Strong and weak quantifiers

PLIN0056 Semantics Research Seminar 2024–25

Lecture 1

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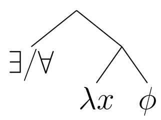
1 Generalized Quantifiers (GQs) in Logic

(1) a. $||\exists x\phi||^g = 1$ iff for some $g' \approx_x g$, $||\phi||^{g'} = 1$

b. $||\forall x\phi||^g = 1$ iff for all $g' \approx_x g$, $||\phi||^{g'} = 1$

(2) $g' \approx_x g$ iff for each variable $v \neq x$, $g'(v) = g(v)$.

These quantifiers can be seen as taking unary predicates with x being the argument.



In the context of GQs, they are called type-$\langle 1 \rangle$ GQs.

(3) a. $||\exists x\phi||^g = 1$ iff for some $g' \approx_x g$, $||\phi||^{g'} = 1$
iff $\{\, g'(x) \mid g' \approx_x g \text{ and } ||\phi||^{g'} = 1 \,\} \neq \varnothing$

b. $||\forall x\phi||^g = 1$ iff for all $g' \approx_x g$, $||\phi||^{g'} = 1$
iff $\{\, g'(x) \mid g' \approx_x g \text{ and } ||\phi||^{g'} = 1 \,\} = D$

Other type-$\langle 1 \rangle$ GQs include:

(4) a. $||\nexists x\phi||^g = 1$ iff for no $g' \approx_x g$, $||\phi||^{g'} = 1$
iff $\{\, g'(x) \mid g' \approx_x g \text{ and } ||\phi||^{g'} = 1 \,\} = \varnothing$

b. $||\exists\!\!\exists x\phi||^g = 1$ iff for some $g', g'' \approx_x g$ such that $g' \neq g''$, $||\phi||^{g'} = ||\phi||^{g''} = 1$
iff $|\,\{\, g'(x) \mid g' \approx_x g \text{ and } ||\phi||^{g'} = 1 \,\}\,| \geqslant 2$

These are expressible in first-order logic, but there are type-$\langle 1 \rangle$ quantifiers that are no first-order definable:

(5) a. $||\dashv x\phi||^g = 1$ iff there are finitely many $g'(x) \approx_x g$ such that $||\phi||^{g'} = 1$
iff $\{\, g'(x) \mid g' \approx_x g \text{ and } ||\phi||^{g'} = 1 \,\}$ is finite

b. $||\mathfrak{R} x\phi||^g = 1$ iff $\left| \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ ||\phi||^{g'} = 1 \end{array} \right\} \right| > \left| D - \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ ||\phi||^{g'} = 1 \end{array} \right\} \right|$

1.1 Type-$\langle n \rangle$ GQs

We can increase the number of variables to bind.

(6) a. $\|\exists x_1 x_2 \phi\|^g = 1$ iff for some $g' \approx_{\{x_1,x_2\}} g$, $\|\phi\|^{g'} = 1$
iff $\{ \langle g'(x_1), g'(x_2)\rangle \mid g' \approx_{x_1,x_2} g$ and $\|\phi\|^{g'} = 1 \} \neq \varnothing$

b. $\|\forall x_1 x_2 \phi\|^g = 1$ iff for all $g' \approx_{\{x_1,x_2\}} g$, $\|\phi\|^{g'} = 1$
iff $\{ \langle g'(x_1), g'(x_2)\rangle \mid g' \approx_{x_1,x_2} g$ and $\|\phi\|^{g'} = 1 \} = D \times D$

(7) $g' \approx_X g$ iff for each variable $v \notin X$, $g'(v) = g(v)$.

These are type-$\langle 2 \rangle$ GQs, essentially taking a relation as argument. Other type-$\langle 2 \rangle$ GQs include:

