PLING228 Research Seminar in Semantics
Week 6: Plural Anaphora 27 February, 2015

1 Problems of Plural Anaphora

1. Max vs. non-max readings

   Classical dynamic semantic systems (File Change Semantics, Discourse Representation Theory, Dynamic Predicate Logic) assign the same meaning to the examples in (1).

   (1) a. John has a^x donkey. He vaccinated it$_x$.
   b. John has a donkey that he vaccinated.

   As we will see shortly, it's not hard to extend this to plural indefinites like (2).

   (2) a. John has two^x donkeys. He vaccinated them$_x$.
   b. John has two donkeys that he vaccinated.

   But not all plural anaphora work this way. For example, (3a) is stronger than (3b).

   (3) a. John has more than two^x donkeys. He vaccinated them$_x$.
   b. John has more than two donkeys that he vaccinated.

   A more accurate paraphrase of (3a) is:

   (4) John has more than two^x donkeys. He vaccinated all of them$_x$.

   This type of anaphora is called a **maximal anaphora**. Generally, indefinites don't require (or maybe cannot have?) maximal readings, while modified numerals do.

   (5) a. John has at least two^x donkeys. He vaccinated them$_x$.
   b. John has at least two donkeys that he vaccinated.

   (6) a. John has at most four^x donkeys. He vaccinated them$_x$.
   b. John has at most four donkeys that he vaccinated.

   Another manifestation of maximal vs. non-maximal readings:

   (7) a. Two students wrote a paper. Perhaps there were other students who did the same.
   b. More than two students wrote a paper. #Perhaps there were other students who did the same. (Nouwen 2003b:35f)

2. Refset and Maxset Anaphora

   Non-indefinite quantifiers can also function as antecedents to pronouns. There are two kinds of interpretations.

(8) Most^x students came to the party. They$_x$ had a good time.
They = the students who came to the party

In this case, the pronoun is interpreted as the maximal set of objects that satisfy the restrictor and nuclear scope of the quantifier *most*. This set is called the **refset**.

(9) Most^x marbles in this bag are red. But exactly three of them$_x$ are black.
Them = the marbles in this bag

In this case, the pronoun is interpreted as the maximal set of objects that satisfy the restrictor. This set is called the **maxset**.

There is a third type of anaphora with quantificational antecedents that refers to the **complement set**.

(10) Few students showed up. They all stayed home.

See Nouwen (2003a,b) for arguments for a position that complement anaphora involves pragmatic bridging inference. Following Nouwen (and others), we do not represent the complement anaphora in the semantics of quantifiers.

3. Singular antecedents

A plural pronoun can be anaphoric to a singular antecedent:

(11) a. **Every^x guest** showed up at 8 pm. **They$_x$** left at 10 pm.
b. **No^x guest** show up. **They$_x$** all stayed home.

These cases are sort of intuitive: Although the noun is singular, the quantifier is about multiple people.

A more complicated case is (12). The antecedent is singular, but *because of another quantifier* (i.e. 'most students'), it gives rise to a plural reading.

(12) Most students wrote **a^x paper**. **They$_x$** weren't very well-written.
(Nouwen 2007:134)

4. Quantificational subordination

A more complicated version of (12) is (13): The pronoun is singular ('it') this time.

(13) **Every^x student** wrote **a^y paper**. **Most of them$_x$/They$_x$** submitted **it$_y$** to a journal. (Nouwen 2007:127)

Notice that the second sentence is about specific student-paper pairs, namely each student and the paper they wrote. So it cannot mean that a majority of students submitted some other student's paper to a journal.

Also compare this to (14), where the second sentence has no plural subject.

(14) **Every^x student** wrote **a^y paper**. I marked **#it/them$_y$** yesterday.

2 A Very Quick Introduction to Plurality

- We assume that there are plural objects in addition to singular objects. Singular objects are singleton sets of objects and plural objects are non-singleton sets.
  - Singular objects: $\{a\}, \{b\}, \{c\}, \{d\}, \ldots$
  - Plural objects: $\{a,b\}$, $\{a,c\}$, $\{b,d\}$, $\{a,b,c\}, \ldots$
- Three types of predicates.
  - **Distributive predicates** are inherently about singular objects.

