

1 Presuppositions of Quantified Sentences

- (1) Andy quit smoking recently. $\leadsto$ Andy was a smoker.

What do quantified sentences presuppose?

- (2) Projection out of nuclear scope:
- a. A student of mine
 - b. Every student of mine
 - c. No student of mine
 - d. Most students of mine
- } quit smoking recently.
- (3) Projection out of restrictor:
- a. A student of mine
 - b. Every student of mine
 - c. No student of mine
 - d. Most students of mine
- } who quit smoking recently is/are happier.

Different theoretical possibilities:

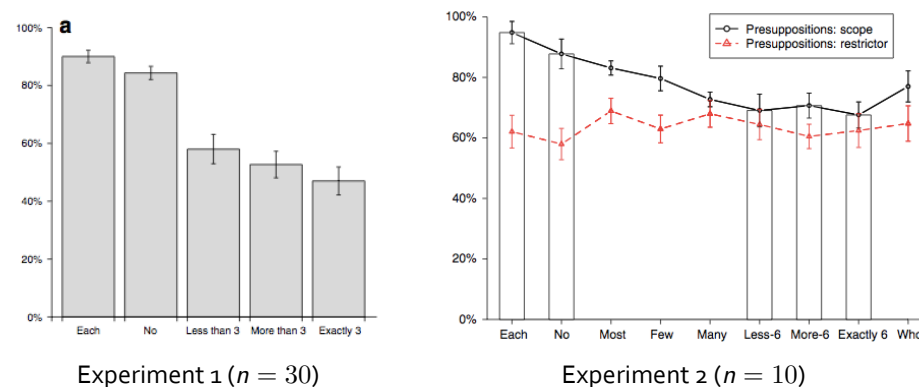
- **Universal presuppositions only:** Heim (1983) predicts all of (2) and (3) presuppose that each of my students was a smoker. This seems to be too strong.
- **Existential presuppositions only:** Beaver (1994, 2001) contends that there's no evidence for any quantified sentence that it has a universal presupposition. According to him, all of the above sentences presuppose that at least one student was a smoker (see also Van der Sandt 1992, Kadmon 2001 for similar remarks).
- Trivalent theory makes more nuanced predictions.

2 Chemla (2009)

2.1 Experiment 1

Question: Does *S* suggest *p*?

- (4) *S*: None of these 10 students knows that his father is going to receive a congratulation letter.
p: The father of each of these 10 students is going to receive a congratulation letter.



Binary choice: Yes (*Oui*) or No (*Non*)

Quantifiers: *each* (*chaqun*), *no* (*aucun*), *less than three* (*moins de 3*), *more than three* (*plus de 3*), *exactly three* (*exactement 3*)

Presupposition triggers: *know*, *be aware*, *stop*, *continue*, *his*

2.2 Experiment 2

Graded judgments

Quantifiers: *each* (*chacun*), *no* (*aucun*), *most* (*la plupart*), *few* (*peu*), *many* (*beaucoup*), *less than six* (*moins de 6*), *more than six* (*plus de 6*), *exactly six* (*exactement 6*), *who* (*qui*)

- (5) Projection out of restrictor (NP)
- S*: Among these 20 students, most who know that their father is going to receive a congratulation letter take English lessons.
p: The father of each of these 20 students is going to receive a congratulation letter.

2.3 Summary

- *No* tends to give rise to a universal inference (possibly not a presupposition) more robustly than *less than three*, *more than three* and *exactly three*.
- (No interesting differences among presupposition triggers)

3 Zehr, Bill, Tieu, Romoli & Schwarz (2016)

Zehr, Bill, Tieu, Romoli & Schwarz (2016) investigated presuppositions triggered in the nuclear scope of *none*. They tested three possible readings:

- (6) None of the bears won the race.
- EXISTENTIAL: At least one of the bears participated (and no bear won)
 - UNIVERSAL: All of the bears participated (and no bear won)
 - PRESUPPOSITIONLESS: (No bear both participated and won)

Context sentence + picture followed by a covered box choice task

Four types of pictures:

- (7) a. TRUECONTROL: All the bears participated, none of them won
b. ONLYSOME: Only some of the bears participated, none of them won
c. NORUNNER: None of the bears participated
d. FALSECONTROL: All the bears participated, one won

[During the morning race, these three bears did really well, and in the end, one of them won. I thought they would do the same later in the day as well, but...] (audio)



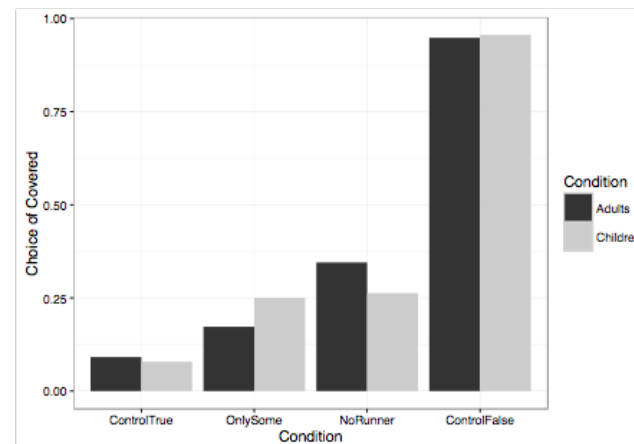
Press any key to continue.

