Introduction to Semantics

Lecture 4: Generalized Quantifier Theory II

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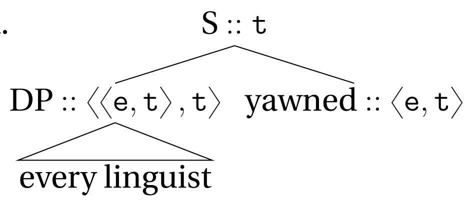
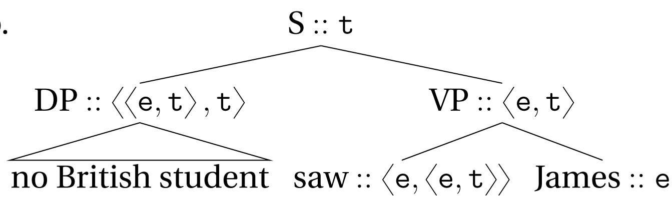
1 Quantificational determiners

Our analysis so far:

(1) a.


b.


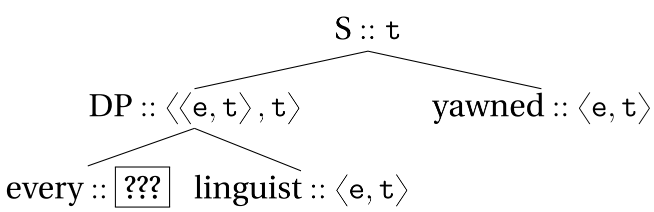
(2) a. $[\![\texttt{every linguist}]\!]_{\mathcal{M}} = \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ 1$ iff for every linguist x in $\mathcal{M}$, $f(x) = 1$
b. $[\![\texttt{no British student}]\!]_{\mathcal{M}} = \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ 1$ iff for no British student x in $\mathcal{M}$, $f(x) = 1$

(3) a. $\mathsf{SET}([\![\texttt{every linguist}]\!]_{\mathcal{M}}) = \{\, S \subseteq D \mid S$ contains every linguist in $\mathcal{M}\,\}$
b. $\mathsf{SET}([\![\texttt{no British student}]\!]_{\mathcal{M}}) = \{\, S \subseteq D \mid S$ contains no British student in $\mathcal{M}\,\}$

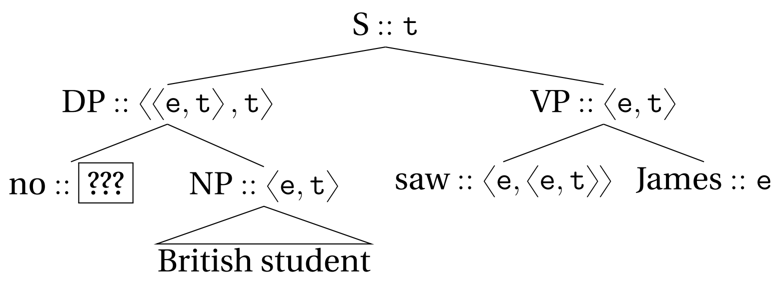
Given that NPs denote type-$\langle \texttt{e}, \texttt{t}\rangle$ functions, we can work out the semantic type of the quantificational determiners.

(4) For any model $\mathcal{M}$,
a. $[\![\texttt{linguist}]\!]_{\mathcal{M}} = \lambda x \in D.\ 1$ iff x is a linguist in $\mathcal{M}$
b. $[\![\texttt{British student}]\!]_{\mathcal{M}} = \lambda x \in D.\ 1$ iff x is a British student in $\mathcal{M}$

First, let's figure out the semantic type.

(5) a.


b.



If a quantificational determiner had type $\texttt{e}$, it could combine with an NP denotation via Functional Application, but the result would be type $\texttt{t}$, rather than $\langle\langle \texttt{e}, \texttt{t}\rangle, \texttt{t}\rangle$. Also, a quantificational determiner clearly does not refer to a particular individual.

So, the only possibility we are left with is type $\langle\langle \texttt{e}, \texttt{t}\rangle, \langle\langle \texttt{e}, \texttt{t}\rangle, \texttt{t}\rangle\rangle$. This looks complex, but don't forget that any functional type has only two parts, the input type on the left of the comma—$\langle \texttt{e}, \texttt{t}\rangle$ in this case—and the output type on the right of the comma—$\langle\langle \texttt{e}, \texttt{t}\rangle, \texttt{t}\rangle$ in this case.