    (15) a. John sat down.
    b. Bill sat down.

    Distributive predicates are only true of singular individuals. For sentences like (16), we assume a covert distributivity operator δ (to be defined below).

    (16) John and Bill δ sat down.
  - **Collective predicates** are inherently about plural objects.

    (17) a. *John is a couple.
    b. *Bill is a couple.
    c. John and Bill are a couple.
  - **Mixed predicates** are compatible with both singular and plural objects.

    (18) a. John carried the box.
    b. Bill carried the box.
    c. John and Bill carried the box.

    (18c) can be used to describe two different kinds of situations:

    ○ John and Bill carried the box separately.
    ○ John and Bill carried the box together.

    The distributive reading is captured by δ:

    (19) John and Bill δ carried the box.

3 Plural Indefinites

Plural indefinites can be easily added to Classical Heim by assuming that assignments map variables to sets of objects. Here's an example (Because the predicate is distributive, we assume δ, which universally quantifies over the singleton subsets of the plural subject):

(20) $$F[\text{two}^x \text{ men } \delta_x \text{ came in}] = \left\{ \langle g', w \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g, w \rangle \in F \\ g[x]g' \text{ and } |g'(x)| = 2 \text{ and} \\ \text{each member of } g'(x) \\ \quad \text{is a man who came in in } w \end{array} \right\}$$

But this only derives the non-maximal reading.

(21) $$F[\textbf{they}_x\ \delta_x\ \textbf{sat down}] = \left\{ \langle g, w\rangle \in F \;\middle|\; \begin{array}{l} g(x) \text{ is a plural object and} \\ \text{each member of } g(x) \text{ sat down in } w \end{array} \right\}$$

So the sentences in (22) are synonymous.

(22) a. Two^x men came in. They$_x$ sat down.
b. There are two men who came in and sat down.

This is good for indefinites, but not for modified numerals. The following analysis does not derive the desired maximal reading.

(23) $$F[\textbf{more than two}^x\ \textbf{men}\ \delta_x\ \textbf{came in}] = \left\{ \langle g', w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g, w\rangle \in F \\ g[x]g' \text{ and } |g'(x)| > 2 \text{ and} \\ \text{each member of } g'(x) \text{ is a man who came in in } w \end{array} \right\}$$

This predicts the following two sentences to be synonymous.

(24) a. More than two^x men came in. They$_x$ sat down.
b. There are more than two men who came in and sat down.

In order to derive the maximal reading, we analyse *more than two* with the **max**-operator.

4 Selective Generalised Quantifiers with Max

- Van den Berg (1996) introduces the **max**-operator (cf. the Σ-operator of Kamp & Reyle 1993, Nouwen 2003b, 2007).
- The idea is that with *more than two*, g' assigns to x a maximally large plural object that has more than two singular members and satisfies the predicates.

  (25) $$F[\textbf{more than two}^x\ \textbf{men}\ \delta\ \textbf{came in}] = \left\{ \langle g', w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g, w\rangle \in F \\ g[x]g' \text{ and } |g'(x)| > 2 \text{ and} \\ g'(x) \text{ is the \textbf{largest set} such that} \\ \text{each member of } g'(x) \text{ is a man who came in in } w \end{array} \right\}$$

  Then what *they* refers to in the subsequent discourse is the set consisting of **all** the men who came in, i.e. the maximal reading.
- We can use the same notion of maximality to account for cross-sentential anaphora with selective generalised quantifiers.

  (26) All^x linguists came in. They$_x$ sat down.

  Chierchia's (1995) selective quantifiers do not introduce a discourse referent (they are only internally dynamic) (see also Kanazawa 1993, 1994):

(27) $$F[\textsf{all}^x\ \phi\ \psi] = \left\{ \langle g,w\rangle \in F \;\middle|\; \begin{array}{l} \{\, o \in \mathcal{O} \mid \{\langle g[x/o],w\rangle\}\,[\phi] \neq \varnothing \,\} \\ \subseteq \{\, o \in \mathcal{O} \mid \{\langle g[x/o],w\rangle\}\,[\phi][\psi] \neq \varnothing \,\} \end{array} \right\}$$

We can assume that every quantifier introduces the refset, represented in (28) as $g_s(x)$ (we'll come back to the maxset later).