(8) a. $\left\|\exists^{\forall} x_1 x_2 \phi\right\|^g = 1$ iff for some $g' \approx_{x_1} g$, and for all $g'' \approx_{x_2} g'$, $\|\phi\|^{g''} = 1$

$$\text{iff} \left\{ g'(x_1) \;\middle|\; \begin{array}{l} g' \approx_{x_1} g \text{ and} \\ \left\{ g''(x_2) \;\middle|\; \begin{array}{l} g'' \approx_{x_2} g' \text{ and} \\ \|\phi\|^{g''} = 1 \end{array} \right\} = D \end{array} \right\} \neq \varnothing$$

b. $\left\|\forall^{\exists} x_1 x_2 \phi\right\|^g = 1$ iff for all $g' \approx_{x_1} g$ and for all $g'' \approx x_2 g'$, $\|\phi\|^{g''} = 1$

$$\text{iff} \left\{ g'(x_1) \;\middle|\; \begin{array}{l} g' \approx_{x_1} g \text{ and} \\ \left\{ g''(x_2) \;\middle|\; \begin{array}{l} g'' \approx_{x_2} g' \text{ and} \\ \|\phi\|^{g''} = 1 \end{array} \right\} \neq \varnothing \end{array} \right\} = D$$

More generally, the type-$\langle n \rangle$ versions of $\exists$ and $\forall$ look like:

(9) a. $\|\exists x_1 \ldots x_n \phi\|^g = 1$ iff for some $g' \approx_{\{x_1,\ldots,x_n\}} g$, $\|\phi\|^{g'} = 1$

$$\text{iff} \left\{ \langle g'(x_1), \ldots, g'(x_n) \rangle \;\middle|\; \begin{array}{c} g' \approx_{\{x_1,\ldots,x_2\}} g \\ \text{and } \|\phi\|^{g'} = 1 \end{array} \right\} \neq \varnothing$$

b. $\|\forall x_1 \ldots x_n \phi\|^g = 1$ iff for all $g' \approx_{\{x_1,\ldots,x_n\}} g$, $\|\phi\|^{g'} = 1$

$$\text{iff} \left\{ \langle g'(x_1), \ldots, g'(x_n) \rangle \;\middle|\; \begin{array}{c} g' \approx_{\{x_1,\ldots,x_2\}} g \\ \text{and } \|\phi\|^{g'} = 1 \end{array} \right\} = D^n$$

1.2 Type-$\langle 1, \ldots, 1 \rangle$ GQs

We can increase the number of formulas to take.

(10) a. $\|\exists x(\phi; \psi)\|^g = 1$ iff for some $g' \approx_x g$, $\|\phi\|^{g'} = \|\psi\|^{g'} = 1$
iff $\{ g'(x) \mid g' \approx_x g$ and $\|\phi\|^{g'} = \|\psi\|^{g'} = 1 \} \neq \varnothing$

b. $\|\forall x(\phi; \psi)\|^g = 1$ iff for all $g' \approx_x g$, $\|\phi\|^{g'} = \|\psi\|^{g'} = 1$
iff $\{ g'(x) \mid g' \approx_x g$ and $\|\phi\|^{g'} = \|\psi\|^{g'} = 1 \} = D$

These are type-$\langle 1, 1 \rangle$ GQs. Other type-$\langle 1, 1 \rangle$ GQs include:

(11) a. $\|\texttt{EVERY} x(\phi; \psi)\|^g = 1$ iff for all $g' \approx_x g$, if $\|\phi\|^{g'} = 1$, then $\|\psi\|^{g'} = 1$

$$\text{iff} \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\phi\|^{g'} = 1 \end{array} \right\} \subseteq \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\psi\|^{g'} = 1 \end{array} \right\}$$

b. $\|\texttt{HALF}x(\phi;\psi)\|^g = 1$ iff for half of $g' \approx_x g$ such that $\|\phi\|^{g'} = 1$, $\|\psi\|^{g'} = 1$ iff

$$\left|\left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\phi\|^{g'} = \|\psi\|^{g'} = 1 \end{array} \right\}\right| > \left|\left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\phi\|^{g'} = 1 \text{ and } \|\psi\|^{g'} = 0 \end{array} \right\}\right|$$

More generally, the type-$\langle\underbrace{1,\ldots,1}_{n \text{ times}}\rangle$ versions of $\exists$ and $\forall$ look like:

(12) a. $\|\exists x(\phi_1;\ldots;\phi_n)\|^g = 1$ iff for some $g' \approx_x g$, $\|\phi_1\|^{g'} = \cdots = \|\phi_n\|^{g'} = 1$

$$\text{iff } \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\phi_1\|^{g'} = \cdots = \|\phi_n\|^{g'} = 1 \end{array} \right\} \neq \varnothing$$

b. $\|\forall x(\phi_1;\ldots;\phi_n)\|^g = 1$ iff for all $g' \approx_x g$, $\|\phi_1\|^{g'} = \cdots = \|\phi_n\|^{g'} = 1$

$$\text{iff } \left\{ g'(x) \;\middle|\; \begin{array}{l} g' \approx_x g \text{ and} \\ \|\phi_1\|^{g'} = \cdots = \|\phi_n\|^{g'} = 1 \end{array} \right\} = D$$

1.3 Type-$\langle n_1,\ldots,n_m\rangle$ GQs

We can also increase the number of variables and number of formulas at the same time.