From this semantic type, we can know what the denotations have to look like, namely:

(6) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff } \boxed{???}$

b. $[\![\texttt{no}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff } \boxed{???}$

At this point, let's go back to the denotations of the quantificational DPs:

(7) For any model $\mathcal{M}$,

a. $[\![\texttt{every linguist}]\!]_{\mathcal{M}} = \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for every linguist } x \text{ in } \mathcal{M},\ f(x) = 1$

b. $[\![\texttt{no British student}]\!]_{\mathcal{M}} = \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for no British student } x \text{ in } \mathcal{M},\ f(x) = 1$

We can isolate the semantic contributions of the quantificational determiners by restating these denotations in terms of the denotations of the NPs.

(8) For any model $\mathcal{M}$,

a. $[\![\texttt{linguist}]\!]_{\mathcal{M}} = \lambda x \in D.\, 1 \text{ iff } x \text{ is a linguist in } \mathcal{M}$

b. $[\![\texttt{British student}]\!]_{\mathcal{M}} = \lambda x \in D.\, 1 \text{ iff } x \text{ is a British student in } \mathcal{M}$

This step might not be easy, but here's one way of doing it:

(9) For any model $\mathcal{M}$,

a. $[\![\texttt{every linguist}]\!]_{\mathcal{M}}$

$= \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for every linguist } x \text{ in } \mathcal{M},\ f(x) = 1$

$= \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for every } x \in D \text{ such that } [\![\texttt{linguist}]\!]_{\mathcal{M}}(x) = 1,\ f(x) = 1$

b. $[\![\texttt{no British student}]\!]_{\mathcal{M}}$

$= \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for no British student } x \text{ in } \mathcal{M},\ f(x) = 1$

$= \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\, 1 \text{ iff for no } x \in D \text{ such that } [\![\texttt{British student}]\!]_{\mathcal{M}}(x) = 1,\ f(x) = 1$

From these representations, we can abstract over these specific NPs and get the denotations of the quantificational determiners themselves:

(10) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for every $x \in D$ such that $g(x) = 1$, $f(x) = 1$

b. $[\![\texttt{no}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for no $x \in D$ such that $g(x) = 1$, $f(x) = 1$

As before, we can state the same thing in different ways, e.g.

(11) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for no $x \in D$ such that $g(x) = 1$, $f(x) = 0$

b. $[\![\texttt{no}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for every $x \in D$ such that $g(x) = 1$, $f(x) = 0$

(12) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for every $x \in D$ such that $f(x) = 0$, $g(x) = 0$

b. $[\![\texttt{no}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for no $x \in D$ such that $f(x) = 1$, $g(x) = 0$

2 Quantificational determiners as relations between sets of individuals

The denotations of quantificational determiners are of type $\langle\langle \mathtt{e},\mathtt{t}\rangle,\langle\langle \mathtt{e},\mathtt{t}\rangle,\mathtt{t}\rangle\rangle$, so they do not characterize sets. But their arguments are of type $\langle \mathtt{e},\mathtt{t}\rangle$ and characterize sets of individuals. Let us rewrite the denotations of 'every' and 'no' in terms of sets of individuals:

(13) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for every $x \in D$ such that $g(x) = 1$, $f(x) = 1$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for every $x \in D$ such that $x \in \mathsf{set}(g)$, $x \in \mathsf{set}(f)$

b. $[\![\texttt{no}]\!]_{\mathcal{M}}$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for no $x \in D$ such that $g(x) = 1$, $f(x) = 1$
$= \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff for no $x \in D$ such that $x \in \mathsf{set}(g)$, $x \in \mathsf{set}(f)$

Notice that we have a way to state 'for every $x \in D$ such that $x \in \mathsf{set}(g)$, $x \in \mathsf{set}(f)$' using a symbol, namely, '$\mathsf{set}(g) \subseteq \mathsf{set}(f)$'. Similarly, 'for no $x \in D$ such that $x \in \mathsf{set}(g)$, $x \in \mathsf{set}(f)$' can be restated as '$\mathsf{set}(g) \cap \mathsf{set}(f) = \varnothing$'. So we can rewrite (13) as follows.

