

1 Problems of Plural Anaphora

1. Max vs. non-max readings

Classical dynamic semantic systems (File Change Semantics, Discourse Representation Theory, Dynamic Predicate Logic) assign the same meaning to the sentences in (1).

- (1) a. John has a^x donkey. He vaccinated it_x.
- b. John has a donkey that he vaccinated.

It's not hard to extend this to plural indefinites like (2).

- (2) a. John has two^x donkeys. He vaccinated them_x.
- b. John has two donkeys that he vaccinated.

But not all plural anaphora works this way. (3a) is stronger than (3b).

- (3) a. John has more than two^x donkeys. He vaccinated them_x.
- b. John has more than two donkeys that he vaccinated.

A more accurate paraphrase of (3a) is:

- (4) John has more than two^x donkeys. He vaccinated all of them_x.

This type of anaphora is called a **maximal reading**. Generally, indefinites don't require (or maybe cannot have?) maximal readings, while modified numerals do.

2. Singular antecedents

A plural pronoun can be anaphoric to a singular antecedent:

- (5) a. **Every^x guest** showed up at 8 pm. **They_x** left at 10 pm.
- b. **No^x guest** show up. **They_x** all stayed home.

These cases are sort of intuitive. Although the noun is singular, the quantifier is about multiple people.

A more complicated case is (6). The quantifier is singular, but *because of another quantifier* (i.e. 'most students'), it gives rise to a plural reading.

- (6) Most students wrote a^x **paper**. **They_x** weren't very well-written.
(Nouwen 2007:134)

3. Quantificational subordination

A more complicated version of (6) is (7): The pronoun is singular ('it') this time.

- (7) **Every^x student** wrote **a^y paper**. **Most of them_x/They_x** submitted **it_y** to a journal. (Nouwen 2007:127)

Notice that the second sentence is about specific student-paper pairs, namely each student and the paper they wrote. So it cannot mean that a majority of students submitted some other student's paper to a journal. Also compare this to (8), where the second sentence has no plural subject.

- (8) **Every^x student** wrote **a^y paper**. I marked **#it/them_y** yesterday.

2 A Very Quick Introduction to Plurality

- We assume that there are plural objects in addition to singular objects:
 - Singular objects: $a, b, c, d, \dots$
 - Plural objects: $(a \oplus b), (a \oplus c), (b \oplus d), (a \oplus b \oplus c), \dots$

The ontology of plural objects doesn't matter much. We can think of them as objects with other objects as parts. Or if you like, singular objects can be seen as singleton sets of objects and plural objects as non-singleton sets of objects.
- $\oplus$ is idempotent, commutative and associative. (In the set version, it's $\cup$).
- The number of singular parts of a given plural individual o is denoted by $|o|$.
 - $|a \oplus b| = 2$
 - $|a \oplus b \oplus c \oplus d| = 4$
- Part-of: If $o_1 = o_2 \oplus o_3$, we say $o_2 \sqsubseteq o_1$ (and $o_3 \sqsubseteq o_1$). (This of course corresponds to $\subseteq$ in the set version).
- Three types of predicates.
 - **Distributive predicates** are inherently about singular objects.

(9) a. John sat down.
b. Bill sat down.
c. John and Bill sat down.

(10) John and Bill P iff John P and Bill P .

Distributive predicates are only true of singular individuals. For sentences like (10b), we assume a covert distributivity operator δ , whose function is essentially the same as floating quantifiers *both, each, all* etc.

(11) a. John and Bill δ sat down.
b. John and Bill both sat down.
 - **Collective predicates** are inherently about plural objects.

(12) a. *John met.
b. *Bill met.

- c. John and Bill met.
- **Mixed predicates** are compatible with both singular and plural objects.

- (13) a. John carried the box.
- b. Bill carried the box.
- c. John and Bill carried the box.

(13b) can be used to describe two different kinds of situations:

- John and Bill carried the box separately.
- John and Bill carried the box together.