(28) $$F[\textsf{all}^x\ \phi\ \psi] := \left\{ \langle g_s,w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g,w\rangle \in F, \\ \text{for some } \langle g_r,w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\phi]) \text{ and} \\ \text{for some } \langle g_s,w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\phi][\psi]), \\ g_r(x) \subseteq g_s(x) \end{array} \right\}$$

(only defined if x is a new variable in g)

$g_r(x)$ is a maximal individual that satisfies the restrictor ϕ and $g_s(x)$ is a maximal set that satisfies both the restrictor ϕ and the nuclear scope ψ. Both of these are (typically) plural. The last clause of (28) expresses the meaning of *every* in Classical Generalised Quantifier Theory.

Importantly, the output keeps g_s, which only differs from $\langle g,w\rangle$ in the value of x. So $g_s(x)$ will be plural in the subsequent discourse.

(29) All^x guests came at 7 pm. They$_x$ left at 10 pm.

- The key is the max-operator: (recall $F[x] := \{\langle g',w\rangle \mid \text{for some } \langle g,w\rangle \in F,\ g[x]g'\}$)

(30) $$\textbf{max}^x_{\langle g,w\rangle}([\phi]) := \left\{ \langle g',w\rangle \;\middle|\; \begin{array}{l} \langle g',w\rangle \in \{\langle g,w\rangle\}\,[x][\phi] \text{ and} \\ \text{for all } \langle g'',w\rangle \in \{\langle g,w\rangle\}\,[x][\phi],\ g'(x) \not\subset g''(x) \end{array} \right\}$$

This ensures that there is no superset of $g'(x)$ that results from the update with $[x][\phi]$.

- With **distributive predicates**, there is always a single maximum.

(31) $\textbf{max}^x_{\langle g,w\rangle}([x\ \textsf{students}])$
$= \{\langle g',w\rangle \mid g[x]g' \text{ and } g'(x) \text{ is the set of all students in } w\}$

- With **non-distributive predicates** (collective or mixed), there might be multiple maxima.

(32) $\textbf{max}^x_{\langle g,w\rangle}([x\ \textsf{met}])$
$$= \left\{ \langle g',w\rangle \;\middle|\; \begin{array}{l} g[x]g' \text{ and } g'(x) \text{ met in } w \\ \text{and no proper superset of } g'(x) \text{ met in } w \end{array} \right\}$$

If John and Mary ($\{j,m\}$) met and Mary and Bill ($\{m,b\}$) met (and there was no other meeting), then there will be two assignments in (32), one assigns $\{j,m\}$ to x, one assigns $\{m,b\}$ to x.

But if John and Mary met ($\{j,m\}$), and John, Mary and Bill ($\{j,m,b\}$) met (and there was no other meeting), then there will be one assignment g' such that $g'(x) = \{j,m,b\}$.

This is potentially problematic given that "Exactly two people met" is true in such a situation (cf. Benjamin Spector's talk at UCL last year). But Van den Berg seems to disagree with this judgment. We'll come back to this shortly.

- With distributive quantifiers like *every*, only distributive interpretations are available (you might disagree with the judgments here).

  (33) ?*Every student met.

  We can force this by assuming that the singular noun **student** and the singular verb **met** (although this is obscured in English) are only true of singular objects. This forces the insertion of distributor δ.

  (34) $$F[\textbf{every}^x\ \phi\ \psi] := \left\{ \langle g_s, w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g, w\rangle \in F, \\ \text{for some } \langle g_r, w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\delta_x(\phi)]) \text{ and} \\ \text{for some } \langle g_s, w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\delta_x(\phi)][\delta_x(\psi)]), \\ g_r(x) \subseteq g_s(x) \end{array} \right\}$$

  We define δ as follows.

