow \psi$,

$$\mathsf{a}\ \mathsf{S}_1\ \mathsf{S}_2 \rightsquigarrow \mathsf{SOME}^{\xi}(\varphi)(\psi)$$

$$\begin{aligned}
&[\![\mathsf{SOME}^{\xi}(\varphi)(\psi)]\!]\\
&= \lambda A\colon\ \{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\} \in \mathrm{dom}([\![(\varphi \wedge \psi)]\!]).\ [\![(\varphi \wedge \psi)]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\})\\
&= \lambda A\colon\ \begin{array}{l}\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\} \in \mathrm{dom}([\![\varphi]\!]) \wedge \\ [\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\}) \in \mathrm{dom}([\![\psi]\!])\end{array}.\ [\![\psi]\!]([\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\}))
\end{aligned}$$

- The first conjunct of the presupposition, $\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\} \in \mathrm{dom}([\![\varphi]\!])$, requires the presupposition of the restrictor φ to be true of every individual in $\mathcal{D}$.
- The second conjunct of the presupposition, $[\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D}\,\}) \in \mathrm{dom}([\![\psi]\!])$, requires the presupposition of the nuclear scope ψ to be true of every individual that makes the restrictor φ true.

For instance,

(2) A linguist who quit smoking started vaping.

should presuppose:

- Every individual in the model used to smoke. (projection out of the restrictor)
- Every individual that is a linguist and quit smoking used to not vape. (projection out of the nuclear scope)

Arguably both of these predictions are problematic.
Likewise,

(3) Every linguist who returned to Russia called both of their brothers.

should presupposed:

- Every individual in the model is from Russia.
- Every linguist who returned to Russia has exactly two brothers.

The second prediction might be fine, but the first one is clearly problematic.

3.3.1 Local accommodation

Heim 1982 is aware of this problem, and proposes a fix in terms of *local accommodation*. Local accommodation is conceived of as an operation to turn the presupposition into part of the assertive meaning.
Technically, this can be defined with the Acc-operator:

(4) For any $\varphi \in \mathcal{L}^{\mathsf{UGQ}}_{\mathsf{SAT}}$, $\mathsf{Acc}([\![\varphi]\!]) = \lambda A.\ [\![\varphi]\!](\max_{A' \subseteq A} A' \in \mathrm{dom}([\![\varphi]\!]))$

A potential issue is that local accommodation, if available, will render the following pairs of sentences semantically equivalent.

(5) a. Exactly one student returned to Russia.
b. Exactly one student is from Russia and returned there.

(6) a. No student hates both of their supervisors.
b. No student has exactly two supervisors and hates both of them.

(7) a. No student who hates both of their supervisors has filed a formal complaint against them.
b. No student who has exactly two supervisors and hates both of them has filed a formal complaint against them.

But it's not entirely clear if this is a bad prediction.

3.3.2 *Existential projection*

Beaver 1994, 2001 argues that there is no convincing evidence that any quantifier gives rise to universal presuppositions in English, and proposes an existential denotation.
This can be implemented in the current fragment by simply stipulating an existential presupposition for the presupposition. That is, instead of (8),

(8) $[\![\exists\xi\varphi]\!] = \lambda A\colon\ \{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\} \in \mathrm{dom}([\![\varphi]\!]).\ [\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\})$

we can have:

(9) $[\![\exists\xi\varphi]\!] = \lambda A\colon\ \begin{array}{l}\text{for some } A' \subseteq \{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\} \\ A' \neq \varnothing \text{ and } A' \in \mathrm{dom}([\![\varphi]\!])\end{array}.\ [\![\varphi]\!](\{\, a[\xi \mapsto x] \mid a \in A, x \in \mathcal{D} \,\})$

However, more recent experiment research provides empirical evidence that some quantifiers actually give rise to universal presuppositions.

4 Experiments on presupposition projection in quantified sentences

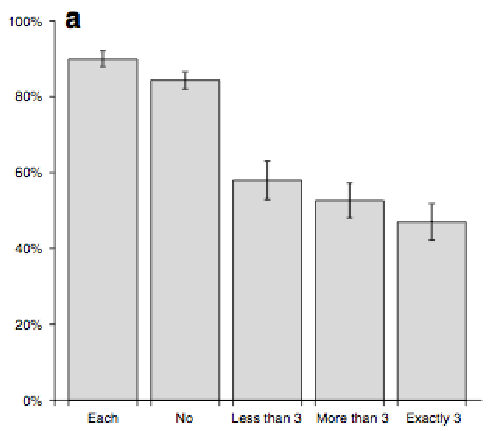
4.1 Chemla 2009

Inference task: 'Does "S" *suggest* p?'