(14) For any model $\mathcal{M}$,

a. $[\![\texttt{every}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff $\mathsf{set}(g) \subseteq \mathsf{set}(f)$

b. $[\![\texttt{no}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\lambda f \in D_{\langle \mathtt{e},\mathtt{t}\rangle}.\ 1$ iff $\mathsf{set}(g) \cap \mathsf{set}(f) = \varnothing$

These representations particularly clearly show us an important fact: The quantificational determiners express relations between two sets of individuals. One set ($\mathsf{set}(g)$) is denoted by the NP and the other set ($\mathsf{set}(f)$) is denoted by the VP. Different determiners express different relations:

(15) a. 'every' says that the NP set is a subset of the VP set.
b. 'no' says that the two sets have no common member.

We will analyze other determiners below.

According to our analysis, therefore, in a sentence with a quantificational subject, the quantificational determiner is in a sense the semantic (transitive) predicate of the sentence. Compare the following quantificational sentence with a sentence with a transitive verb and two proper names.

(16) a. [S :: $\mathtt{t}$ [DP :: $\langle\langle \mathtt{e},\mathtt{t}\rangle,\mathtt{t}\rangle$ [every :: $\langle\langle \mathtt{e},\mathtt{t}\rangle,\langle\langle \mathtt{e},\mathtt{t}\rangle,\mathtt{t}\rangle\rangle$] [linguist :: $\langle \mathtt{e},\mathtt{t}\rangle$]] [yawned :: $\langle \mathtt{e},\mathtt{t}\rangle$]]

b. [S :: $\mathtt{t}$ [Patrick :: $\mathtt{e}$] [VP :: $\langle \mathtt{e},\mathtt{t}\rangle$ [likes :: $\langle \mathtt{e},\langle \mathtt{e},\mathtt{t}\rangle\rangle$] [Andy :: $\mathtt{e}$]]]

In both cases, the main transitive predicate is of type $\langle \tau,\langle \tau,\mathtt{t}\rangle\rangle$, which express a relation, and it combines with two constituents, each having a type-τ denotation, one by one.

3 Presuppositions

The analysis of 'every' above misses the intuition that when the NP set is empty, the sentence sounds odd. Imagine, hypothetically, that I present you with a picture with a bunch of geometrical figures (like triangles, squares, circles) in different colors, but none of which is pink. Then it's strange to say (17).

(17) Every pink figure is a triangle.

Note also that an empty VP is totally fine. The sentence is just not true.

(18) Every triangle is pink.

According to the analysis we have developed, (17) should be true, when there is no pink figure, because $\varnothing$ is a subset of every set!

To put it differently, a sentence containing 'every' is associated with an inference that its NP set is non-empty. It's almost a requirement for a felicitous use of the expression, and it is known to not interact with various kinds of so-called 'non-veridical operators' like (19).

(19) Every pink cat I saw was asleep.
a. I don't think every pink cat I saw was asleep.
b. If every pink cat I saw was asleep, I'll be surprised.

c. Maybe every pink cat I saw was asleep.
d. Was every pink cat I saw asleep?

Inferences that have these characteristics are generally called **presuppositions**. There are a number of other presuppositions that go with specific words and constructions, e.g.

(20) a. Maria went back to Russia.
⇒ Maria was once in Russia.
b. Daniel stopped talking on the phone.
⇒ Daniel was talking on the phone.
c. Kurt fed his cat.
⇒ Kurt had a cat.
d. The billionaire semanticist lives here.
⇒ There is a billionaire semanticist.

All of these inferences 'project' through non-veridical operators like the ones above (examples omitted). Also, pragmatically, they all seem to be somehow taken for granted or backgrounded, when the utterance is made.

Presuppositions are defined to be inferences that have these properties. There are a number of different theories of presuppositions today, and we unfortunately do not have time to delve into this very interesting literature in this course.

But let me mention one major approach to presuppositions that uses a third truth-value. The third truth-value is given different symbols such as $\#$, $*$, $\star$, but let's use 2, just to be neutral with respect to what the third truth-value is supposed to represent (theories differ with respect to this question).

The idea is that the presupposition needs to be true, in order for the sentence to be said to be true or false. In the case of 'every', it will look like:

(21) For any model $\mathcal{M}$,

$$[\![\texttt{every}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathrm{e},\mathrm{t}\rangle}.\lambda f \in D_{\langle \mathrm{e},\mathrm{t}\rangle}. \begin{cases} 1 \text{ iff } \mathsf{set}(g) \neq \varnothing \text{ and } \mathsf{set}(g) \subseteq \mathsf{set}(f) \\ 0 \text{ iff } \mathsf{set}(g) \neq \varnothing \text{ and } \mathsf{set}(g) \nsubseteq \mathsf{set}(f) \\ 2 \text{ iff } \mathsf{set}(g) = \varnothing \end{cases}$$

This ensures that whenever the sentence is true (1) or false (0) the NP set is non-empty. One can analyze non-veridical operators in this framework so that they inherit such inferences, but details are omitted here.