  (35) $$F[\delta_x(\phi)] = \{ \langle g, w\rangle \in F \mid \text{ for each } o \in g(x),\ \{ \langle g[x/\{o\}], w\rangle \}[\phi] \neq \varnothing \ \}$$

  This is externally static, which is clearly problematic (we lose a way to account for donkey anaphora with plural quantifiers, for example). We'll fix this later.

  Importantly, $g_s(x)$ is (typically) a plural object. So the anaphora is plural.

  (36) Every^x linguist came. They$_x$ δ_x sat down.

- Plural quantifiers allow collective predication.

  (37) Most of the students met.

  Collective interpretation of ψ:

  (38) $$F[\textbf{most}^x\ \phi\ \psi] = \left\{ \langle g_s, w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g, w\rangle \in F, \\ \text{for some } \langle g_r, w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\phi]) \text{ and} \\ \text{for some } \langle g_s, w\rangle \in \textbf{max}^x_{\langle g,w\rangle}([\phi][\psi]), \\ \text{most members of } g_r(x) \text{ are in } g_s(x) \end{array} \right\}$$

  In the output file, $g_s(x)$ is the set of students who met. This enables the refset anaphora.

  (39) Most^x of the students met. They$_x$ δ_x were angry.

- Note 1: In order to derive partially distributive interpretation (for (38), there are several meetings by students, and they collectively cover most of the students), another operator is needed.

  (40) $$F[\pi_x([\phi])] = \left\{ \langle g, w\rangle \in F \;\middle|\; \begin{array}{l} g(x) = S_1 \cup \cdots \cup S_n \text{ and} \\ \text{for each } S_i,\ \{ \langle g[x/S_i], w\rangle \}[\phi] \neq \varnothing \end{array} \right\}$$

  π decomposes the set $g(x)$ into several subsets and universally quantifies over them. See Schwarzschild (1996).

- Note 2: There are two ways to do 'maxset anaphora' (anaphora to the restrictor of the quantifier). We can let the quantifier introduce two discourse referents, one for the maxset and one for the refset, rather than just the latter.

(41) $F[\textsf{all}^{x,y}\ \phi\ \psi] := \left\{ \langle g'', w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g,w\rangle \in F, \\ \text{for some } \langle g',w\rangle \in \mathbf{max}^x_{\langle g,w\rangle}([\phi]) \text{ and} \\ \text{for some } \langle g'',w\rangle \in \mathbf{max}^y_{\langle g',w\rangle}([\phi][\psi]), \\ g''(x) \subseteq g''(y) \end{array} \right\}$

Or we can assume that (strong) quantifiers themselves are anaphors and their maxset is referred to by the mechanism similar to the definite article the. If so, the maxset is always discourse salient.

- Downward monotonic quantifiers allow $g_s(x)$ to be null, i.e. $g_s(x) = \varnothing$.

(42) a. None of the students came to class.
b. Fewer than 10 students came to class.

But if there is a pronoun referring to their refset, the refset cannot be null.

(43) Fewer than 10^x students came to class. They$_x$ asked a lot of questions.

This can be accounted for by the presupposition of pronouns that their referent cannot be $\varnothing$.

Consequently, no and none only support the maxset/complment set anaphora.

(44) None^x of the students came to class. They$_x$ stayed home.

- Van den Berg (1996) claims that downward monotonic quantifiers give rise to a problem with collective predicates.

(45) Fewer than 10 students met.

Suppose there are two meetings: one by $\{ s_{11}, s_{12} \}$ and one by $\{ s_1, \cdots, s_{10} \}$. The following entry, which is parallel to the meaning of every and most above, predicts (45) to be true here, due to $\{ s_{11}, s_{12} \}$.

(46) $F[\textsf{fewer than 10}^x\ \phi\ \psi] = \left\{ \langle g_s, w'\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle g,w\rangle \in F, \\ \text{for some } \langle g_r,w'\rangle \in \mathbf{max}^x_{\langle g,w\rangle}([\phi]) \text{ and} \\ \text{for some } \langle g_s,w'\rangle \in \mathbf{max}^x_{\langle g,w\rangle}([\phi][\psi]), \\ \text{fewer than 10 members of } g_r(x) \text{ are in } g_s(x) \end{array} \right\}$

I think this is not a bad prediction (again, cf. Benjamin Spector's seminar talk), but Van den Berg (1996) thinks the sentence should be false in this situation.