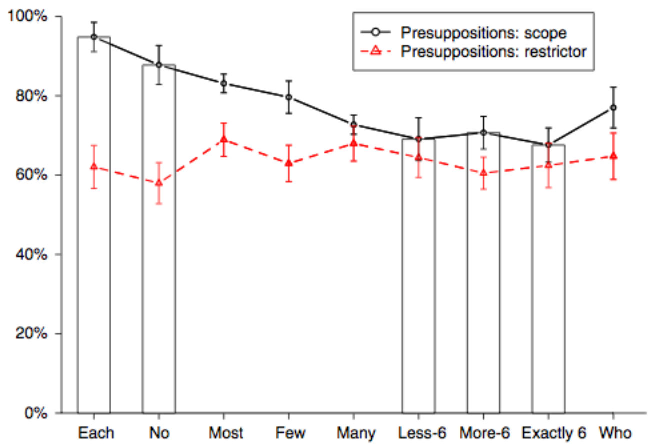
(10) S: None of these 10 students knows that his father is going to receive a congratulation letter.
p: The father of each of these 10 students is going to receive a congratulation letter.

4.1.1 *Experiment 1*

- Binary choice: *Yes (Oui)* or *No (Non)*
- Quantifiers: *each (chaqun), no (aucun), less than three (moins de 3), more than three (plus de 3), exactly three (exactement 3)*



Experiment 1 ($n = 30$)



Experiment 2 ($n = 10$)

- Presupposition triggers: *know, be aware, stop, continue, his*

4.1.2 *Experiment 2*

- Graded judgments (slider)
- Quantifiers: *each (chacun), no (aucun), most (la plupart), few (peu), many (beaucoup), less than six (moins de 6), more than six (plus de 6), exactly six (exactement 6), who (qui)*

(11) Projection out of restrictor (NP)
S: Among these 20 students, most who know that their father is going to receive a congratulation letter take English lessons.
p: The father of each of these 20 students is going to receive a congratulation letter.

4.1.3 *Summary*

- *No* tends to give rise to a universal inference (possibly not a presupposition) more robustly than *less than three, more than three* and *exactly three.*
- (No interesting differences among presupposition triggers)

4.2 Zehr et al. 2016

Zehr et al. 2016 investigated presuppositions triggered in the nuclear scope of *none.* They tested three possible readings:

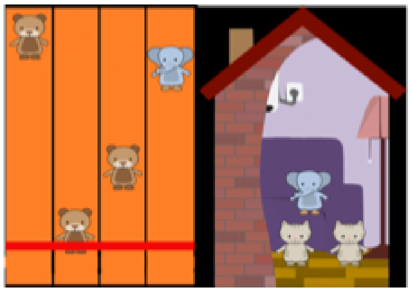
(12) None of the bears won the race.
a. EXISTENTIAL: At least one of the bears participated (and no bear won)
b. UNIVERSAL: All of the bears participated (and no bear won)
c. PRESUPPOSITIONLESS: (No bear both participated and won)

Context sentence + picture followed by a covered box choice task
Four types of pictures:

(13) a. TRUECONTROL: All the bears participated, none of them won

b. ONLYSOME: Only some of the bears participated, none of them won
c. NORUNNER: None of the bears participated
d. FALSECONTROL: All the bears participated, one won

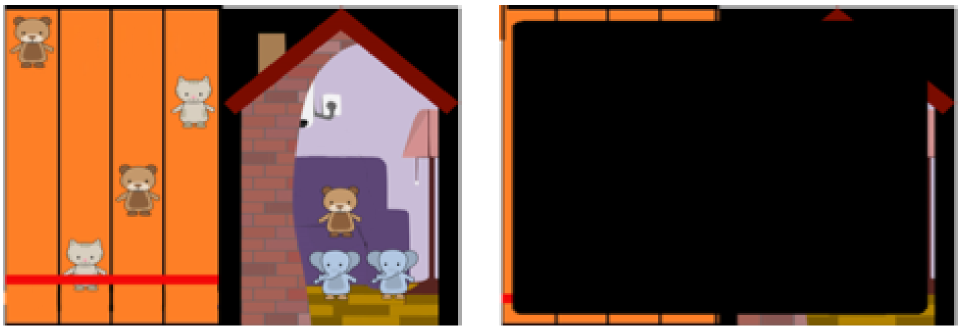
[During the morning race, <u>these three bears</u> did really well, and in the end, one of them won. I thought they would do the same later in the day as well, but...] (audio)

