PLING228 Semantics Research Seminar more epistemic modality de re

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1 Going dynamic

Veltman's [1996] dynamic semantics:

$c[A] = \{w \in c : A \text{ is true in } w\}$
$c[\neg\phi] = \{w \in c : w \notin c[\phi]\}$
$c[\phi \wedge \psi] = c[\phi][\psi]$
$c[\Diamond\phi] = \{w \in c : c[\phi] \neq \emptyset\}$

1.1 Dynamic treatment of epistemic contradiction

Recall Yalcin's observation that 'it's raining but it might not be raining' exhibits a kind of incoherence beyond Moorean paradox:

(1) a. ?Suppose it's raining but it might not be raining.
b. Suppose it's raining but you'd don't know it's raining.

$c[A \wedge \Diamond\neg A]$ $c[\Diamond\neg A][A]$

Fixed point of ϕ, is context c such that $c[\phi] = c$. Neither epistemic contradiction has a fixed point. The thought: not having a fixed point makes a discourse incoherent, so this explains the badness of epistemic contradiction.

1.2 A problem with embeddings

$c[\neg(\Diamond\neg A \wedge A)] = (\text{if } c[\neg A] \neq \emptyset) = c[\neg A]$.

And this is going to yield a fixed point. Since $c[\neg A][\neg(\Diamond\neg A \wedge A)] = c[\neg A]$

So, things do not look rosy for dynamic treatment of epistemic contradictions.

1.3 Yalcin's treatment of updates and embedding

Does Yalcin's semantics have this problem as well? Here is Yalcin's semantics (with a dynamic semantics):

$[\![A]\!]^{w,s}$ is true iff w is in $[\![\alpha]\!]$.
$[\![\neg\alpha]\!]^{w,s}$ is true iff $[\![\alpha]\!]^{w,s}$ is false.
$[\![\alpha\&\beta]\!]^{w,s}$ is true iff $[\![\alpha]\!]^{w,s}$ is true and $[\![\beta]\!]^{w,s[\![\alpha]\!]}$ is true.
$[\![\Diamond\alpha]\!]^{w,s}$ is true iff $\exists w' \in s$ $[\![\Box\alpha]\!]^{w',s}$ is true.

We call $c[\![\alpha]\!]$ the update of c by α, it is defined as follows:

$c[\![\alpha]\!] = \{w \in c : [\![\alpha]\!]^{w,c} \text{ is true}\}$

Klinedinst and Rothschild [2012] proved the following: for any sentence α any context c, and any world $w \in c$, $w \in c[\alpha]$ iff $w \in c[\![\alpha]\!]$.

So does Yalcin have the same problem about epistemic contradictions as dynamic semantics does?

Not quite. More in the spirit of Yalcin [2007] would be the following update condition:

$c[\![\alpha]\!]$ = the maximum c' in c such that $\forall w \in c' : [\![\alpha]\!]^{w,c'}$ is true.

On this definition $c[\![\Diamond\neg A\&A]\!]$ is empty. Because any world w in c' will ensure that $[\![\Diamond\neg A\&A]\!]^{w,c'\cup\{w\}}$ is false. So the *update* associated with either order of epistemic contradiction is correct on Yalcin's semantics.

However, Yalcin is saddled with the same problem with negations of epistemic contradictions. For $[\![\neg\Diamond\neg A\&A]\!]$, is true with respect to all w in any $c \subseteq A$.

Note that we also get a problem with disjunction (for Veltman and Yalcin). Dynamic definition of disjunction is controversial but I think most issues are orthogonal here.

$c[A \vee B] = c[A] \cup c[B]$

(2) (It might be raining and it's not raining) or it's raining.

1.4 Fixed Points in Dynamic Semantics

We can however modify Veltman's update semantics to deal with embedding phenomenon

We put this modification on *all update rules*: if $c[\phi][\phi] \neq c[\phi]$ then $c[\phi] = \emptyset$. This requires all non-crashing updates to be idempotent.