[None of the bears won the afternoon race.] (audio)



There's a difference between ONLYSOME and NORUNNER for adults (but not fo kids). Zehr et al. take this as evidence that the PRESUPPOSITIONLESS reading exists, in addition to the EXISTENTIAL reading.

But is it possible that somehow the ONLYSOME picture was preferred to the NORUNNER picture for unrelated reasons?



42 adults and 22 children (4;00-5;10)

4 Trivalent Theory

4.1 Basics

Three truth-values, # when the presupposition fails.

$$(8) \llbracket \text{Andy quit smoking} \rrbracket = \begin{cases} 1 & \text{if Andy used to smoke and no longer smokes} \\ 0 & \text{if Andy used to smoke and still does} \\ \# & \text{if Andy never smoked before} \end{cases}$$

The presupposition of a sentence amounts to the cases where it does not denote #; or equivalently, the cases where it denotes a classical truth-value (1 or 0), i.e. the disjunction of the first two conditions.

(9) Symmetric Conjunction (Weak Kleene)

$$\llbracket S \text{ and}_w T \rrbracket = \begin{cases} 1 & \text{if } \llbracket S \rrbracket = 1 \text{ and } \llbracket T \rrbracket = 1 \\ \# & \text{if } \llbracket S \rrbracket = \# \text{ or } \llbracket T \rrbracket = \# \\ 0 & \text{otherwise} \end{cases}$$

(10) Symmetric Conjunction (Strong Kleene)

$$\llbracket S \text{ and}_s T \rrbracket = \begin{cases} 1 & \text{if } \llbracket S \rrbracket = 1 \text{ and } \llbracket T \rrbracket = 1 \\ 0 & \text{if } \llbracket S \rrbracket = 0 \text{ or } \llbracket T \rrbracket = 0 \\ \# & \text{otherwise} \end{cases}$$

(11) *Asymmetric Conjunction (Middle Kleene)*

$$\llbracket S \text{ and}_m T \rrbracket = \begin{cases} 1 & \text{if } \llbracket S \rrbracket = 1 \text{ and } \llbracket T \rrbracket = 1 \\ \# & \text{if } \llbracket S \rrbracket = \#, \text{ or } \llbracket S \rrbracket = 1 \text{ and } \llbracket T \rrbracket = \# \\ 0 & \text{otherwise} \end{cases}$$

4.2 'Every'

One-place predicates map entities to truth-values.

$$(12) \llbracket \text{quit smoking} \rrbracket (x) = \begin{cases} 1 & \text{if } x \text{ was a smoker but is a non-smoker now} \\ 0 & \text{if } x \text{ was a smoker and still is} \\ \# & \text{if } x \text{ was not a smoker} \end{cases}$$

A universal quantifier has the following meaning:

$$(13) \llbracket \text{Everyone VP} \rrbracket = \begin{cases} 1 & \text{if for every } x, \llbracket \text{VP} \rrbracket (x) = 1 \\ 0 & \text{if for at least one } x, \llbracket \text{VP} \rrbracket (x) = 0 \\ \# & \text{otherwise} \end{cases}$$

$$(14) \llbracket \text{Every student quit smoking} \rrbracket = \begin{cases} 1 & \text{if every student used to smoke but no longer does} \\ 0 & \text{if at least one student used to smoke and still does} \\ \# & \text{otherwise} \end{cases}$$

The predicted presupposition of (14) is the disjunction of the first two conditions: either every student used to smoke and stopped, or at least one student used to smoke and still smokes.

In this case, the sentence entails that every student used to smoke, so in an experiment like Chemla's it is predicted that it always gives rise to a 'universal inference' (if not a universal presupposition).

But how does the following sound to you?

- (15) A: Every student of mine went back to China for holidays.
B: Hey wait a minute! I didn't know that all of your students are from China!

Fox (2012) suggests that one tends to accommodate something stronger than this disjunctive presupposition *per se*, namely, every student used to smoke. He suggests that this inference is of the same nature as the proviso inference.