(13) a. $\left\|\exists[x_1^1,\ldots x_i^1;\ldots;x_1^n,\ldots,x_j^n](\phi_1;\ldots;\phi_n)\right\|^g = 1$
iff for some $g_1' \approx_{\{x_1^1,\ldots,x_i^1,\ldots,x_1^n,\ldots x_j^n\}} g$, $\|\phi_1\|^{g_1'} = \cdots = \|\phi_n\|^{g_n'} = 1$

b. $\left\|\forall[x_1^1,\ldots x_i^1;\ldots;x_1^n,\ldots,x_j^n](\phi_1;\ldots;\phi_n)\right\|^g = 1$
iff for all $g' \approx_{\{x_1^1,\ldots x_i^1,\ldots,x_1^n,\ldots x_j^n\}} g$, $\|\phi_1\|^{g'} = \cdots = \|\phi_n\|^{g'} = 1$

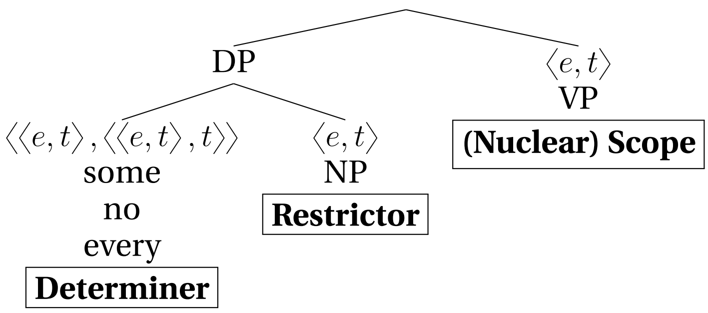
2 Natural language quantifiers

Natural language quantifiers are analyzed as GQs (Montague 1973, Barwise & Cooper 1981, Keenan & Stavi 1986; for overviews, see Peters & Westerståhl 2006, Westerståhl 2024).
Determiners are type-$\langle 1,1\rangle$ GQs. Ignoring presuppositions for now:

(14) a. $[\![\textbf{some}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ for some $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\ \{\, x \mid P(x) = 1 \,\} \cap \{\, x \mid Q(x) = 1 \,\} \neq \varnothing$

b. $[\![\textbf{no}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ for no $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\ \{\, x \mid P(x) = 1 \,\} \cap \{\, x \mid Q(x) = 1 \,\} = \varnothing$

c. $[\![\textbf{every}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ for every $x \in D$ such that $P(x) = 1$, $Q(x) = 1$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}. \{\, x \mid P(x) = 1 \,\} \subseteq \{\, x \mid Q(x) = 1 \,\}$



DPs are Type-$\langle 1 \rangle$ GQs.

(15) a. $[\![\textbf{some dog}]\!]^g = \lambda Q_{\langle e,t \rangle}.$ for some dog $x \in D$, $Q(x) = 1$
$= \lambda Q_{\langle e,t \rangle}.\ \{ x \mid x \text{ is a dog} \} \cap \{ x \mid Q(x) = 1 \} \neq \varnothing$

b. $[\![\textbf{no dog}]\!]^g = \lambda Q_{\langle e,t \rangle}.$ for no dog $x \in D$, $Q(x) = 1$
$= \lambda Q_{\langle e,t \rangle}.\ \{ x \mid x \text{ is a dog} \} \cap \{ x \mid Q(x) = 1 \} = \varnothing$

c. $[\![\textbf{every dog}]\!]^g = \lambda Q_{\langle e,t \rangle}.$ for every dog $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda Q_{\langle e,t \rangle}.\ \{ x \mid x \text{ is a dog} \} \subseteq \{ x \mid Q(x) = 1 \}$

Referring DPs can be lifted to type-$\langle 1 \rangle$ GQs.

(16) $[\![\textbf{John}]\!]^g = \lambda Q_{\langle e,t \rangle}.\ Q(j) = 1$

Ignoring presuppositions, type-$\langle 1 \rangle$ GQs are isomorphic to sets of sets of individuals.

