

1 Epistemic Modality De Re

Imagine the following situation. You and your spouse have three sons. One of them broke a vase. Your spouse is very anxious to find out who did it. Both you and your spouse know that your eldest didn't do it, he was playing outside when it must have happened. Actually, you are not interested in the question who broke the vase. But you are looking for your eldest son to help you do the dishes. He might be hiding somewhere.

Compare:

- (1) Someone who might be guilty is in the closet
- (2) Someone is hiding in the closet. He might be guilty.

Getting first examples seems to require talking about narrow scope epistemic modality under quantifiers. Similarly Yalcin notes. That if there is a 2/3 chance that a marble is white, we can say:

- (3) This marble is probably white.

However, we cannot say:

- (4) ?Some marble which is probably white is black.

First try to capture this on standard semantics of epistemic modals:

- (5) Some marble which I think is possible to be black is in fact white.

Seems impossible. Similarly ? fails to capture this, as Yalcin notes.

2 Going dynamic

Basic idea. Sentences are instructions to update contexts. Epistemic modals are tests of context. Conversation proceeds by update unless there is a crash:

An information state is a set c consisting of worlds.

For atomic sentence A

$$c[A] = \{w \in c : A \text{ is true in } w\}$$

$$c[\neg\phi] = \{w \in c : w \notin c[\phi]\}$$

$$c[\phi \wedge \psi] = c[\phi][\psi]$$

$$c[\Diamond\phi] = \{w \in c : c[\phi] \neq \emptyset\}$$

Examples:

$$c[A \wedge \Diamond\neg A] \quad c[\Diamond\neg A][A]$$

Fixed point of ϕ , is context c such that $c[\phi] = c$

3 GSV's System (simplified)

Information states are sets of pairs of partial assignment functions and worlds (for short, assignment worlds). So an information state s is a set of pairs $\langle w, f \rangle$ where w is an interpretation function for first order logic (i.e. a set of assignments to constants and predicates) and f is a partial function from variables V , to objects in the domain D and for each i and i' in S the assignment functions f in i and f' in i' have the same domain. We assume one large domain one set of V of variables common to all information states.

Let's start with some simple notions: f' is an extension of f (or $f' \geq f$) if f' agrees with f everywhere (though it may also have more assignments). We also say an assignment world i' is an extension of i iff $i = \langle w, f \rangle$ and $i' = \langle w, f' \rangle$ and $f' \geq f$.

We call the variable domain of s the set of variables such that are in the domain of every assignment function f in any i in s . We will also define the *free extension* of s with respect to a variable x , $s[x]$, which is only defined if x is not in the variable domain of s . $s[x] = \{\langle w, f \rangle : (w, f') \in s \text{ and the domain of } f \text{ is the domain of } f' \text{ union } \{x\}\}$

We say $\langle w, f \rangle \models R(x_1, \dots, x_n)$ iff the interpretation function of w makes $R(f(x_1) \dots f(x_n))$ true.

Information states are updated as follows:

$$s[R(x_1 \dots x_n)] = \{i \in S : i \models R(x_1, \dots x_n)\}$$

$$s[\exists x] = s[x]$$

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[\neg\phi] = \{i \in s : \neg\exists i' \in s[\phi] \text{ s.t. } i' \geq i\}$$

$$s[\Diamond\phi] = \{i \in s : s[\phi] \neq \emptyset\}$$

Some more notions

Consistency : An information state s is consistent with ϕ if $s[\phi] \neq \emptyset$. ϕ is consistent if there is some s consistent with it.

Coherence : An information state s supports ϕ if $\forall i \in s \exists i' \in s[\phi] \text{ s.t. } i' \geq i$. i is coherent if some s supports it.

Examples:

$P \wedge \Diamond\neg P$. inconsistent

$\Diamond\neg P \wedge P$. consistent but not coherent.

$\exists x Fx \wedge \Diamond\neg Fx$. inconsistent

$\exists x \Diamond\neg Fx \wedge Fx$. consistent and coherent!?!

$s[\neg\Diamond\neg P \wedge P] = s$ if s allows P , and otherwise $= s[\neg P]$!?!