4.3 'No'

$$(16) \llbracket \text{No one VP} \rrbracket = \begin{cases} 1 & \text{if for every } x, \llbracket \text{VP} \rrbracket (x) = 0 \\ 0 & \text{if for at least one } x, \llbracket \text{VP} \rrbracket (x) = 1 \\ \# & \text{otherwise} \end{cases}$$

$$(17) \llbracket \text{No student quit smoking} \rrbracket = \begin{cases} 1 & \text{if every student used to smoke and still does} \\ 0 & \text{if at least one student used to smoke but no longer does} \\ \# & \text{otherwise} \end{cases}$$

The predicted presupposition is: either every student used to smoke and still does, or at least one student used to smoke but stopped. This is weaker than a universal presupposition, but again, it is entailed that every student used to smoke.

The PRESUPPOSITIONLESS reading can be explained by the optional *A*-operator, which turns presuppositions into assertions (LOCAL ACCOMMODATION):

$$(18) \llbracket A \text{ VP} \rrbracket (x) = \begin{cases} 1 & \text{if } \llbracket \text{VP} \rrbracket (x) = 1 \\ 0 & \text{if } \llbracket \text{VP} \rrbracket (x) = 0 \text{ or } \llbracket \text{VP} \rrbracket (x) = \# \end{cases}$$

$$(19) \llbracket A \text{ quit smoking} \rrbracket (x) = \begin{cases} 1 & \text{if } x \text{ was a smoker but is a non-smoker now} \\ 0 & \text{if } x \text{ has been a smoker or } x \text{ has been a non-smoker} \end{cases}$$

'No student *A* quit smoking' will then not entail that every student used to smoke:

$$(20) \llbracket \text{No student } A \text{ quit smoking} \rrbracket = \begin{cases} 1 & \text{if every student either has been a smoker or has been a non-smoker} \\ 0 & \text{if at least one student was a smoker but is not a smoker now} \end{cases}$$

It is still predicted that 'Every student *A* quit smoking' entails that every student used to smoke.

$$(21) \llbracket \text{Every student } A \text{ quit smoking} \rrbracket = \begin{cases} 1 & \text{if every student was a smoker but is not anymore} \\ 0 & \text{if at least one student has been a smoker or has been a non-smoker} \end{cases}$$

This explains the (slight) difference between *every* and *no* in Chemla's experiments.

Note that this does not straightforwardly explain the difference between ONLYSOME and NORUNNER in Zehr, Bill, Tieu, Romoli & Schwarz's (2016) experiment with adult participants: The reading is true with respect to the overt picture in both conditions. But again,

it's not entirely clear to me if this difference should be taken as evidence for the EXISTENTIAL reading.

4.4 Existential Quantifiers

$$(22) \llbracket A \text{ NP VP} \rrbracket = \begin{cases} 1 & \text{if for at least one } x, \llbracket \text{NP} \rrbracket(x) = 1 \text{ and } \llbracket \text{VP} \rrbracket(x) = 1 \\ 0 & \text{if for every } y \text{ such that } \llbracket \text{NP} \rrbracket(y) = 1, \llbracket \text{VP} \rrbracket(y) = 0 \\ \# & \text{otherwise} \end{cases}$$

$$(23) \llbracket \text{A student quit smoking} \rrbracket = \begin{cases} 1 & \text{if at least one student used to smoke and does not anymore} \\ 0 & \text{if every student used to smoke and still does} \\ \# & \text{otherwise} \end{cases}$$

The predicted presupposition is: at least one student used to smoke and quit, or every student used to smoke and still does. Beaver & Krahmer (2001) seem to think that this is a good prediction, but would you infer this disjunction from this sentence?

Fox (2012) suggests that something stronger tends to be accommodated, namely, every student used to smoke. But this does not seem to happen with Heim's example. I don't think anyone infers from (24) that all fat men have bicycles (or is this really less likely than that all students all smoke?).

(24) A fat man was pushing his bicycle.

But this might be due to implicit *domain restriction*. (Westerståhl 1984, von Stechow 1994, Gawron 1996, Geurts & Van der Sandt 1999, Stanley & Szabó 2000, Rothschild 2011). We know that natural language determiners can implicitly restrict their domain of quantification:

(25) When I entered the room, everyone was asleep.

If a in (24) ranges over some specific group of fat men, the presupposition is going to be about these fat men only. When strengthened, it amounts to that each fat man *in this group* has a bicycle, which is a much weaker inference than all fat men in the world have bicycles.

In an extreme case, maybe the domain can be narrowed down to just one person (cf. Schwarzschild 2002). Then the universal presupposition will be essentially identical to the existential presupposition.

Or at least, the domain can be narrowed down to the set of fat men who own bicycles. This might not be so outlandish, if you assume that the presupposition of the VP could be used to signal the domain of quantification.