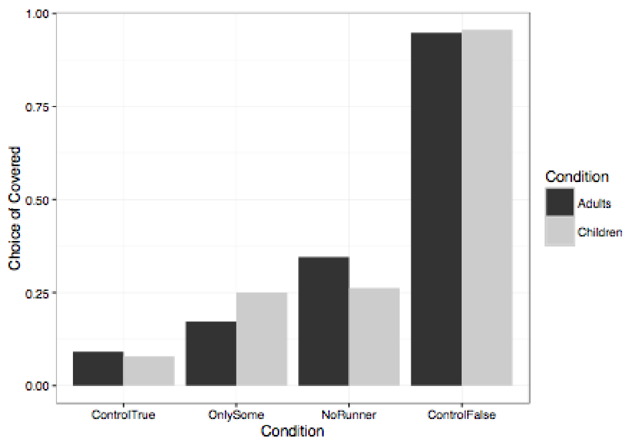


Press any key to continue.

[None of the bears won the afternoon race.] (audio)

F **J**





42 adults and 22 children (4;00-5;10)

- There's a statistically significant difference between ONLYSOME and NORUNNER for adults (but not fo kids). Zehr et al. take this as evidence that the PRESUPPOSITIONLESS reading exists, in addition to the EXISTENTIAL reading.
- But is it possible that somehow the ONLYSOME picture was preferred to the NORUNNER picture for unrelated reasons? It would be good to test the results against some baseline (e.g., sentences with no presuppositions).

4.3 Creemers, Zehr & Schwarz 2018

Creemers, Zehr & Schwarz 2018 tested two quantifiers, *at least one* and *every*, with respect to two presupposition triggers *aware* and *again*.[1]

(14) a. At least one alien is aware that he is blue.
b. At least one alien is blue again.

(15) a. Every alien is aware that he is blue.
b. Every alien is blue again.

(Note that *again* gives rise to a potential scope ambiguity in (14b) with respect to the subject quantifier, but since the results are not so different from (14a), we won't worry about this.)

They consider the following five readings:

(16) *Aware*

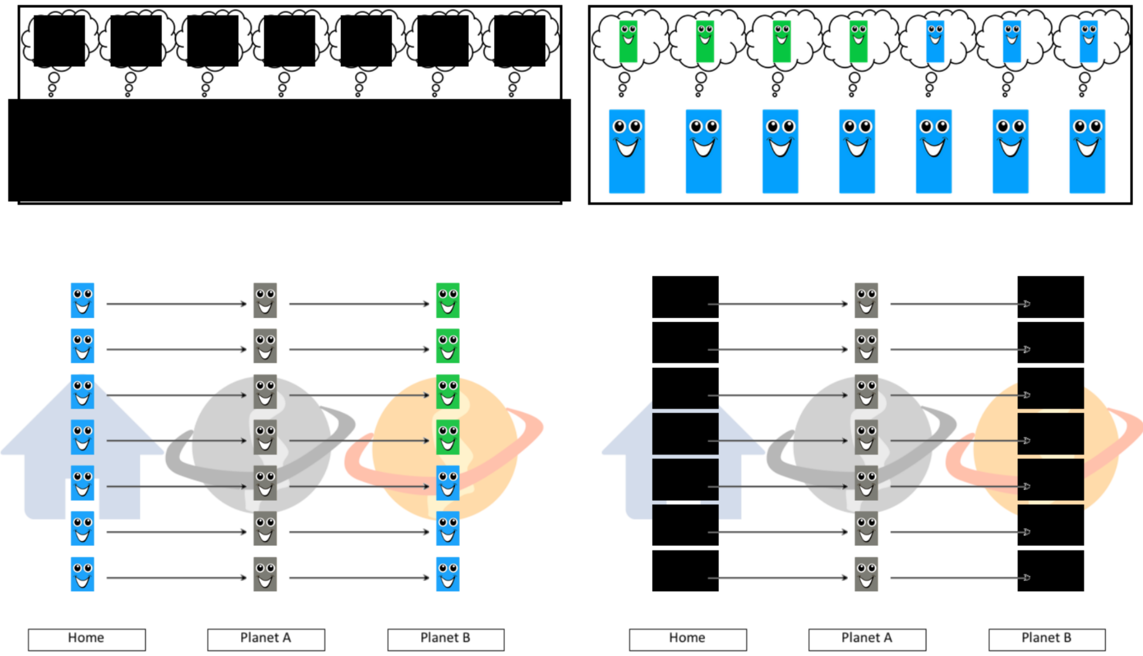
a. UNIVERSAL
   (i) Presupposition: All alien are blue.
   (ii) Assertion: At least one/Every alien thinks he is blue.
b. EXISTENTIAL[+ENTAILMENT]
   (i) Presupposition: At least one alien is blue.
   (ii) Assertion: At least one/Every alien is blue and thinks he is blue.
c. DOMAINRESTRICTION
   (i) Presupposition: none (the existential presupposition for *every*)
   (ii) Assertion: At least one/Every blue alien is blue.
d. EXISTENTIAL[−ENTAILMENT]
   (i) Presupposition: At least one alien is blue.
   (ii) Assertion: At least one/Every alien thinks he is blue.
e. PRESUPPOSITIONLESS
   (i) Presupposition: none
   (ii) Assertion: At least one/Every alien thinks he is blue.