I should also remark that (21) is not quite right, because it doesn't take into account the possibility that other presuppositions come from g and/or f. So the actual analysis needs to be very complex. There is actually a lot of debate about how presuppositions 'project' with respect to quantificational determiners, both theoretically and empirically, so I won't even try to spell out a more realistic analysis of 'every' here (see, e.g., Beaver 1994, 2001, George 2008, Chemla 2009, Fox 2012, Sudo 2012, Zehr et al. 2016, Creemers, Zehr & Schwarz 2018).

Another common approach to presuppositions, which some textbooks like Heim & Kratzer 1998 adopt, sees presuppositions as definedness conditions for the function. Recall that in λ-notation, the space between the semi-colon ':' and the dot '.' states what inputs are admissible. We could use this to encode presuppositions as in (22).

(22) For any model $\mathcal{M}$,
$[\![\texttt{every}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}\colon\ \mathsf{set}(g) \neq \varnothing.\ \lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \subseteq \mathsf{set}(f)$

This doesn't look like it's using a third truth-value, but it is simply hidden in the background, because the theory implicit has a representation for when this function receives an input that is not in its domain. Formally, this approach could be recast in terms of a third truth-value like (22).

3.1 Other quantificational determiners with presuppositions

There are a number of determiners that 'trigger' presuppositions like 'every'. Some of them take plural nouns or mass nouns, but just for the sake of presentation, let's simply assume that they have the same kind of type-$\langle \texttt{e}, \texttt{t}\rangle$ denotations as singular count nouns (although that is evidently problematic).

- 'Both' and 'neither' presuppose that the NP set has exactly two members.
- 'The', when combining with a singular count noun, presupposes that the NP set has exactly one member.
- 'The', when combining with a plural noun or a mass noun, presupposes that the NP set is not empty.

We could, for instance, analyze 'both' as a version of 'every' with a stronger presupposition. Under the trivalent approach we can represent it as (23).

(23) For any model $\mathcal{M}$,

$$[\![\texttt{both}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ \begin{cases} 1 \text{ iff } |\mathsf{set}(g)| = 2 \text{ and } \mathsf{set}(g) \subseteq \mathsf{set}(f) \\ 0 \text{ iff } |\mathsf{set}(g)| = 2 \text{ and } \mathsf{set}(g) \nsubseteq \mathsf{set}(f) \\ 2 \text{ iff } |\mathsf{set}(g)| \neq 2 \end{cases}$$

Arguably, not all quantificational determiners have presuppositions. For example:

(24) a. At least three pink cats were asleep.
b. Jake saw 10 or more pink cats.
c. Claire ordered between 10 and 15 Korean pálinkas.

Or do they???

3.2 Anti-presuppositions

One last thing to mention about 'every' is that it is actually odd, unless there are three or more things in the NP set.

(25) a. #Every current president of the US is male.
b. #You are using every hemisphere of your brain all the time.

Then do we want to say that 'every' actually presupposes that the NP set has at least three members?

There are reasons to think that the **existence** presupposition that the NP set is non-empty is a presupposition, while the inference that it has three or more members is a different type of inference, called an **anti-presupposition**.

Anti-presuppositions are extremely interesting and are a hot topic these days, but we do not have time to talk about them in detail in this course, unfortunately. You can check out an overview article by Bade 2021.

Nonetheless, I'd like to mention one difference between the presupposition and anti-presupposition of 'every'. Presuppositions and anti-presuppositions are very similar in many respects, but they show divergent behavior when embedded under certain operators like another quantifier. To see this concretely, consider the sentence in (26).

(26) Every professor$_i$ emailed every Hungarian student they$_i$ are supervising.

Under the bound reading of the pronoun, this sentence has a presupposition that every professor is supervising at least one Hungarian student, which is ultimately coming from the existence presupposition of the object 'every'. The existence presupposition somehow 'universally projects' through 'every professor' and gives rise to the presupposition that the existence presupposition is true for every professor.

Now, if you think about the anti-presupposition of the object 'every', it doesn't seem to project universally. That is, (26) does not seem to have an inference that every professor is supervising three or more Hungarian students. Rather, it seems to only suggest that at least one professor is supervising three or more Hungarian students, but the other professors could well be supervising one or two Hungarian students. In other words, the anti-presupposition only 'existentially projects' through 'every professor'.