Van den Berg proposes to deal with downward monotonic quantifiers as the negations of upward monotonic quantifiers.

(47) a. fewer than 10^x $\phi\ \psi$ = not(more than 10^x $\phi\ \psi$)
b. no^x $\phi\ \psi$ = not(some^x $\phi\ \psi$)
c. few^x $\phi\ \psi$ = not(most^x $\phi\ \psi$)

But a drawback of this is that we cannot account for the following, because negation blocks anaphora.

(48) a. $F[\textsf{not}\ \phi] = F \ominus F[\phi]$
b. $F \ominus F[\phi] := \{ \langle g,w\rangle \in F \mid \text{there is no } \langle g',w\rangle \in F' \text{ such that } g \leqslant g' \}$

Van den Berg (1996) defines a negation that does not block anaphora, but then it's not very clear how to deal with (49):

(49) *John doesn't have a^x car. It$_x$ is expensive.

To be fair, the alternative version of negation accounts for double-negation like (50).

(50) It's not the case that John doesn't have a^x car. It$_x$ is in the garage.

- To sum up,
  - Assignments are partial functions from $\mathcal{V}$ to $\wp(\mathcal{O})$ (including $\varnothing$).
  - Indefinites introduce sets of objects that are potentially non-maximal.

(51) a. $F[\mathbf{a}^x\ \phi\ \psi] = \left\{ \langle g', w\rangle \;\middle|\; \begin{array}{l}\text{for some } \langle g,w\rangle \in F[x],\\ |g(x)| = 1 \text{ and}\\ \langle g',w\rangle \in \{\langle g(x),w\rangle\}[\phi][\psi]\end{array} \right\}$

b. $F[\mathbf{two}^x\ \phi\ \psi] = \left\{ \langle g', w\rangle \;\middle|\; \begin{array}{l}\text{for some } \langle g,w\rangle \in F[x],\\ |g(x)| = 2 \text{ and}\\ \langle g',w\rangle \in \{\langle g(x),w\rangle\}[\phi][\psi]\end{array} \right\}$

  - Generalised quantifiers express relations between two maximal sets and register the refset.

(52) $F[\mathbf{Q}^x\ \phi\ \psi] = \left\{ \langle g_s, w'\rangle \;\middle|\; \begin{array}{l}\text{for some } \langle g,w\rangle \in F,\\ \text{for some } \langle g_r,w\rangle \in \mathbf{max}^x_{\langle g,w\rangle}([\phi]) \text{ and}\\ \text{for some } \langle g_s,w\rangle \in \mathbf{max}^x_{\langle g,w\rangle}([\phi][\psi]),\\ \mathbb{Q}(g_r(x), g_s(x))\end{array} \right\}$

The key is the maximality operator:

(53) $\mathbf{max}^x_{\langle g,w\rangle}([\phi]) = \left\{ \langle g', w\rangle \;\middle|\; \begin{array}{l}\langle g',w\rangle \in \{\langle g,w\rangle\}[x][\phi] \text{ and}\\ \text{for all } \langle g'',w\rangle \in \{\langle g,w\rangle\}[x][\phi],\ g'(x) \not\subset g''(x)\end{array} \right\}$

As discussed above, this might or might not cause a problem with collective predication. I think it's a problem (contra Van den Berg). That is, it is predicted that (54) is false in a situation where $\{j,m\}$ met and $\{j,m,b,c,k\}$ met.

(54) a. Exactly two people met.
b. Between 1 and 3 people met.

Cf. Van den Berg predicts (55) to be true (correctly, I think) in the same situation. I think there's no big difference among these sentences.

(55) Two people met.

There are two things we could do:

- Introduce event quantification. E.g., (54b) means there is an event of meeting where the maximal individual involved in the meeting is between 1 and 3. This allows there to be a different event involving more people.
- Use Benjamin Spector's method of optional maximisation.