The distributive reading is forced by a floating quantifier.

- (14) John and Bill both carried the box.

3 Plural Indefinites

Plural indefinites can be easily added to Classical Heim.

$$(15) \quad F[\text{two}^x \text{ men came in}] = \left\{ \langle g', w \rangle \left| \begin{array}{l} \text{for some } \langle g, w \rangle \in F \\ g[x]g' \text{ and } g'(x) = o_1 \oplus o_2 \text{ such that} \\ o_1 \text{ and } o_2 \text{ are both men who came in in } w \end{array} \right. \right\}$$

But this only derives the non-maximal reading.

$$(16) \quad F[\text{they}_x \text{ sat down}] = \left\{ \langle g, w \rangle \in F \left| \begin{array}{l} g(x) \text{ is a plural object and} \\ \text{each singular part of } g(x) \text{ sat down in } w \end{array} \right. \right\}$$

So the sentences in (17) are synonymous.

- (17) a. Two^x men came in. They_x sat down.
- b. There are two men who came in and sat down.

This is good for indefinites, but not for modified numerals. The following analysis does not derive the desired maximal reading.

$$(18) \quad F[\text{more than two}^x \text{ men } \delta \text{ came in}] \\ = \left\{ \langle g', w \rangle \left| \begin{array}{l} \text{for some } \langle g, w \rangle \in F \\ g[x]g' \text{ and } |g'(x)| > 2 \text{ and} \\ \text{each singular part of } g'(x) \text{ is a man who came in in } w \end{array} \right. \right\}$$

This predicts the following two sentences to be synonymous.

- (19) a. More than two^x men came in. They_x sat down.
- b. There are more than two men who came in and sat down.

In order to derive the maximal reading, we analyse *more than two* with the **max**-operator.

4 Selective Generalised Quantifiers with Max

- Van den Berg (1996) introduces the **max**-operator (cf. the Σ -operator of Kamp & Reyle 1993, Nouwen 2003b, 2007).
- The idea is that with *more than two*, g' assigns to x a maximally large plural object that has more than two singular parts and satisfies the predicates.

$$(20) \quad F[\text{more than two}^x \text{ men } \delta \text{ came in}] \\ = \left\{ \langle g', w \rangle \left| \begin{array}{l} \text{for some } \langle g, w \rangle \in F \\ g[x]g' \text{ and } |g'(x)| > 2 \text{ and} \\ g'(x) \text{ is the maximal plural object } o \text{ such that} \\ \text{each singular part of } o \text{ is a man who came in in } w \end{array} \right. \right\}$$

Then what *they* refers to in the subsequent discourse is the plural object consisting of all the men who came in, i.e. the maximal reading.

- We can use the same notion of maximality to account for cross-sentential anaphora with selective generalised quantifiers.

$$(21) \quad \text{Every}^x \text{ linguist came in. They}_x \text{ sat down.}$$

Chierchia's selective quantifiers do not introduce a discourse referent (they are only internally dynamic):

$$(22) \quad F[\text{every}^x \phi \psi] = \left\{ \langle g, w \rangle \in F \left| \begin{array}{l} \{o \in \mathcal{O} \mid \{\langle g[x/o], w \rangle\} [\phi] \neq \emptyset\} \\ \subseteq \{o \in \mathcal{O} \mid \{\langle g[x/o], w \rangle\} [\phi][\psi] \neq \emptyset\} \end{array} \right. \right\}$$

We can make every quantifier introduce a new discourse referent.

$$(23) \quad F[\text{every}^x \phi \psi] = \left\{ \langle g_s, w' \rangle \left| \begin{array}{l} \text{for some } \langle g, w \rangle \in F, \\ \text{for some } \langle g_r, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x([\phi]) \text{ and} \\ \text{for some } \langle g_s, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x([\phi][\psi]), \\ g_r \sqsubseteq g_s \end{array} \right. \right\}$$

g_r assigns x a maximal individual that satisfies the restrictor ϕ and g_s assigns x a maximal individual that satisfies both the restrictor ϕ and the nuclear scope ψ . Both of these are (typically) plural.

