SEMANTICS RESEARCH SEMINAR: TRIVALENT ACCOUNTS OF PRESUPPOSITIONS

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January 26, 2018

1 Trivalence

We don't just assign true or false to sentences (at worlds) but rather also a third truth-value (undefined).[1]

1.1 Weak Kleene

Undefinedness anywhere causes entire sentence to be undefined.

$\neg$	
1	0
0	1
#	#

$\wedge$	1	0	#
1	1	0	#
0	0	0	#
#	#	#	#

$\vee$	1	0	#
1	1	1	#
0	1	0	#
#	#	#	#

In some ways, extension of basic notion of syntactic 'correctness': all parts need to be syntactically well-formed for whole to be, likewise all parts of sentence need to semantically non-deviant for sentence to be.

[1] Equivalent to a trivalent semantics we can just have a partial assignment on truth and falsity [Blamey, 1980]. Note that we can no longer associated sentences with sets of worlds, but rather need a tripartition.

1.2 Strong Kleene

Give truth values where all replacements of # by T or F lead to the same truth value.

$\wedge$	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

$\vee$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

2 Conditionals and trivalence

Strong Kleene:

$\rightarrow$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

3 Validity and logical consequence

When is a formula valid on a trivalent semantics? Again there are a number of ways of extending the standard notions to trivalent logics.

Standard validity: $\models \phi$ iff for all I, $v_I(\phi) = 1$. Note that in this case there are not a lot of valid formulas!

Standard entailments: $\phi \models \psi$ iff for all I, such that $v_I(\phi) = 1$, $v_I(\psi) = 1$.

Strawson entailment: $\phi \models \psi$ iff for all I, such that $v_I(\phi) = 1$, $v_I(\psi) = 1$ or $v_I(\psi) = \#$[2]

4 Supervaluations

An interesting property of Weak Kleene logic: it makes $A \vee \neg A$ and $A \rightarrow A$ false, and even Strong Kleene makes them invalid.

In the literature on vagueness (and elsewhere) there is a standard formalism that avoids this problem [van Fraassen, 1966, Fine, 1975]. We now introduce a set of interpretations $\mathscr{I}$ rather than just one. We start with a standard trivalent partial interpretation function I. So just like in the above three-valued logics we think fundamentally of a trivalent interpretation as being given by a language. We let $\mathscr{I}$ consists of every bivalent (i.e. non-partial) interpretation that agrees with I in each case where I assigns 1 or 0 to a sentence in $\mathscr{A}$, call $\mathscr{I}$ the set of *precisifications* or *bivalent extensions* of I.

A sentence is *supertrue* if it is true on every interpretation in $\mathscr{I}$ it is *superfalse* if it is false on every interpretation in $\mathscr{I}$.

Can we give truth tables for this? Consider one for $\vee$. Some entries are clear, but some are not:

$\vee$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	?

Supertruth and superfalsity play the role of normal truth and falsity, anything else is considered 'truth-valueless'.

5 Presupposition as truth-value gap

Let's compare two semantic entries for the definite article:

Russell: $[\![\text{The } F \text{ is } G]\!]^w = \begin{cases} 1 \text{ if there is exactly one } F \text{ and all } F\text{'s are } G \\ 0 \text{ otherwise} \end{cases}$

[2]This gets applied to the NPI literature by von Fintel [1999].

Frege/Strawson-ish: $[\![\text{The } F \text{ is } G]\!]^w = \begin{cases} 1 \text{ if there is exactly one } F \text{ and all } F\text{'s are } G \\ 0 \text{ if there is exactly one } F \text{ and no } F\text{'s are } G \\ \# \text{ otherwise} \end{cases}$

Original motivation for this: intuitive judgments of truth values.[3]

(1) The king of France is bald.

Weirdly in the philosophical world this was largely seen as the *only* motivation for a trivalent theory of presupposition projection.

Trivalence and intuitive gaps A sentence s is judged to be neither true nor false at w if $[\![s]\!]^w = \#$

As philosophers were quick to note, it's a weak motivation for a trivalent theory:

(2) The king of France is not in this room.

For a very interesting if fairly vague discussion of intuitive judgments of truth-value gap see von Fintel [2004]. Some presuppositional expressions also don't seem to yield truth value gaps at all.

(3) True or false?: I stopped being a billionaire yesterday.

6 Cancellation

Trivalent accounts allow for cancellation. We can define an operator that eliminates presuppositions (Exercise: give truth-table for this).

Assumption general is that presupposition failures lead to falsity:

(4) The king of France isn't bald, because there's no king of France.

But not in all cases:

[3]For empirical study of this see Abrusan and Szendroi [2013].

(5) Albert didn't vote for the Bill too.

Yablo [2014] suggest an inference scheme for inferring non-presuppositional content. Whereas, we could also just have a 4-valued logic as Beaver suggests, essentially having two dimensions of meaning.