This also relates to the issue of downward quantifiers + collective predication. The first method does not generate a stronger reading. This requires something like Spector's method, I think.

We account for the following:

1. Maximal vs. non-maximal readings:

(56) a. Two^x men came in. They$_x$ sat down.
b. More than two^x men came in. They$_x$ sat down.

2. Quantificational antecedents:

(57) a. Every^x man came in. They$_x$ sat down.
b. Most^x of the men came in. They$_x$ sat down.

But not quantificational subordination:

(58) a. Every^x student wrote a^y paper. They$_y$ weren't interesting.
b. Every^x student wrote a^y paper. They$_x$ submitted it$_y$ to a^z journal.

or the following type of anaphora:

(59) Most^x students wrote a^y paper. They$_y$ were not very good.

Also, we need a way to do donkey anaphora with plural quantifiers + distributive predication.

(60) Most$_x$ farmers who own a^y donkey beat it$_y$.

These are not accounted for because the δ-operator is externally static. It's actually not hard to redefine it with a dynamic meaning, but the question is how it should be done. Or more precisely, what do we want to store in y in the above examples? Notice that we somehow want to keep the dependency between x and y, but if we keep $\{a,b,c\}$ in x and $\{d,e,f\}$ in y, we cannot keep track of which objects are related to which. A system with plural files allows us to capture this kind of dependency.

5 Quantificational Subordination with Plural Files

- Plural files offers two kinds of plurality.
  - **Local plurality**: assignment g maps x to a plural object $\{o_1, o_2\}$.
  - **Global plurality**: each assignment $g \in G$ maps x to a simple object o, but collectively, the assignments in G maps x to more than one such object, e.g. $G = \{[x \mapsto o_1], [x \mapsto o_2]\}$.

  Let's assume a system only with global plurality, i.e. as before that assignments map variables to simple objects (not sets).[1]
- We can reformulate the selective generalised quantifiers in terms of global plurality as follows.
  - $g[x]g'$ iff g and g' differ at most in the value of x.

[1] Brasoveanu (2008), unlike Van den Berg (1996) and Brasoveanu (2010a), allows for both kinds of plurality in a single system. We discuss the simpler option here.

- $G[x]G'$ iff for each $g \in G$, there is $g' \in G'$ such that $g[x]g'$ and for each $g' \in G$ there is $g \in G$ such that $g[x]g'$.
- $F[x] := \{ \langle G', w\rangle \mid$ for some $\langle G, w\rangle \in F$, $G[x]G' \}$
- $G(x) = \{ o \in \mathcal{O} \mid$ for some $g \in G$, $g(x) = o \}$

Let's also dissociate $[x]$ and **MAX** (not crucial).

(61) $$F[\mathsf{all}^x\ \phi\ \psi] = \left\{ \langle G_s, w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle G, w\rangle \in F, \\ \text{for some } \langle G_r, w\rangle \in \{ \langle G, w\rangle \}\,[x][\mathbf{MAX}_x([\phi])], \\ \text{for some } \langle G_s, w\rangle \in \{ \langle G, w\rangle \}\,[x][\mathbf{MAX}_x([\phi][\psi])], \\ G_r(x) \subseteq G_s(x) \text{ and} \end{array} \right\}$$

(62) $$F[\mathbf{MAX}_x([\phi])] = \left\{ \langle G', w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle G, w\rangle \in F, \\ \langle G', w\rangle \in \{ \langle G, w\rangle \}\,[\phi] \text{ and} \\ \text{for all } \langle G'', w\rangle \in \{ \langle G, w\rangle \}\,[\phi],\ G'(x) \not\subset G''(x) \end{array} \right\}$$

For example, $\mathbf{MAX}([x\ \mathsf{students}])$ is the set of $\langle G', w'\rangle$ such G' that covers all the students. Suppose that there are only three students, s_1, s_2, s_3 in w. Then,

$$\begin{aligned} &\{ \langle G, w\rangle \}\,[x][\mathbf{MAX}_x([x\ \mathsf{students}])] \\ =&\langle \{ [x \mapsto s_1], [x \mapsto s_2], [x \mapsto s_3] \} \Cup G, w\rangle \end{aligned}$$

(where $\Cup$ is point-wise union).