The output keeps g_s . So $g_s(x)$ will be a plural individual in the subsequent discourse.

$$(24) \quad \text{Every}^x \text{ guest showed up at 7 pm. They}_x \text{ left at 10 pm.}$$

- The key is the max-operator: (recall $F[x] = \{\langle g', w \rangle \mid \text{for some } \langle g, w \rangle \in F, g[x]g'\}$)

$$(25) \quad \mathbf{max}_{\langle g, w \rangle}^x([\phi]) = \left\{ \langle g', w' \rangle \left| \begin{array}{l} \langle g', w' \rangle \in \{\langle g, w \rangle\} [x][\phi] \text{ and} \\ \text{for all } \langle g'', w'' \rangle \in \{\langle g, w \rangle\} [x][\phi], g'(x) \not\sqsubseteq g''(x) \end{array} \right. \right\}$$

(only defined if x is a new variable in g)

- With **distributive predicates** max returns a singleton set.

$$(26) \quad \mathbf{max}_{\langle g, w \rangle}^x ([x \text{ student}])$$

$$= \{ \langle g', w \rangle \mid g[x]g' \text{ and } g'(x) \text{ is the plural object consisting of all students in } w \}$$

- With **non-distributive predicates** (collective or mixed), there might be multiple maxima.

$$(27) \quad \mathbf{max}_{\langle g, w \rangle}^x ([x \text{ met}])$$

$$= \left\{ \langle g', w \rangle \mid \begin{array}{l} g[x]g' \text{ and } g'(x) \text{ met in } w \\ \text{and no plurality containing } g'(x) \text{ met in } w \end{array} \right\}$$

If John and Mary met ($j \oplus m$) and Mary and Bill ($m \oplus b$) met (and there was no other meeting), then there will be two assignments in (27), one assigns $j \oplus m$ to x , one assigns $m \oplus b$ to x .

(But if John and Mary met ($j \oplus m$), and John, Mary and Bill ($j \oplus m \oplus b$) met (and there was no other meeting), then it will be a singleton with $g'(x) = j \oplus m \oplus b$.)

- With distributive quantifiers like *every*, only distributive interpretations are available.

$$(28)?^* \text{Every student met.}$$

We can force this by assuming that the singular noun *student* and the singular verb *met* (although this is obscured in English) are only true of singular objects. This forces the insertion of distributor δ .

$$(29) \quad \text{Every}^x (\delta_x(x \text{ student})) (\delta_x(\text{met}))$$

$$(30) \quad F[\delta_x(\phi)] = \{ \langle g, w \rangle \in F \mid \text{for each singular part } o \sqsubseteq g(x), \{ \langle g[x/o], w \rangle \} [\phi] \neq \emptyset \}$$

- Plural quantifiers allow collective predication.

$$(31) \quad \text{Most of the students met.}$$

Collective interpretation of ψ :

$$(32) \quad F[\mathbf{most}^x \phi \psi] = \left\{ \langle g_s, w' \rangle \mid \begin{array}{l} \text{for some } \langle g, w \rangle \in F, \\ \text{for some } \langle g_r, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x ([\phi]) \text{ and} \\ \text{for some } \langle g_s, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x ([\phi][\psi]), \\ \text{most singular parts of } g_r \text{ are also parts of } g_s \end{array} \right\}$$

In the output file, $g_s(x)$ is the set of students who met. A pronoun can refer to this plural object in the subsequent discourse.

$$(33) \quad \text{Most}^x \text{ of the students met. They}_x \text{ were all angry.}$$

(Distributive readings can be derived by using δ_x)

- Note 1: In order to derive partially distributive interpretation (for (32), there are several meetings by students, and they collectively cover most of the students), another operator is needed.

$$(34) \quad F[\pi_x([\phi])] = \left\{ \langle g, w \rangle \in F \mid \begin{array}{l} g(x) = o_1 \oplus \dots \oplus o_n \text{ and} \\ \text{for each } o_i, \{ \langle g[x/o_i], w \rangle \} [\phi] \neq \emptyset \end{array} \right\}$$

π decomposes the plurality into parts (not necessarily singular) and universally quantifies over them.