Now we get that:

$c[\neg(\Diamond\neg A \wedge A)] = \emptyset$.

So an interesting and unexplored hypothesis about dynamic semantics: all updates must ensure idempotence when there are tests in the semantics system al la Veltman [1996].

Not clear how to modify Yalcin's semantics in the right way: seems like non-standard treatment of negation is necessary.

2 Modifying Yalcin?

We need a new semantics for negation or conjunction.

We don't want $[\![\neg(\Diamond\neg A \wedge A)]\!]$ to be false in contexts in which c includes a $\neg A$ world and A is true in w.

Well, derivatively, we don't want $[\![\Diamond\neg A \wedge A]\!]^{w,c}$ to be true in contexts in which c include a $\neg A$ world and A is true. So conjunction seems to be the target.

But what do we do? Well, brute force:

$[\![\phi \wedge \psi]\!]^{w,c} = 1$ iff $[\![\phi]\!]^{w,c} = 1$ and $[\![\psi]\!]^{w,c} = 1$ and $[\![\phi]\!]^{w,c[\psi]} = 1$, and $[\![\psi]\!]^{w,c[\phi]} = 1$

3 Epistemic modality de re

Recall we want to deal with: $\exists x \Diamond Fx \wedge \neg Fx$. Yalcin's semantics (in unmodified form) does not handle this.

3.1 A dynamic semantics with quantification *and* modality

My version of Groenendijk et al. [1996]:

First we take the notion of an information state (or a file) due in essence to Heim [1982] and Kamp [1981].

Information states are sets of pairs of partial assignment functions and worlds (for short, assignment worlds). So an information state s is a set of pairs $\langle w, f \rangle$ where w is an interpretation function for first order logic (i.e. a set of assignments to constants and predicates) and f is a partial function from variables V, to objects in the domain D and for each i and i' in S the assignment functions f in i and f' in i' have the same domain. We assume one large domain D, and one set of V of variables common to all information states.

Let's start with some simple notions: f' is an extension of f (or $f' \geq f$) if f' agrees with f everywhere (though it may also have more assignments). We also say an assignment world i' is an extension of i iff $i = \langle w, f \rangle$ and $i' = \langle w, f' \rangle$ and $f' \geq f$.

We call the variable domain of s the set of variables such that are in the domain of every assignment function f in any i in s. We will also define the *free extension* of s with respect to a variable x, $s[x]$, which is only defined if x is not in the variable domain of s. In this case: $s[x] = \{\langle w, f \rangle : (w, f') \in s$ and the domain of f is the domain of f' union $\{x\}\}$

We say $\langle w, f \rangle \models R(x_1, \ldots x_n)$ iff the interpretation function of w makes $R(f(x_1) \ldots f(x_n))$ true.

Information states are updated as follows:

$s[R(x_1 \ldots x_n)] = \{i \in S : i \models R(x_1, \ldots x_n)$

$s[\exists x] = s[x]$

$s[\phi \wedge \psi] = s[\phi][\psi]$

$s[\neg\phi] = \{i \in s : \neg\exists i' \in s[\phi]$ s.t. $i' \geq i\}$

$s[\Diamond\phi] = \{i \in s : s[\phi] \neq \emptyset\}$

(note we assume that $s[\exists x Fx]$ is short for $s[\exists x][\phi]$))

3.2 Quantified epistemic contradictions

Some more notions:

Consistency : An information state s is consistent with ϕ if $s[\phi] \neq \emptyset$. ϕ is consistent if there is some s consistent with it.

Coherence : An information state s is supports ϕ if $\forall i \in s \exists i' \in s[\phi]$ s.t. $i' \geq i$. i is coherent if some s supports it.

3.2.1 Successes

$P \wedge \Diamond\neg P$. inconsistent

$\Diamond\neg P \wedge P$. consistent but not coherent.

$\exists x Fx \wedge \Diamond\neg Fx$. inconsistent

3.2.2 Problems

First we inherit the problems discussed above with Veltman's system.