To bring about these judgments more sharply, consider the following examples.

(27)
a. #Every professor that has no Hungarian student or has 1 or 2 Hungarian students emailed every one of them.
b. Every professor that has between 1 and 3 Hungarian students emailed every one of them.
c. Every professor that has 1 or more Hungarian students emailed every one of them.
d. Every professor that has 3 or more Hungarian students emailed every one of them.

The current theories of anti-presuppositions all assume that they are not lexically encoded, but come from competition with some alternative expression that has a stronger presupposition. In the case of 'every', it has two competitors:

(28) Every Hungarian student is happy.
a. The Hungarian student is happy.
b. Both Hungarian students are happy.

The alternative with a singular definite, (28a), presupposes that there is exactly one Hungarian student, and the alternative with 'both', (28b), presupposes that there are exactly two Hungarian students. Both of these alternatives have presuppositions that are stronger than the existence presupposition of (28), which only says that there is at least one Hungarian student.

Now, the idea is that whenever 'every' is used, it needs to be made sure that the presuppositions of the alternatives with a singular definite and 'both' are not satisfied (I would have to explain what it means for a presupposition to be satisfied, but there's no time for that now).

This competition-based approach is the standard approach today, and it would be a good exercise to apply it to the examples in (27). You should also see if the example in (29) poses any issue to the approach.

(29) #Every professor that has 1 or 2 Hungarian students emailed every one of them.

4 Scalar implicatures

Let us now turn to other determiners like the ones in (30). For the sake of discussion, we'll keep assuming that number-marking on nouns has no semantic consequences, but keep in mind that this is wrong. Ultimately, we need a theory of the semantics of number-marking and adjust our theory of quantification accordingly.

(30)
a. Some students failed.
b. Two students failed.
c. At least two students failed.
d. More than one student failed.

Let's discuss 'some' first. It is standardly analyzed as the negation of 'no'.

(31) For any model $\mathcal{M}$,
a. $[\![\texttt{no}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) = \varnothing$
b. $[\![\texttt{some}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t}\rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t}\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) \neq \varnothing$

According to this analysis, (30a) is true if and only if the intersection of the set of all students and the set of all people that failed is non-empty, which is to say that at least one student failed.

Note that (30a) entails that more than one student failed. It seems reasonable to attribute this to the plural number marking of the noun 'students', but since we are ignoring its semantic contribution here, it's natural that we cannot account for this entailment. It should also be remarked that 'some' is compatible with singular count nouns and mass nouns, in such a case, there is no plurality inference.

(32)
a. Some student failed.
b. Some luggage went missing.

The other thing to note about 'some' is that (30a) has an extra inference called a **scalar im-**

plicature. I believe this particular example can be understood in two different ways in this respect. One possible scalar implicature of (30a) is that not all the students failed. Another possible scalar implicature is that the number of failed students is not that large.

Either way, the point here is that the semantics given in (31b) does not encode such inferences, so according to it, the sentence would be true if there are 120 students and all of them failed. The intuition, however, is that (30a) does not sound straightforwardly true. Is this a problem?

It is a problem but the standard view is that it does not require us to revise the semantics in (31b). Rather, we just need to get the scalar implicature as an extra inference on top of that. Here too, we find a lot of different theories of scalar implicatures in the current literature and we unfortunately do not have time to discuss them in detail.

But let me mention one common idea that many (if not all) theories of scalar implicatures are built on. Specifically, a scalar implicature is the negation of a semantically stronger alternative (recall from Lecture 1, 'stronger' means 'entails but is not entailed by'). In the case of (30a), one could consider the following sentences as potentially relevant alternatives.

(33) a. All students failed.
b. Many students failed.
c. Several students failed.

Importantly, scalar implicatures are not considered to be part of the lexically encoded meaning of expressions like 'some', in part because they are not consistently observed in all uses of 'some' and also because they seem to be weaker in some sense than real entailments. But the tricky thing is that often it's not immediately clear what is a scalar implicature and what is an entailment of a given linguistic expression, and one would have to run linguistic tests to tease them apart.

Theories of scalar implicatures differ, among other things, with respect to what alternatives are considered, and how exactly they are negated. The literature is particularly large, involving not only semantics, but also pragmatics, philosophy of language, and experimental linguistics, but there are a number of different overview articles like Chierchia, Fox & Spector 2012, Sauerland 2012, Chemla & Singh 2014a,b, Cheirchia 2017 to name a few.