(17) *Again*

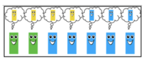
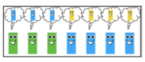
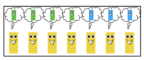
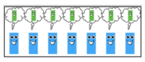
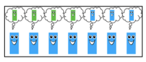
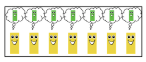
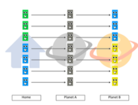
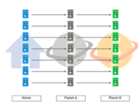
a. UNIVERSAL
   (i) Presupposition: All alien were blue before.
   (ii) Assertion: At least one/Every is blue now.
b. EXISTENTIAL[+ENTAILMENT]
   (i) Presupposition: At least one alien was blue before.
   (ii) Assertion: At least one/Every alien was blue before and is blue now.
c. DOMAINRESTRICTION
   (i) Presupposition: none (the existential presupposition for *every*)
   (ii) Assertion: At least one/Every alien that was blue before is blue now.
d. EXISTENTIAL[−ENTAILMENT]
   (i) Presupposition: At least one alien was blue before.
   (ii) Assertion: At least one/Every alien is blue now.
e. PRESUPPOSITIONLESS
   (i) Presupposition: none
   (ii) Assertion: At least one/Every alien is blue now.

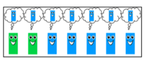
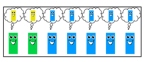
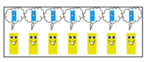
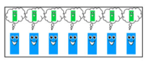
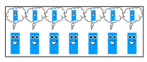
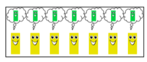
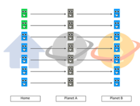
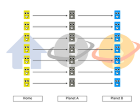
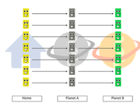
[1] They were interested in potential difference between entailed and non-entailed presuppositions and chose these presupposition triggers to test it, but only minor difference were present in the results, which we will not delve into here.

The task is a covered choice task with pictures (similar to Zehr et al. 2016). Example trials looked like the following for the two triggers.

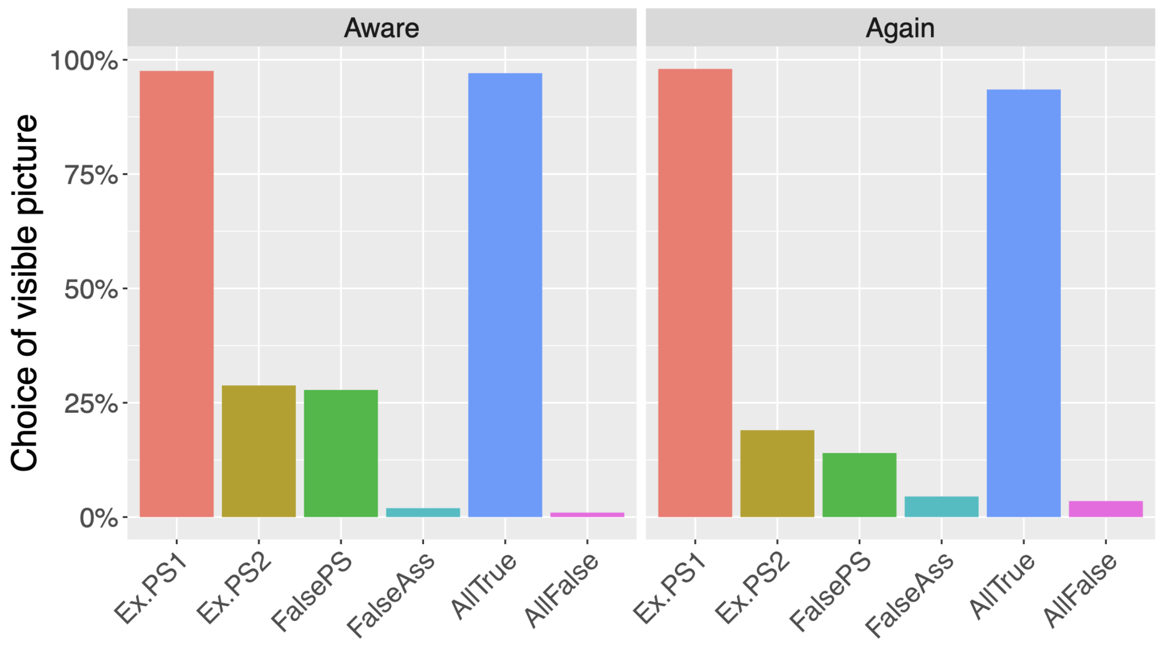


There were six conditions in terms of the five readings are predicted to be true with respect to the overt picture.