Even worse:

$\exists x \Diamond\neg Fx \wedge Fx$. consistent and coherent!?!

Why is this? Well, what happens is we first create new free variable x, then we ensure that there is some x can possibly be an F, then we remove that possibility. But this allows every i in the original s to have an extension $i' \geq i$ in $s[\exists x \Diamond\neg Fx \wedge Fx]$.

How do we fix this? A first thought. Require idempotence as before. However this does not work as, in general, $c[\exists x \phi] \neq c[\exists x \phi][\exists x \phi]$ since the second update is always undefined.

We could remove that condition on $s[x]$ allowing it to be defined whether or not x is in the domain of s (and just doing the same thing: i.e. creating a new free variable and reassigning its values. (This is actually what GSV do). However, if we do this then $c[\exists x \Diamond\neg Fx \wedge Fx] = c[\exists x \Diamond\neg Fx \wedge Fx][\exists x \Diamond\neg Fx \wedge Fx]$ since each update just starts anew and does the same thing (i.e. ensures that x is assigned to an F and there is some non-F in the domain.)

3.3 Heim to the rescue

How do we solve this problem? Well this is seems like a mere technical problem, so there should be a mere technical solution. Note first that actually GSV get a failure one one syntactic parse: $c[\exists x(\Diamond\neg Fx \wedge Fx)]$, here the interior update is non-idempotent. But they don't get a failure on this (semantically equivalent parse) $c[(\exists x \Diamond\neg Fx) \wedge Fx]$. What we need is a new syntax-semantics mapping that eliminates this problem.

Go back to the early eighties with the first dynamic semantics. In Heim's [1982] semantics existential quantifiers are absent, and rather existential quantification happens automatically. Let's go through the semantics:

We have the exact same notion of information state as before, but we simply make existential quantification implicit in our definition of atomic sentences (this is not what Heim does, but I think is equivalent in the end and better suits our purposes):

Here is the new semantics for atomic clauses:

$s[Fx] = \{i \in s' : i \models Fx$, where $s' = s[x]$, if x is not in the domain of s, and $s' = s$ otherwise.

Note now that where x is not in the domain of s, $s[Fx]$ is just like $s[\exists x Fx]$ on the old semantics.

This semantics respects the idea due to Heim that 'a man is tall', 'the man is tall', 'he is tall' are semantically the same (assuming masculine = man) even though they are syntactically distinct and invoke different presuppositions on whether the variable they introduce is in the domain or not (it must be in for definites, but not for indefinites).

Now look at $s[\Diamond\neg Fx \wedge Fx]$. If x is not in domain of s this will return $s[Fx]$.

However, $s[Fx \wedge \Diamond\neg Fx] = \emptyset$. But in this case $s[\Diamond\neg Fx \wedge Fx][\Diamond\neg Fx \wedge Fx] = \emptyset$ and so it is not idempotent.

So Heim's treatment of indefinites + idempotence requirement on updates + Veltman's semantics of epistemic modality = solution to problem of epistemic modality de re.

References

Jeroen Groenendijk, Martin Stokhof, and Frank Veltman. Corefrence and modality. In Shalom Lappin, editor, *Handbook of Contemporary Semantic Theory*. Blackwell, 1996.

Irene Heim. *The Semantics of Definite and Indefinite Noun Phrases*. PhD thesis, University of Massachusetts, Amherst, 1982.

Hans Kamp. A theory of truth and semantic representation. In J. Groenendijk et al., editor, *Formal Methods in the Study of Language*, pages 277–322. Mathematish Centrum, 1981.

Nathan Klinedinst and Daniel Rothschild. Connectives without truth-tables. *Natural Language Semantics*, 20:137–175, 2012.

Frank Veltman. Defaults in update semantics. *Journal of Philosophical Logic*, 25(3):221–261, 1996.

Seth Yalcin. Epistemic modals. *Mind*, 116:983–1026, 2007.