But we know that this kind of domain restriction cannot apply to other quantifiers. This is especially prominent with proportional quantifiers, which are non-monotonic with respect to the restrictor.

(26) Most fat men were pushing their bicycles.

Suppose there were 50 fat men.

- 40 didn't have bicycles.
- Among the 10 fat men who had bicycles, 8 were pushing them.

(26) sounds false in this situation.

Conclusion: Domain restriction is real, but it does not seem to give a general explanation of the non-universal presupposition of existential sentences.

4.5 'Fewer Than 3'

$$(27) \llbracket \text{Fewer than 3 NP VP} \rrbracket = \begin{cases} 1 & \text{if there are zero, one or two } x \text{ such that } \llbracket \text{NP} \rrbracket(x) = 1 \text{ and } \llbracket \text{VP} \rrbracket(x) = 1, \\ & \text{and for all other } y \text{ such that } \llbracket \text{NP} \rrbracket(y) = 1, \llbracket \text{VP} \rrbracket(y) = 0 \\ 0 & \text{if there are four } x \llbracket \text{NP} \rrbracket(x) = 1 \text{ and } \llbracket \text{VP} \rrbracket(x) = 1 \\ \# & \text{otherwise} \end{cases}$$

$$(28) \llbracket \text{Fewer than three students quit smoking} \rrbracket = \begin{cases} 1 & \text{if there are zero, one or two students who used to smoke and quit,} \\ & \text{and the rest of the students used to smoke and did not quit} \\ 0 & \text{if there are four or more students who used to smoke and quit} \\ \# & \text{otherwise} \end{cases}$$

This meaning entails that every student used to smoke, but Chemla's experimental results indicate that *exactly n* does not give rise to a universal inference robustly.

One could resort to $\mathcal{A}$, but a question remains as to why it is not used as often for *no*.

$$(29) \llbracket \text{Fewer than three students } \mathcal{A} \text{ quit smoking} \rrbracket = \begin{cases} 1 & \text{if there are zero, one or two students who used to smoke and quit,} \\ & \text{and the rest of the students have been smokers or non-smokers} \\ 0 & \text{otherwise} \end{cases}$$

An alternative analysis would be to assume that the lexical semantics of *fewer than three* contains the $\mathcal{A}$ -operator:

$$(30) \llbracket \text{Fewer than 3 NP VP} \rrbracket$$

$$= \begin{cases} 1 & \text{if there are zero, one or two } x \text{ such that } \llbracket \text{NP} \rrbracket(x) = 1 \text{ and } \llbracket \text{VP} \rrbracket(x) = 1, \\ & \text{and for all other } y \text{ such that } \llbracket \text{NP} \rrbracket(y) = 1, \llbracket \text{VP} \rrbracket(y) = 0 \text{ or } \llbracket \text{VP} \rrbracket(y) = \# \\ 0 & \text{if there are four } x \llbracket \text{NP} \rrbracket(x) = 1 \text{ and } \llbracket \text{VP} \rrbracket(x) = 1 \end{cases}$$

But again, the prediction is that there would be no presupposition in the following sentences.

- (31) a. I don't think fewer than ten of my students have quit smoking.
b. If fewer than ten of my students have quit smoking, I need to talk to them again.

4.6 'Not Every'

$$(32) \llbracket \text{not } S \rrbracket = \begin{cases} 1 & \text{if } \llbracket S \rrbracket = 0 \\ 0 & \text{if } \llbracket S \rrbracket = 1 \\ \# & \text{if } \llbracket S \rrbracket = \# \end{cases}$$

$$(33) \llbracket \text{Not every student quit smoking} \rrbracket = \begin{cases} 1 & \text{if at least one student used to smoke and still does} \\ 0 & \text{if every student used to smoke but no longer does} \\ \# & \text{otherwise} \end{cases}$$

This does not entail that every student used to smoke. And the predicted presupposition is weaker than universal: either at least one student has been a smoker, or every student is a former smoker.

But intuitively, (33) has a universal presupposition, I think:

- (34) A: Not every student of mine knows that I hate them.
B: Hey wait a minute! I didn't know that you hate all your students!

One might wonder if the scalar implicature is relevant but it won't give you a universal inference: it only entails that at least two students used to smoke.

$$(35) \llbracket \text{Not every student quit smoking} \rrbracket^{\text{SI}} = \begin{cases} 1 & \text{if at least one student used to smoke and still does,} \\ & \text{and at least one student used to smoke and quit} \\ 0 & \text{if every student used to smoke but no longer does} \\ \# & \text{otherwise} \end{cases}$$

4.7 Further readings on quantifiers in trivalent theory

Beaver & Krahmer (2001)

George (2008b,a): Order effects in quantified sentences

Sudo, Romoli, Hackl & Fox (2011), Fox (2012): 'Proviso inferences' in quantified sentences

5 Satisfaction Theory

Sentences denote Context Change Potentials (CCPs). Presuppositions are pre-conditions for CCPs to apply successfully. Let's write ' $c \triangleright S$ ' to mean ' c satisfies the presupposition of S '.