Aware						
Again						
PS reading	**∃PS1**	**∃PS2**	**FalsePs**	**FalseAss.**	**AllTrue**	**AllFalse**
∀	✗	✗	✗	✗	✓	✗
∃+Ent.	✓	✗	✗	✗	✓	✗
DR	✓	✗	✗	✗	✓	✗
∃−Ent.	✓	✓	✗	✗	✓	✗
PS-less	✓	✓	✓	✗	✓	✗

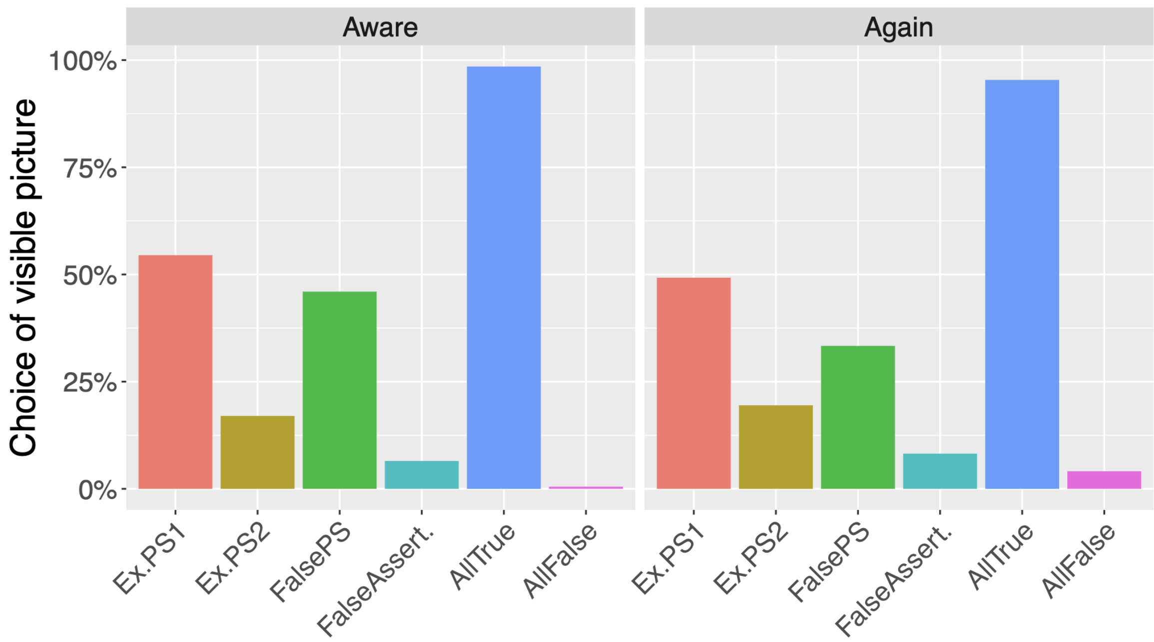
Aware						
Again						
PS reading	**∃PS1**	**∃PS2**	**FalsePs**	**FalseAss.**	**AllTrue**	**AllFalse**
∀	✗	✗	✗	✗	✓	✗
∃+Ent.	✗	✗	✗	✗	✓	✗
DR	✓	✓	✗	✗	✓	✗
∃−Ent.	✓	✗	✗	✗	✓	✗
PS-less	✓	✗	✓	✗	✓	✗

Results for *at least*:



- Significant difference between ∃PS1 and ∃PS2 ⇒ either due to a reading where the presupposition is entailed by the asserted meaning (could be via local accommodation), or due to domain restriction.
- Significant difference between ∃PS2 and FALSEPS ⇒ a reading where the presupposition is not part of the asserted meaning is available.

Results for *every*:



- Significant difference between ∃PS1 and ALLTRUE ⇒ Either due to universal presupposition, or due to entailed presupposition.
- Significant difference between ∃PS1 and ALLFALSE ⇒ Either due to domain restriction, or due to a presuppositionless reading.

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