(17) a. $\{ S \subseteq D \mid \{ x \mid x \text{ is a dog} \} \cap S \neq \varnothing \}$
b. $\{ S \subseteq D \mid \{ x \mid x \text{ is a dog} \} \cap S = \varnothing \}$
c. $\{ S \subseteq D \mid \{ x \mid x \text{ is a dog} \} \subseteq S \}$
d. $\{ S \subseteq D \mid j \in S \}$

Some determiners trigger presuppositions. (Ignoring plurality, homogeneity, and presupposition projection)

(18) a. $[\![\textbf{both}]\!]^g = \lambda P_{\langle e,t \rangle} : \underline{|\{ x \mid P(x) = 1 \}| = 2}.\lambda Q_{\langle e,t \rangle}.$
for every $x \in D$ such that $P(x) = 1$, $Q(x) = 1$

b. $[\![\textbf{neither}]\!]^g = \lambda P_{\langle e,t \rangle} : \underline{|\{ x \mid P(x) = 1 \}| = 2}.\lambda Q_{\langle e,t \rangle}.$
for no $x \in D$ such that $P(x) = 1$, $Q(x) = 1$

c. $[\![\textbf{the}]\!]^g = \lambda P_{\langle e,t \rangle} : \underline{|\{ x \mid P(x) = 1 \}| = 1}.\lambda Q_{\langle e,t \rangle}.$
for every $x \in D$ such that $P(x) = 1$, $Q(x) = 1$

Determiners with existence presuppositions:

(19) a. $[\![\textbf{every}]\!]^g = \lambda P_{\langle e,t \rangle} : \underline{|\{ x \mid P(x) = 1 \}| \geqslant 1}.\lambda Q_{\langle e,t \rangle}.$
for every $x \in D$ such that $P(x) = 1$, $Q(x) = 1$

b. $[\![\textbf{most}]\!]^g = \lambda P_{\langle e,t \rangle} : \underline{|\{ x \mid P(x) = 1 \}| \geqslant 1}.\lambda Q_{\langle e,t \rangle}.$
for most $x \in D$ such that $P(x) = 1$, $Q(x) = 1$

(anti-presuppositions)
Without plurality, numerals look like.

(20) a. $[\![\textbf{three}]\!]^g = \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.$ for three $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.\ |\{ x \mid P(x) = 1 \} \cap \{ x \mid Q(x) = 1 \}| \geqslant 3$

b. $[\![\textbf{more than three}]\!]^g = \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.$ for three $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.\ |\{ x \mid P(x) = 1 \} \cap \{ x \mid Q(x) = 1 \}| > 3$

c. $[\![\textbf{fewer than three}]\!]^g = \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.$ for three $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.\ |\{ x \mid P(x) = 1 \} \cap \{ x \mid Q(x) = 1 \}| < 3$

d. $[\![\textbf{exactly three}]\!]^g = \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.$ for three $x \in D$, $P(x) = 1$ and $Q(x) = 1$
$= \lambda P_{\langle e,t \rangle}.\lambda Q_{\langle e,t \rangle}.\ |\{ x \mid P(x) = 1 \} \cap \{ x \mid Q(x) = 1 \}| = 3$

2.1 Plurality

For plurality, let's use i(ndividual)-sums (Link 1983).

- If $x, y \in D$, then $x \sqcup y \in D$.
- $x \sqcup x = x$, $x \sqcup y = y \sqcup x$, $(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)$
- $x \sqsubseteq y$ iff $x \sqcup y = y$
- $A = \min_{\sqsubseteq} D$
- $x \sqsubseteq_a y$ iff $x \sqsubseteq y$ and $x \in A$

(21) a. $[\![\textbf{dog}]\!]^g = \lambda x_e.\ x$ is a dog
b. $[\![\textbf{dogs}]\!]^g = \lambda x_e.$ for each $y \sqsubseteq_a x$, $[\![\textbf{dog}]\!]^g(y) = 1$

(22) a. $[\![\textbf{fell asleep}]\!]^g = \lambda x_e.$ for each $y \sqsubseteq_a x$, y fell asleep
b. $[\![\textbf{met}]\!]^g = \lambda x_e.\ x$ met
c. $[\![\textbf{wrote a paper}]\!]^g = \lambda x_e.\ x$ wrote a paper