To deal with anaphora and quantification, we model contexts as sets of pairs $\langle g, w \rangle$ where g is a (partial) assignment function and w is a possible world. We assume that the domains of assignment functions in a given context are identical.

5.1 Non-quantified sentences

A plain sentence eliminates those pairs from the context that falsify it.

$$(36) c[\text{it is raining in London}] = \{ \langle g, w \rangle \in c \mid \text{it is raining in London in } w \}$$

w_1 : it is raining in London and it is snowing in NYC
 w_2 : it is not raining in London and it is snowing in NYC
 w_3 : it is raining in London and it is not snowing in NYC

$$\left\{ \begin{array}{l} \langle [3 \rightarrow a, 7 \rightarrow b], w_1 \rangle, \\ \langle [3 \rightarrow b, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow b, 7 \rightarrow c], w_2 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow c], w_2 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow b], w_3 \rangle, \\ \langle [3 \rightarrow d, 7 \rightarrow b], w_3 \rangle, \\ \langle [3 \rightarrow c, 7 \rightarrow a], w_3 \rangle \end{array} \right\} [\text{it is raining in London}] = \left\{ \begin{array}{l} \langle [3 \rightarrow a, 7 \rightarrow b], w_1 \rangle, \\ \langle [3 \rightarrow b, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow d, 7 \rightarrow b], w_3 \rangle, \\ \langle [3 \rightarrow c, 7 \rightarrow d], w_3 \rangle \end{array} \right\}$$

$$(37) c \triangleright \text{It stopped raining in London} \text{ iff for each } \langle g, w \rangle \in c, \text{ it was raining in London in } w.$$

For complex sentences, the presupposition is compositionally computed with respect to the local context of each update:

$$(38) (\text{Presupposition satisfaction in complex sentences})$$

$$c \triangleright S \text{ when } \begin{cases} c \triangleright T_1 & \text{if } S = \text{not } T_1 \\ c \triangleright T_1 \text{ and } c[T_1] \triangleright T_2 & \text{if } S = T_1 \text{ and } T_2 \\ c \triangleright T_1 \text{ and } c[\text{not } T_1] \triangleright T_2 & \text{if } S = T_1 \text{ or } T_2 \\ c \triangleright T_1 \text{ and } c[T_1] \triangleright T_2 & \text{if } S = \text{if } T_1 \text{ then } T_2 \end{cases}$$

5.2 Indefinites and pronouns

Indefinites like *a student* introduces a new discourse referent by updating the assignment functions in the context.¹

$$(39) \quad c[a^5 \text{ boy cried}] = \left\{ \langle g[5 \rightarrow x], w \rangle \mid \begin{array}{l} \langle g, w \rangle \in c, \\ x \text{ is a boy in } w \text{ and cried in } w \end{array} \right\}$$

w_1 : Only Fred and George are boys who cried.
 w_2 : Only Fred is a boy who cried.
 w_3 : No boy cried.

$$\left\{ \begin{array}{l} \langle [3 \rightarrow a, 7 \rightarrow b], w_1 \rangle, \\ \langle [3 \rightarrow b, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow b, 7 \rightarrow c], w_2 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow c], w_2 \rangle, \\ \langle [3 \rightarrow a, 7 \rightarrow b], w_3 \rangle, \\ \langle [3 \rightarrow d, 7 \rightarrow b], w_3 \rangle, \\ \langle [3 \rightarrow c, 7 \rightarrow c], w_3 \rangle, \end{array} \right\} [a^5 \text{ boy cried}] = \left\{ \begin{array}{l} \langle [3 \rightarrow a, 5 \rightarrow \text{Fred}, 7 \rightarrow b], w_1 \rangle, \\ \langle [3 \rightarrow a, 5 \rightarrow \text{George}, 7 \rightarrow b], w_1 \rangle, \\ \langle [3 \rightarrow b, 5 \rightarrow \text{Fred}, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow b, 5 \rightarrow \text{George}, 7 \rightarrow a], w_1 \rangle, \\ \langle [3 \rightarrow b, 5 \rightarrow \text{Fred}, 7 \rightarrow c], w_2 \rangle, \\ \langle [3 \rightarrow a, 5 \rightarrow \text{Fred}, 7 \rightarrow c], w_2 \rangle \end{array} \right\}$$

Pronouns presuppose that assignment functions are defined for their indices.