- By assumption, collective predication is the default.

(63) $F[P(x)] := \{ \langle G, w\rangle \in F \mid G(x) \in w(P) \}$

More generally,

(64) $F[P^n(x_1, \ldots, x_n)] = \{ \langle G, w\rangle \in F \mid \langle G(x_1), \ldots, G(x_n)\rangle \in w(P^n) \}$

(We could assume that predicates presuppose that their arguments are not null, but we don't need to encode this explicitly)

- Pronouns refer to the set of all values under G, i.e. $G(x)$. We assume that singular pronouns presuppose that $|G(x)| = 1$, while plural pronouns presuppose that $|G(x)| > 1$ (or $\geqslant 1$). This prevents pronominal reference to $\varnothing$.
- As before the distributivity operator δ generates distributive readings.

(65) $$F[\delta_x(\phi)] = \left\{ \langle G', w\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle G, w\rangle \in F, \\ G(x) = G'(x) \text{ and} \\ \text{for each } d \in G(x),\ \langle G'{\restriction}_{x=d}, w\rangle \in \{ \langle G{\restriction}_{x=d}, w\rangle \}\,[\phi] \end{array} \right\}$$

(66) For any $o \in \mathcal{O}$, $G{\restriction}_{x=o} = \{ g \in G \mid g(x) = o \}$

This mechanism introduces a dependency. That is, if ϕ contains a new discourse referent y, each $d \in G(x)$ will have a potentially different object o for y, and also each $g' \in G'$ stores a particular combination of d and o.

- Example

(67) Two^x students δ_x wrote a^y paper.

Situation:

	w_1	w_2	w_3	w_4
student	$\{s_1,s_2,s_3\}$	$\{s_2,s_3,s_4\}$	$\{s_2,s_3\}$	$\varnothing$
paper	$\{p_1,p_2,p_3\}$	$\{p_1,p_2,p_3,p_4\}$,	$\{p_1,p_2\}$	$\{p_1\}$
wrote	$\left\{\begin{array}{c}\langle s_1,p_1\rangle,\\ \langle s_2,p_2\rangle,\\ \langle s_3,p_3\rangle\end{array}\right\}$	$\left\{\begin{array}{c}\langle s_2,p_1\rangle,\\ \langle s_2,p_2\rangle,\\ \langle s_3,p_3\rangle,\\ \langle s_4,p_4\rangle\end{array}\right\}$	$\varnothing$	$\varnothing$

Derivation (for any variable v, $\{g_\varnothing\}(v) = \varnothing$):

$$\{\langle\{g_\varnothing\},w_1\rangle,\langle\{g_\varnothing\},w_2\rangle,\langle\{g_\varnothing\},w_3\rangle,\langle\{g_\varnothing\},w_4\rangle\}$$

$$\boxed{\text{two}^x \text{ students}}$$

$$\left\{\begin{array}{lll}\langle\{[x\mapsto s_1],[x\mapsto s_2]\},w_1\rangle, & \langle\{[x\mapsto s_2],[x\mapsto s_3]\},w_2\rangle, & \langle\{[x\mapsto s_2],[x\mapsto s_3]\},w_3\rangle,\\ \langle\{[x\mapsto s_2],[x\mapsto s_3]\},w_1\rangle, & \langle\{[x\mapsto s_3],[x\mapsto s_4]\},w_2\rangle, & \\ \langle\{[x\mapsto s_1],[x\mapsto s_3]\},w_1\rangle, & \langle\{[x\mapsto s_2],[x\mapsto s_4]\},w_2\rangle & \end{array}\right\}$$