(23) $[\![\Delta]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda x_e.$ for each $y \sqsubseteq_a x$, $P(y) = 1$

(We ignore non-atomic distributivity)

(24) a. $[\![\textbf{three dogs}]\!]^g = \lambda Q_{\langle e,t\rangle}.$ for some $x \in D$,
for each $y \sqsubseteq_a x$, y is a dog and $|\{\, y \mid y \sqsubseteq_a x \,\}| = 3$ and $Q(x) = 1$
b. $[\![\textbf{three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$for some $x \in D$,
$P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| = 3$ and $Q(x) = 1$

- (24) derives an at-least reading.
- The restrictor might contain a non-distributive predicate, *dogs that hate each other*.

(25) a. $[\![\textbf{more than three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$for some $x \in D$,
$P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| > 3$ and $Q(x) = 1$
b. $[\![\textbf{fewer than three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$for no $x \in D$,
$P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| \geqslant 3$ and $Q(x) = 1$
c. $[\![\textbf{exactly three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$for some $x \in D$,
$P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| = 3$ and $Q(x) = 1$ and
for no $x \in D$, $P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| > 3$ and $Q(x) = 1$

These can be restated more neatly as follows:

(26) a. $[\![\textbf{more than three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$
$\{\, y \mid y \sqsubseteq_a \bigsqcup \{\, x \mid P(x) = Q(x) = 1 \,\}\,\} > 3$
b. $[\![\textbf{fewer than three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$
$\{\, y \mid y \sqsubseteq_a \bigsqcup \{\, x \mid P(x) = Q(x) = 1 \,\}\,\} < 3$
c. $[\![\textbf{exactly three}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$
$\{\, y \mid y \sqsubseteq_a \bigsqcup \{\, x \mid P(x) = Q(x) = 1 \,\}\,\} = 3$

Generally determiners only count atomic individuals.

- If there are two dogs, a and b, $|\{x \mid [\![\textbf{dogs}]\!]^g(x) = 1\}| = |\{a, b, a \sqcup b\}| = 3$.
- If there are three dogs, $|\{x \mid [\![\textbf{dogs}]\!]^g(x) = 1\}| = 7$, 4 of which are non-atomic.

If you believe *both* and *neither* are distributive quantifiers with respect to the nuclear scope:

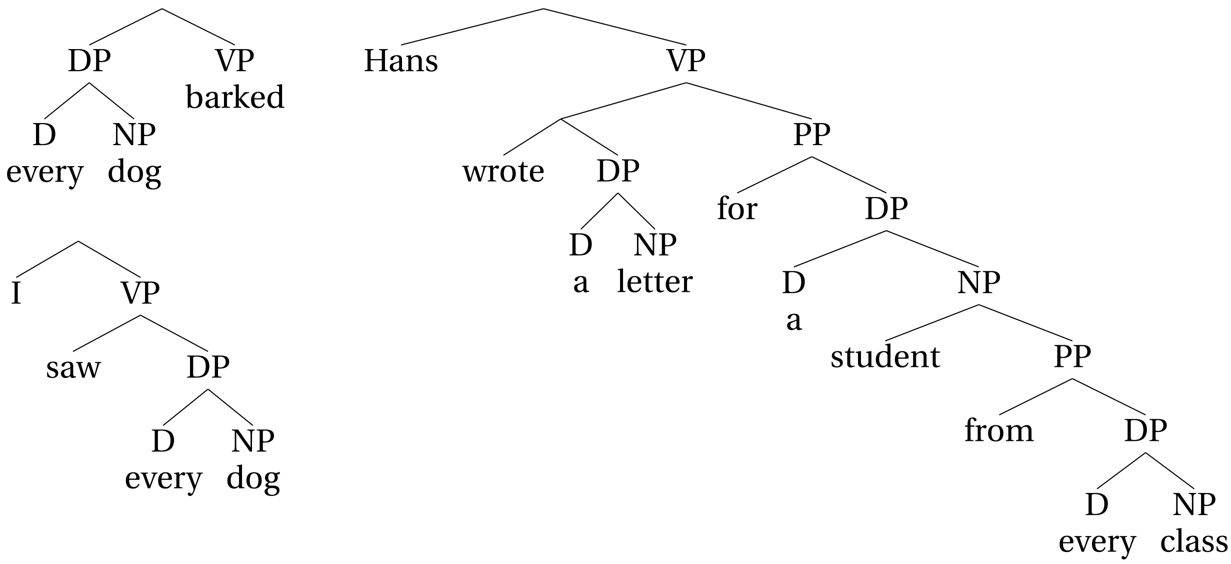
(27) a. $[\![\textbf{both}]\!]^g = \lambda P_{\langle e,t\rangle} \colon \underline{|\min_{\sqsubseteq}\{x \mid P(x) = 1\}| = 2}.\lambda Q_{\langle e,t\rangle}.$ for each $x \in \{x \mid P(x) = 1\}$, for each $y \sqsubseteq_a x$, $Q(y) = 1$