(40) $c \triangleright \text{he}_5 \text{ was angry}$ iff for each $\langle g, c \rangle$, $5 \in \text{dom}(g)$ (and $g(5)$ is male in w).

(41) whenever $c \triangleright \text{he}_5 \text{ was angry}$,
 $c[\text{he}_5 \text{ was angry}] = \{ \langle g, w \rangle \in c \mid g(5) \text{ was angry in } w \}$

w_1 : Only Fred and George were angry.
 w_2 : Only John was angry.
 w_3 : No one was angry.

$$\left\{ \begin{array}{l} \langle [1 \rightarrow a, 5 \rightarrow \text{Fred}], w_1 \rangle, \\ \langle [1 \rightarrow a, 5 \rightarrow \text{George}], w_1 \rangle, \\ \langle [1 \rightarrow a, 5 \rightarrow \text{Ben}], w_1 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{Fred}], w_1 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{George}], w_1 \rangle, \\ \langle [1 \rightarrow a, 5 \rightarrow \text{Fred}], w_2 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{George}], w_2 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{John}], w_2 \rangle, \\ \langle [1 \rightarrow c, 5 \rightarrow \text{George}], w_3 \rangle, \\ \langle [1 \rightarrow c, 5 \rightarrow \text{George}], w_3 \rangle \end{array} \right\} [\text{he}_5 \text{ was angry}] = \left\{ \begin{array}{l} \langle [1 \rightarrow a, 5 \rightarrow \text{Fred}], w_1 \rangle, \\ \langle [1 \rightarrow a, 5 \rightarrow \text{George}], w_1 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{Fred}], w_1 \rangle, \\ \langle [1 \rightarrow b, 5 \rightarrow \text{George}], w_1 \rangle, \\ \langle [1 \rightarrow a, 5 \rightarrow \text{John}], w_2 \rangle \end{array} \right\}$$

It is possible to see indefinites as sentences that perform **random assignment**:

$$(42) \quad c[a^5] = \{ \langle g[5 \rightarrow x], w \rangle \mid \langle g, w \rangle \in c, x \in D \}$$

The NP and VP can be seen as sentences (with implicit subjects with variables) that eliminate the unwanted values:

$$\begin{aligned} c[a^5][x_5 \text{ boy}] &= \{ \langle g[5 \rightarrow x], w \rangle \mid \langle g, w \rangle \in c, x \in D \} [x_5 \text{ boy}] \\ &= \{ \langle g[5 \rightarrow x], w \rangle \mid \langle g, w \rangle \in c, x \in D, g[5 \rightarrow x](5) \text{ is a boy in } w \} \\ c[a^5][x_5 \text{ boy}][x_5 \text{ cried}] &= \{ \langle g[5 \rightarrow x], w \rangle \mid \langle g, w \rangle \in c, x \in D, g[5 \rightarrow x](5) \text{ is a boy in } w \} [x_5 \text{ cried}] \\ &= \left\{ \langle g[5 \rightarrow x], w \rangle \mid \begin{array}{l} \langle g, w \rangle \in c, x \in D, \\ g[5 \rightarrow x](5) \text{ is a boy in } w \\ g[5 \rightarrow x](5) \text{ cried in } w \end{array} \right\} \\ &= \left\{ \langle g[5 \rightarrow x], w \rangle \mid \begin{array}{l} \langle g, w \rangle \in c, \\ x \text{ is a boy in } w \text{ and cried in } w \end{array} \right\} \end{aligned}$$

¹Heim (1983) assumes that indefinites presuppose that the variables they introduce are 'new' in c , but let's ignore this; see Heim (2011) for discussion.

$$\left\{ \begin{array}{l} \langle [2 \rightarrow a], w_1 \rangle, \\ \langle [2 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow a], w_2 \rangle \end{array} \right\} [a^5] = \left\{ \begin{array}{l} \langle [2 \rightarrow a, 5 \rightarrow a], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow c], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow d], w_1 \rangle, \\ \vdots \\ \langle [2 \rightarrow b, 5 \rightarrow a], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow c], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow d], w_1 \rangle, \\ \vdots \\ \langle [2 \rightarrow a, 5 \rightarrow a], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow b], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow c], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow d], w_2 \rangle, \\ \vdots \end{array} \right\}$$

w_1 : only b and d are boys

w_2 : only c is a boy

$$\left\{ \begin{array}{l} \langle [2 \rightarrow a, 5 \rightarrow a], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow c], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow d], w_1 \rangle, \\ \vdots \\ \langle [2 \rightarrow b, 5 \rightarrow a], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow c], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow d], w_1 \rangle, \\ \vdots \\ \langle [2 \rightarrow a, 5 \rightarrow a], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow b], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow c], w_2 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow d], w_2 \rangle, \\ \vdots \end{array} \right\} [x_5 \text{ boy}] = \left\{ \begin{array}{l} \langle [2 \rightarrow a, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow a, 5 \rightarrow d], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow b], w_1 \rangle, \\ \langle [2 \rightarrow b, 5 \rightarrow d], w_1 \rangle, \\ \vdots \\ \langle [2 \rightarrow a, 5 \rightarrow c], w_2 \rangle \end{array} \right\}$$