7 Relating trivalence to pragmatic presupposition

Pragmatic presupposition: what we take for granted.

A sentence s has a linguistic presupposition p, if an assertion of s is felicitous only if p is pragmatically presupposed

How does this relate to truth-value gaps. Stalnaker [1973] writes the following:

> The relation between the semantic notion of presupposition and the pragmatic notion of presupposition requirement is not, of course, just accidental. Among the reasons that a pragmatic presupposition might be required by the use of a sentence, by far the most obvious and compelling reason would be that the semantical rules for the sentence failed to determine a truth value for the sentence in possible worlds in which the required presupposition is false. Since the whole point of expressing a proposition is to divide the relevant set of alternative possible situations– the presupposition set–into two parts, to distinguish those in which the proposition is true from those in which the proposition is false, it would obviously be inappropriate to use a sentence which failed to do this. Thus, that a proposition is presupposed by a sentence in the technical semantic sense provides a reason for requiring that it be presupposed in the pragmatic sense whenever the sentence is used. This explains where the semantic notion gets its name, and why linguists and philosophers have been tempted to identify presupposition in general with this semantic relation.

Let's extract the following:

Stalnaker's bridge principle if we assert (or question) a trivalent sentence s in a context c then our assertion is infelicitous iff s is either true or false in all worlds in c

Stalnaker also makes clear that we can maintain the umbrella notion of pragmatic presupposition and have semantic presuppositions (i.e. trivalence) be *one* source of sentential presuppositions.[4]

Actually Stalnaker's principle seems not to be 'obvious' but rather ad hoc. Trivalence is used in semantic theory (and philosophy of language) to model vagueness [Fine, 1975, Kamp, 1975], paradoxical sentences (Kripke), conditionals [Belnap, 1970], homogeneity [Križ, 2015] among other things. But it is not clear that you cannot assert/question that John is tall unless it is common ground, that John is not a borderline height.[5]

8 Projection: connectives

One of the advantages of trivalent theories is that in conjunction with Stalanker's principle they generate precise predictions about projection.

The weak-Kleene table is associated with uniform projection. While the strong Kleene table gives something like symmetric versions of the standard Karttunen/Heim predictions.

Karttunen Heritage Rules:

- $P(\neg A) = P(A)$
- $P(A \wedge B) = P(A) \wedge (A \rightarrow P(B))$
- $P(A \vee B) = (P(A) \wedge (\neg A \rightarrow B))$

Symmetric versions:

- $P(\neg A) = P(A)$
- $P(A \wedge B) = (P(A) \wedge (A \rightarrow B)) \vee (P(B) \wedge (B \rightarrow A))$
- $P(A \vee B) = (P(A) \wedge (\neg A \rightarrow B)) \vee (P(B) \wedge (\neg B \rightarrow A))$

Projection in supervaluationist semantics: more complex as logical truths now don't

[4] Other sources, for Stalnaker, include something like Gricean implicatures. Note that despite the name, the theory of *pragmatic presuppositions* as espoused in Stalnaker [1974] does not assume a pragmatic source for sentential presuppositions.

[5] Spector [2015] suggests a multivalent semantics to handle this.

project.

(6) Either John regrets going to India or he doesn't regret going to India.
⇝ John went to India.

Seems wrong prediction. Does this *contrast* with vagueness:

(7) If John is bald, then John is bald.

but:

(8) ?Either John is bald or he isn't bald.

Peters [1979] proposed truth-tables that give us exactly Karttunen's rules:

$\wedge$	1	0	
1	1	0	#
0	0	0	0
#	#	#	#

Working with ideas in Schlenker [2006] and as specifically advocated by George [2007] and Fox [2008] we might see these tables as derived by strong Kleene (or supervalautionist) connectives plus an order constraint.

Thus we can reproduce the success and weaknesses of the standard dynamic semantics for connectives in a trivalent system. It is not clear that this system is any more complex or ad hoc than Heim's system.

It also seems to fall prey to the proviso problem, but unlike Heim's theory it gives a two-directional disjunction:

(9) Either there's no bathroom or the bathroom's under the stairs
⟷ The bathroom's under the stairs or there's no bathroom.

9 Projection: quantifiers

Quantifiers are more complex, as we must go beyond what is explicitly part of the Strong Kleene semantics (or Peter's semantics).

Most naturally a strong Kleene 'Every F is G' would be defined iff either a) there is some object that is an F and not a G or b) every object that is an F is a G and every object that is not a G is not an F.

This is a very strange presupposition for quantifiers to have. Fox [2012] suggests we might though strengthen them to the normal predictions akin to proviso problem.

10 Trivalence and triggering

Does trivalent account commit us to lexical stipulation of presupposition, thus deny pragmatic origins? Not clear: we might just think the mapping of sentential truth values (or lack thereof) partly determined pragmatically (by context).

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