b. $[\![\textbf{neither}]\!]^g = \lambda P_{\langle e,t\rangle} \colon \underline{|\min_{\sqsubseteq}\{x \mid P(x) = 1\}| = 2}.\lambda Q_{\langle e,t\rangle}.$ for each $x \in \{x \mid P(x) = 1\}$, for each $y \sqsubseteq_a x$, $Q(y) = 0$

(28) $[\![\textbf{the}]\!]^g = \lambda P_{\langle e,t\rangle} \colon \underline{|\max_{\sqsubseteq}\{x \mid P(x) = 1\}| = 1}.\lambda Q_{\langle e,t\rangle}.$ for every $x \in \max_{\sqsubseteq}\{x \mid P(x) = 1\}$, $Q(x) = 1$

2.2 Quantifier Scope

English syntax allows the scope to be in all sorts of positions.



Since all DPs are uniformly type-$\langle 1\rangle$ GQs, we can just type-lift the verbs.

(29) a. $[\![\textbf{barked}]\!]^g = \lambda Q_{\langle\langle e,t\rangle,t\rangle}.Q(\lambda x_e.\ x \text{ barked})$

b. $[\![\textbf{saw}]\!]^g = \lambda Q_{\langle\langle e,t\rangle,t\rangle}.\lambda R_{\langle\langle e,t\rangle,t\rangle}.\ R(\lambda y_e.\ Q(\lambda x_e.\ y \text{ saw } x))$

But this is not enough to account for complex examples. Also, (29b) assigns the subject quantifier wide scope.

(30) A child saw every dog.

One way to derive the inverse scope reading, is by using a different entry for the verb.

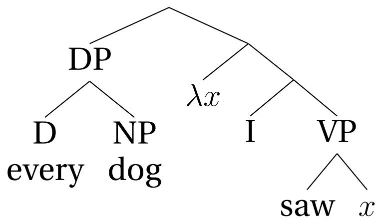
(31) $[\![\textbf{saw}_2]\!]^g = \lambda Q_{\langle\langle e,t\rangle,t\rangle}.\lambda R_{\langle\langle e,t\rangle,t\rangle}.\ Q(\lambda x_e.\ R(\lambda y_e.\ y \text{ saw } x))$

Or we can change the denotation of the quantifier

(32) $[\![\textbf{every}^2\ \textbf{dog}]\!]^g = \lambda R_{\langle e,\langle e,t\rangle\rangle}.\lambda Q_{\langle\langle e,t\rangle,t\rangle}.\ Q(\lambda x_e.\ \text{for every dog } y,\ R(x)(y))$

These changes in meaning are systematic and formulated as general type-shifting rules. Then we don't need to see scope ambiguity as lexical ambiguity, but 'derivational ambiguity'.

Alternatively, one can assume a syntactic representation different from the surface structure and move things around via Quantifier Raising to make sure that we always have $D(R)(S)$.



3 Strong and weak quantifiers

GQT treats all quantifiers on a par.

There are linguistic phenomena that seem to distinguish so-called weak and strong quantifiers.

- *There*-sentences
- Stage-level vs. individual-level predicates

3.1 *There*-sentences

Only some quantifiers can appear in *there*-sentences in English (Milsark 1977).

(33) $$\text{There is} \left\{ \begin{array}{l} \text{a} \\ \text{no} \\ \text{*the} \\ \text{*every} \\ \text{*each} \\ \text{*Maria's} \end{array} \right\} \text{solution to this problem.}$$

(34) $$\text{There are} \left\{ \begin{array}{l} \text{some} \\ \text{three} \\ \text{many} \\ \text{no} \\ \text{*the} \\ \text{*all} \\ \text{*most} \\ \text{*both} \\ \text{*neither} \end{array} \right\} \text{solutions to this problem.}$$

Note that definites and proper names are allowed in the 'list' interpretation.