$$\boxed{\delta_x \text{ wrote a}^y \text{ paper}}$$

$$\left\{\begin{array}{ll}
\left\langle\left\{\begin{bmatrix}x\mapsto s_1\\ y\mapsto p_1\end{bmatrix},\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_2\end{bmatrix}\right\},w_1\right\rangle, & \left\langle\left\{\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_1\end{bmatrix},\begin{bmatrix}x\mapsto s_3\\ y\mapsto p_3\end{bmatrix}\right\},w_2\right\rangle,\\
 & \left\langle\left\{\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_2\end{bmatrix},\begin{bmatrix}x\mapsto s_3\\ y\mapsto p_3\end{bmatrix}\right\},w_2\right\rangle,\\
\left\langle\left\{\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_2\end{bmatrix},\begin{bmatrix}x\mapsto s_3\\ y\mapsto p_3\end{bmatrix}\right\},w_1\right\rangle, & \left\langle\left\{\begin{bmatrix}x\mapsto s_3\\ y\mapsto p_3\end{bmatrix},\begin{bmatrix}x\mapsto s_4\\ y\mapsto p_4\end{bmatrix}\right\},w_2\right\rangle,\\
\left\langle\left\{\begin{bmatrix}x\mapsto s_1\\ y\mapsto p_1\end{bmatrix},\begin{bmatrix}x\mapsto s_3\\ y\mapsto p_3\end{bmatrix}\right\},w_1\right\rangle, & \left\langle\left\{\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_1\end{bmatrix},\begin{bmatrix}x\mapsto s_4\\ y\mapsto p_4\end{bmatrix}\right\},w_2\right\rangle,\\
 & \left\langle\left\{\begin{bmatrix}x\mapsto s_2\\ y\mapsto p_2\end{bmatrix},\begin{bmatrix}x\mapsto s_4\\ y\mapsto p_4\end{bmatrix}\right\},w_2\right\rangle
\end{array}\right\}$$

The resulting state not only says which objects x and y can be mapped to in which world, but also the dependency between x and y in each world.

(68) They$_x$ δ_x are proud of it$_y$.

For each $\langle G,w\rangle$ in the final state above, *they*$_x$ refers to $G(x)$, which is a set of two students. δ_x decomposes G to $G\restriction_{x=o}$ for each student s in the set. Then checks whether o is proud of $G\restriction_{x=o}(y)$. Notice that $G\restriction_{x=o}(y)$ is a single paper in each of these cases, even for s_2 in w_2, who wrote two papers. Also notice that $G\restriction_{x=o}(y)$ is a paper that o wrote, so the dependency is preserved.

So we can account for the following examples:

(69) Most^x students δ_x wrote a^y paper. They$_y$ δ_x weren't very good.

(70) a. Two^x students δ_x wrote a^y paper. They$_x$ δ_x are proud of it$_y$.
b. Every^x student wrote a^y paper. They$_x$ δ_x submitted it$_y$ to a journal.

Importantly, the singular anaphora in (70) is made possible by δ_x.

(71) Most^x students δ_x wrote a^y paper. *I marked it$_y$ yesterday.

(Actually, nothing so far prevents δ_x from occurring in the second sentence of (71), sort of vacuously. We could maybe say this is a syntactic restriction)

- δ_x allows us to account for donkey anaphora as well.

  (72) Every^x farmer who owns a^y donkey beats it$_y$.

  However, this only generates a $\exists$-reading. Brasoveanu (2010b) proposes a very ad hoc way of deriving $\forall$-readings, which I think has a number of unwelcome results. Brasoveanu (2008), on the other hand, proposes (i) that assignments map variables to sets of objects (local plurality), and (ii) that the $\forall$-reading is generated with a version of indefinites that uses **MAX**. But it's not clear if its predictions are correct. For instance, it predicts that (73) only has a $\forall$ reading.

  (73) Every man who has at least two^y credit cards used them$_x$ to pay for the bill.

- Nouwen (2003b, 2007) points out an undergeneration problem for Van den Berg's (1996) system, which we inherit here. Under the scope of δ_x, it is no longer possible to refer to the collection of all the papers. But we want to be able to refer to this.

  (74) They$_x$ δ_x wrote an^z essay about them$_y$.

  Nouwen develops a system where the values are never overwritten, using stacks, instead of assignments. But alternatively, we could account for the collective reading like (74) by QRing *them*$_y$.

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