Similar complications arise with numeral quantifiers like the following.

(34) a. Two students failed.
b. At least two students failed.
c. More than one student failed.

What is the meaning of 'two'? One possibility is to analyze it as meaning 'exactly two' as follows.

(35) For any model $\mathcal{M}$,
$$[\![\texttt{two}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \texttt{e},\texttt{t} \rangle}.\lambda f \in D_{\langle \texttt{e},\texttt{t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| = 2$$

But then you might analyze 'exactly two' this way, and say that 'two' means something else, because if the semantics of 'two' is (35), what would 'exactly' be doing? Some people say that 'two' actually means 'two or more'.

(36) For any model $\mathcal{M}$,
$[\![\texttt{two}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 2$

Certain, 'two' often means 'exactly two'. For this, one could say that it is due to a scalar implicature that higher numerals are not true.

(37) For any model $\mathcal{M}$,

a. $[\![\texttt{three}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 3$
b. $[\![\texttt{four}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 4$
c. $[\![\texttt{five}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 5$

Ultimately, we should apply tests to see if there are any implicatures involved in the meaning of 'two', but since I haven't introduced such tests, I'll skip that. You can check out overview articles like Spector 2013, Bylinina & Nouwen 2020.

Another question about (36) would be, how would 'two' be different from 'two or more' and 'at least two'? It seems reasonable to assume the same semantics as (36).

(38) For any model $\mathcal{M}$,

a. $[\![\texttt{two or more}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 2$
b. $[\![\texttt{at least two}]\!]_{\mathcal{M}} = \lambda g \in D_{\langle \mathtt{e,t} \rangle}.\lambda f \in D_{\langle \mathtt{e,t} \rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| \geqslant 2$

One big difference between bare numerals like 'two' on the one hand, and modified numerals like 'two or more' and 'at least two' is that the latter never mean 'exactly two'. If you believe 'two' comes to mean 'exactly two' with scalar implicatures, you should also say something about why similar implicatures don't arise for 'two or more' and 'at least two'. In fact, no matter what your analysis of 'two' is, this arises as a question.

Conversely, there are scalar implicatures that the modified numerals have but the bare numeral doesn't. Namely, 'two or more' and 'at least two' are typically associated with ignorance inferences that the speaker considers that it is possible that 'exactly two' is the case, but it is also possible that some other numeral is the case. More concretely, both sentences in (39) suggest that the speaker doesn't know the exact number of students that failed, and it might be two, it might be more than two.

(39) a. Two or more students failed.
b. At least two students failed.

These ignorance inferences are often considered to be a kind of scalar implicatures, but there is some debate about this in the current literature. We cannot delve into this debate right now, but if you are interested, you should read the aforementioned overview articles, as well as papers like Geurts & Nouwen 2007, Coppock & Brochhagen 2013, Mayr 2013, Schwarz 2016, Mendia 2018, Cohen & Krifka 2014.

Lastly, there is also numerals like 'more than one', which arguably has the same semantics as those in (39), but has differences with respect to pragmatics. See Enguehard 2018 for some interesting discussion.

5 Vagueness

The last thing I should mention is that some determiners are inherently vague. There are many of them, e.g., 'several', 'many', 'most', 'few', etc. Intuitively, they are simply not about a particular quantify. To make the discussion more concrete, take the sentence in (40).

(40) Most of the students failed.

What are the truth-conditions of this sentence? That is, when exactly is it true and when exactly is it false? There are many concrete cases where one can confidently say whether (40) is true or false. For instance, let's say there are 30 students, and 27 of them failed. Then (40) is clearly false. Or, when only 4 out of the 30 students failed, (40) is clearly false.

Now, what if 16 out of the 30 students failed? The common intuition seems to be that it's not really false, but it's not really true either.

Such judgments are discussed under the rubric of **vagueness**. Our current fragment has no way of dealing with it, because in it, every sentence is either simply true or simply false (or its presupposition fails).

Vagueness is actually one of the oldest issues in semantics/pragmatics, and consequently has a long history as well as many competing theories. I will simply mention a couple of overview papers, Kennedy 2011, Solt 2015, Sorensen 2018. I would also like to mention an excellent book on this topic, Van Deemter 2010, which assumes no expert knowledge and is fun to read, but also introduces some important formal ideas.

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