5.3 Presupposition projection through indefinites: nuclear scope

(43) $a^1(x_1 \text{ boy})(x_1 \text{ was pushing his}_1 \text{ bike})$

We know for each $\langle g, w \rangle \in c[a^1][x_1 \text{ boy}]$, $g(1)$ is a boy in w .

By assumption, the nuclear scope has the following semantics:

- (44) a. $c \triangleright x_1 \text{ was pushing his}_1 \text{ bike}$ iff for each $\langle g, w \rangle \in c$, $1 \in \text{dom}(g)$ and $g(1)$ had a bike in w and $g(1)$ was male in w
b. $c[x_1 \text{ was pushing his}_1 \text{ bike}] = \{ \langle g, w \rangle \in c \mid g(1) \text{ was pushing } g(1)\text{'s bike in } w \}$

Then:

- (45) $c[a^1][x_1 \text{ boy}] \triangleright x_1 \text{ was pushing his}_1 \text{ bike}$ iff for each $\langle g, w \rangle \in c$, for each $\langle g[1 \rightarrow b], w \rangle$ where b is a boy in w , b had a bike in w (and b is male in w).

So, all possible boys must have bikes to satisfy the presupposition!

This is arguably too strong. Heim (1983) suggests that local accommodation can happen before computing the nuclear scope, which turns the presupposition of the nuclear scope into an at-issue meaning. Effectively, what is predicted is the same as:

- (46) A boy owned a bike and was pushing it.

But as Soames (1989) points out, this predicts that there should be no presupposition when the sentence is embedded in projective (non-veridical) contexts:

- (47) a. I don't think a boy was pushing his bike.
b. If a boy was pushing his bike, it was Mark.
c. Was a boy pushing his bike?

Do these sentences presuppose anything?

5.4 Presupposition projection through indefinites: restrictor

A similar problem arises with a presupposition triggered in the restrictor.

Let us assume that ' $x_1 \text{ refilled glass}$ ' presupposes that x_1 was filled before:

- (48) a. $c \triangleright x_1 \text{ refilled}$ iff for each $\langle g, w \rangle \in c$, $1 \in \text{dom}(g)$ and $g(1)$ was filled before in w .
b. $c[x_1 \text{ refilled glass}] = \{ \langle g, w \rangle \in c \mid g(1) \text{ is filled in } w \}$

- (49) $A^1(x_1 \text{ refilled glass})(x_1 \text{ is on the table})$

Then:

- (50) $c[a^1] \triangleright x_1 \text{ refilled glass}$ iff for each $\langle g, w \rangle \in c[a^1]$, $g(1)$ was filled before in w

Because a^1 performs random assignment, it requires all individuals in the model to be filled before. This is way too strong.

Heim's (1983) suggestion is that here too, you perform local accommodation, so before

the update with the restrictor, you insert an additional update so that the presupposition will be satisfied:

$$(51) \quad c[a^1][x_1 \text{ was a filled glass}][x_1 \text{ refilled glass}]$$

But the prediction here is that there is no presupposition in the following sentences:

- (52) a. I don't think a refilled glass is on the table now.
 b. If there a refilled glass on the table?
 c. If there is a refilled glass on the table, you need to pay for it.

5.5 Beaver's Fix

Beaver (1994, 2001) proposes a version of Satisfaction Theory that predicts existential presuppositions for all quantified sentences.

The trick is to divide up the context according to which value the assignment functions assign to a given index. For example:

$$(53) \quad c \upharpoonright_{9 \rightarrow \text{John}} = \{ \langle g, w \rangle \in c \mid g(9) = \text{John} \}$$

Then we require the presupposition to be satisfied universally in for some restricted context:

$$(54) \quad c \triangleright x_1 \text{ is pushing his}_1 \text{ bike iff there is some individual } a \in D \text{ such that for each } \langle g, w \rangle \in c \upharpoonright_{1 \rightarrow a}, a \text{ has a bike in } w.$$

That is, what is required is merely that there be at least one individual a is known to own a bike.