(35) Who can help us?

a. There's the new lecturer.
b. There's Maria.

But these become worse in questions/non-assertions(?).

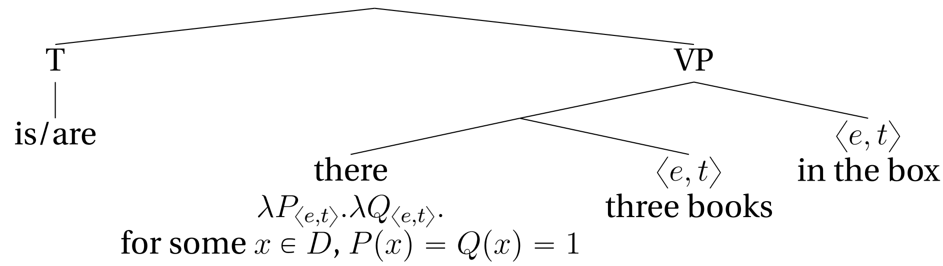
3.1.1 *Milsark's 1977 analysis*

Milsark 1977 called those noun phrases that can appear in *there*-sentences 'weak' and those that cannot 'strong'.
Milsark 1977 analyzes weak noun phrases as cardinality predicates, so non-quantifiers.

(36) a. $[\![\textbf{three}_{\textbf{wk}}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| = 3$
b. $[\![\textbf{some}_{\textbf{wk}}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}|$ is 'small'

The *there*-construction itself denotes the existential quantifier. One way to implement this is something like (37).

(37)



Or the existential quantifier can be somewhere else, with *there* being completely meaningless.
On this analysis, a strong quantifier cannot be the pivot for type-reasons.
Note that for negative weak NPs, we need to separate the negation. The following are wrong (why?).

(38) a. $[\![\textbf{no}_{\textbf{wk}}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| = 0$
b. $[\![\textbf{fewer than three}_{\textbf{wk}}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ P(x) = 1$ and $|\{\, y \mid y \sqsubseteq_a x \,\}| < 3$

We have to decompose these into negation and a positive weak NP, and have negation obligatorily take scope over the existential quantifier.

3.1.2 *Complications*

McNally's 2019 observations (see also **mcnallyvangeenhoven:97**, **mcnally:16** for related data and discussion):

- Strong quantifiers are allowed if they are about kinds/types.

(39) a. There was **every reason** to leave.
b. Students will be able to compute how many days during the week there was **each type of weather**.

- Non-unique definites can appear in *there*-sentences.

(40) We finally discovered that there was **the lid to a Cool-Whip container** in the pie underneath the filling!

- Partitive quantifiers are not always bad.

(41) *There are **some of the people** in the bathroom. (Milsark 1977)

(42) Ida wondered if there was some of the marines' homebrewed engine juice around. (McNally 2019)

3.2 Stage-level vs. individual-level predicates

The coda of a *there*-sentence has to be a **stage-level predicate** describing a non-permanent property, and cannot be an **individual-level predicate** (Milsark 1977 called them 'state-descriptive predicates' and 'property predicates' respectively).

(43) There were some/no people $\left\{\begin{array}{l}\text{in the classroom}\\ \text{drunk}\\ \text{*intelligent}\\ \text{*tall}\\ \text{*young}\end{array}\right\}$

NB: As Milsark 1977 points out, a complex AP like tall enough to play basketball may appear as part of the pivot.

(44) There are [some people tall enough to play basketball] [CODA].

Milsark 1977 proposes that individual-level predicates only combine with strong quantifiers. Note that some of the following are grammatical:

(45) a. ??A student was tall.
b. Students were tall.
c. Some students were tall.
d. Many students were tall.
e. Few students were tall.
f. No students were tall.

But the acceptable cases are either generic, proportional (*many, few*), or partitive (cf., *some of the students*), and not purely existential like (46). For Milsark 1977, they are strong quantifiers (or strong uses of these words).

(46) a. There was an tall student.
b. There were tall students.
c. There were some tall students.
d. There were many tall students.
e. There were few tall students.
f. There were no tall students.

One way of accounting for these observations is by postulating strong versions of the weak NPs, which are universal, proportional, or partitive GQs.

Ideally, we want to have a general way of turning a weak NP to a strong quantifier (and/or a strong quantifier to a weak NP).

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