- Note 2: We don't represent complement anaphora:

(35) Few students showed up. They all stayed home.

See Nouwen (2003a,b) for arguments for a position that complement anaphora involves pragmatic bridging inference.

- Note 3: There are two ways to do 'maxset anaphora' (anaphora to the restrictor of the quantifier). We can let the quantifier introduce two discourse referents, one for the maxset and one for the refset, rather than just the latter.

Or we can assume that (strong) quantifiers themselves are anaphors and their maxset is referred to by the mechanism similar to the definite article **the**. If so, the maxset is always discourse salient already.

- Downward monotonic quantifiers require a little twist, because they allow $g_s(x)$ to be null.

(36) a. None of the students came to class.
b. Fewer than 10 students came to class.

Furthermore, with collective predicates, downward monotonic quantifiers give rise to strong readings.

(37) Fewer than 10 students met.

Suppose there are two meetings: one by $s_1 \oplus s_2$ only and one by $s_1 \oplus \dots \oplus s_1 0$. In this situation, we want (37) to be false, but the following entry, which is parallel to the meaning of every and most above, predicts it to be true due to $s_1 \oplus s_2$.

$$(38) \quad F[\text{fewer than } 10^x \phi \psi] = \left\{ \left\langle g_s, w' \right\rangle \left| \begin{array}{l} \text{for some } \langle g, w \rangle \in F, \\ \text{for some } \langle g_r, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x([\phi]) \text{ and} \\ \text{for some } \langle g_s, w' \rangle \in \mathbf{max}_{\langle g, w \rangle}^x([\phi][\psi]), \\ \text{fewer than 10 singular parts of } g_r \text{ are} \\ \text{also parts of } g_s \end{array} \right. \right\}$$

Van den Berg (1996) proposes to deal with downward monotonic quantifiers as the negations of upward monotonic quantifiers.

(39) a. fewer than $10^x \phi \psi = \text{not}(\text{more than } 10^x \phi \psi)$
b. $\text{no}^x \phi \psi = \text{not}(\text{some}^x \phi \psi)$
c. $\text{few}^x \phi \psi = \text{not}(\text{most}^x \phi \psi)$

A drawback of this is that we cannot account for the following, because negation blocks anaphora.

(40) Fewer than 10^x students came to class. They_x asked a lot of questions.

- (41) a. $F[\text{not } \phi] = F \ominus F[\phi]$
 b. $F \ominus F[\phi] := \{ \langle g, w \rangle \in F \mid \text{there is no } \langle g', w \rangle \in F' \text{ such that } g \leq g' \}$

Van den Berg (1996) defines a negation that does not block anaphora, but then it's not very clear how to deal with (42):

- (42) *John doesn't have a^x car. It_x is expensive.

To be fair, the alternative version of negation accounts for double-negation like (43).

- (43) It's not the case that John doesn't have a^x car. It_x is in the garage.

• The analysis so far accounts for the following:

- (44) a. Two^x men came in. They_x sat down.
 b. More than two^x men came in. They_x sat down.
 c. Every^x man came in. They_x sat down.
 d. Most^x of the men came in. They_x sat down.

But not the following.

- (45) a. Every^x student wrote a^y paper. They_y weren't interesting.
 b. Every^x student wrote a^y paper. They_y submitted it_y to a^z journal.

This can be dealt with in a system with plural files (sets of assignments). This is for Week 6.

References

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