Whenever the presupposition is satisfied, each such 'slice' of c is updated and the resulting slices are put together:

$$(55) \quad c[x_1 \text{ is pushing his}_1 \text{ bike}] = \bigcup \left\{ c' \mid \begin{array}{l} \text{for some } a \in D, \\ \text{for each } c \upharpoonright_{1 \rightarrow a}, a \text{ has a bike in } w, \\ \text{and } c' = \{ \langle w, g \rangle \in c \upharpoonright_{1 \rightarrow a} \mid a \text{ is pushing } a\text{'s bike in } w \} \end{array} \right\}$$

Beaver believes that all quantificational sentences have existential presuppositions, but experimental data suggest otherwise.

5.6 Universal quantifiers

There are several theories of generalised quantifiers in dynamic semantics (see Chierchia 1995, Van den Berg 1996, Nouwen 2003, 2007, Brasoveanu 2007, 2008, 2010).

One analysis for *every*:

$$(56) \quad c[\text{every}^i \phi \psi] = \left\{ \langle g, w \rangle \in c \mid \begin{array}{l} \{ g'(i) \mid \langle g', w \rangle \in c[a^i][\phi] \wedge g \leq g' \} \\ \subseteq \{ g''(i) \mid \langle g'', w \rangle \in c[a^i][\phi][\psi] \wedge g \leq g'' \} \end{array} \right\}$$

$$(57) \quad g \leq g' \text{ iff for each } i \in \text{dom}(g), g(i) = g'(i).$$

$$(58) \quad \text{Whenever defined, } c[\text{Every}^3 (x_3 \text{ girl}) (x_3 \text{ clapped})] = \{ \langle g, w \rangle \in c \mid \text{the set of girls in } w \text{ is a subset of the set of girls who clapped in } w \}$$

What if the restrictor ϕ and/or nuclear scope ψ have presuppositions?

As you can see, you need to compute $c[a^i][\phi]$ and $c[a^i][\phi][\psi]$. Thus, the predictions are the same as in the case of indefinites: the presupposition of ϕ needs to be satisfied in c with respect to all individuals; the presupposition of ψ needs to be satisfied with respect to all individuals that made ϕ true.

For instance, (59) is predicted to presuppose that every student has a paper and will have their paper accepted by $L\&P$.

$$(59) \quad \text{Every student is aware that their paper will be accepted by } L\&P.$$

This prediction looks reasonable. On the other hand, the prediction for restrictors is wrong. E.g. (60) is predicted to presuppose that every individual, linguist or not, is female.

$$(60) \quad \text{Every pregnant linguist gave a talk.}$$

5.7 Other Generalised Quantifiers

General recipe for generalised quantifiers:

$$(61) \quad \begin{array}{l} \text{a. } c[Q^i \phi \psi] \text{ is defined only if } i \notin \text{dom}(c) \\ \text{b. Whenever defined, } c[Q^i \phi \psi] \\ = \left\{ \langle g, w \rangle \in c \mid \begin{array}{l} \mathbb{Q}(\{ g'(i) \mid \langle g', w \rangle \in c[a^i][\phi] \wedge g \leq g' \}) \\ (\{ g''(i) \mid \langle g'', w \rangle \in c[a^i][\phi][\psi] \wedge g \leq g'' \}) \end{array} \right\} \end{array}$$

Q here is a dynamic quantifier, and $\mathbb{Q}$ is its classical counterpart, which is a relation between two sets. The semantics of *every* above is an instance of this. *Most* looks (roughly) like (62).

$$(62) \quad \begin{array}{l} \text{a. } c[\text{most}^i \phi \psi] \text{ is defined only if } i \notin \text{dom}(c) \\ \text{b. Whenever defined, } c[\text{most}^i \phi \psi] \\ = \left\{ \langle g, w \rangle \in c \mid \frac{|\{ g'(i) \mid \langle g', w \rangle \in c[a^i][\phi] \wedge g \leq g' \}|}{|\{ g''(i) \mid \langle g'', w \rangle \in c[a^i][\phi][\psi] \wedge g \leq g'' \}|} \gg \frac{1}{2} \right\} \end{array}$$

According to analysis, the predictions about presupposition projection are the same for

all dynamic generalised quantifiers, because in order to compute $c[Q^i \phi \psi]$, you need to compute $c[a'][\phi]$ and $c[a'][\phi][\psi]$. Arguably, these predictions are not correct in the general case, especially for restrictors.

6 Other Theories

- Discourse Representation Theory systematically makes weaker predictions than Satisfaction Theory. But it's not true that they never predict universal presuppositions. Read Geurts & Van der Sandt (1999).
- Chemla (2009)
- Sudo (2014)

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