Specific indefinites and dynamic presuppositions*

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Abstract

This paper puts forward a novel presuppositional theory of specific indefinites that makes crucial use of *dynamic presuppositions*—presuppositions with anaphoric content in addition to propositional content. It is shown that the theory provides a straightforward explanation for the exceptional scope properties of specific indefinites with respect to different operators, including quantifiers, in terms of presupposition projection and anaphora. It is also proposed that puzzling interpretive properties of *certain* are explained as dynamic presuppositions with functional anaphora.

1 Introduction

This paper offers a novel theory of 'specific indefinites'. As is often remarked (Farkas 1994, 2002a,b, Farkas & Brasoveanu 2021, among others), related but qualitatively different interpretive properties of indefinite noun phrases have been discussed under the rubric of 'specificity'. For the bulk of the present paper, we will be concerned with the issue of indefinites with exceptional wide scope, so 'specific indefinites' are meant to be simply indefinites with exceptional wide scope most of the time. However, since the proposed theory has consequences beyond their scopal properties, so we will stick to the broader, but potentially confusing, term.

The topic of specific indefinites has been copiously written about, especially since the 1980s (pioneering works include Farkas 1981 and Fodor & Sag 1982), and a host of different analyses have been put forward. I will not review the background literature in this paper, as excellent overviews already exist (e.g., Ruys 2001, Ruys & Spector 2017, Ebert 2021, Farkas & Brasoveanu 2021). In addition to a number of different theories, the past research also has identified an array of empirical facts to be understood, and the present paper has very little to add to what is already known. The

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key empirical properties of specific indefinites to be accounted will be reviewed in the next section, but the main contribution of the present paper will be theoretical in nature.

In brief, it is proposed that peculiar interpretive properties of specific indefinites naturally fall out once they are understood as having presuppositions with non-trivial anaphoric content, or what are called *dynamic presuppositions* here. Let us briefly describe this idea without introducing too much formal detail.

We adopt the Stalnaker-Heim view of presuppositions, according to which sentence meanings are modelled by functions over contexts (also known as 'Context Change Potentials') and the presupposition π of a sentence S restricts the domain of the function that S denotes to the set of contexts that 'satisfy' π (in the sense to be made clear later). This is schematically represented in (1) as a general update rule. ⌜$c + S$⌝ denotes the result of updating context c by asserting (declarative) sentence S, with α being the assertive content of S, which is by assumption a function over contexts, and π the presuppositional content of S, a proposition. Following the standard practice in dynamic semantics, postfix notation is used for updates as in ⌜$c[\alpha]$⌝, rather than ⌜$[\alpha](c)$⌝.

(1) $$c + S = \begin{cases} c[\alpha] & \text{if } \pi \text{ is satisfied with respect to } c \\ \# & \text{otherwise} \end{cases}$$

It bears emphasising that this standard implementation of the Stalnaker-Heim view of presuppositions holds that the presupposition π is only used for determining the domain of the function over contexts. More specifically, π is modelled as a (potentially assignment-sensitive) proposition and is said to be satisfied with respect to c iff π is true everywhere in c, or equivalently, c entails π. Importantly, in this implementation, π only has propositional content.

I propose to change the general update rule as follows. Firstly, we regard π as not merely a proposition but a function over contexts, which is a richer semantic object that embodies both propositional and anaphoric content. Consequently, under the new theory, the assertive content α and the propositional content π are modelled by the same type of semantic objects—functions over contexts. Secondly, the presuppositional content π plays two interpretive roles: It is used once to check if the input context c is an admissible one—which is the same as (1)—but not only that, it is crucially also used to update c, before the update with α, as schematized in (2). The newly added bit of the update rule is underscored in (2). Following the tradition in the literature, we omit parentheses for context updates, which associate to the left, e.g., $c[\pi][\alpha] = (c[\pi])[\alpha]$.

(2) $$c + S = \begin{cases} c\underline{[\pi]}[\alpha] & \text{if } \pi \text{ is satisfied with respect to } c \\ \# & \text{otherwise} \end{cases}$$

Because presuppositions in this setup are fully dynamic, I call them *dynamic presuppositions*. What dynamic presuppositions crucially buy us is *cross-dimensional anaphora* from π to α. As remarked above, whenever π is satisfied with respect to c, its propositional content, is by definition, true everywhere in c, meaning that the propositional content of π is bound to be entirely inert in the update of c with π. However, π may still have non-trivial anaphoric content, and in that case it will have non-trivial update effect on c that can feed the interpretation of α (and subsequent updates).

Or to put it conversely, the proposed change in the interpretative role of presuppositions will make a detectable difference only for sentences whose presuppositional content π has non-trivial anaphoric meaning. I propose that what makes indefinites specific is precisely such dynamic presuppositions that introduce new discourse referents in the presupposition that are referred back to in the assertive content. A particularly important consequence of this is that specific indefinites acquire wide scope as a result of presupposition projection. Therefore, there is no need for theoretical machinery specific to the scopal properties of specific indefinites (such as choice functions). The theory only makes use of two mechanisms—anaphora and presupposition projection—which are clearly both needed for independent reasons.

Two remarks are in order regarding the novelty of the proposal. Firstly, the idea of dynamic presuppositions is in fact not entirely novel (Beaver 1992, Elliott & Sudo 2021; also see Appendix B for a related theory formulated in Discourse Representation Theory), but I believe it is fair to say that it is non-standard at this point, especially among theories based on the Stalnaker-Heim view of presupposition. To the best of my knowledge, Elliott & Sudo 2021 is the only study that directly argues for it with an empirical argument, to which we will come back later. Against this backdrop, what is achieved in this paper can be seen as further support for the idea of dynamic presuppositions.

Secondly, the idea that specific indefinites acquire wide scope via presupposition projection has been previously explored by a number of authors (Cresti 1995, Van Geenhoven 1998, Yeom 1998, Jäger 2007, Geurts 2010, Onea 2015). However, none of the presuppositional theories of specific indefinites is based on the idea of dynamic presuppositions in the above sense. Appendix B contains a detailed comparison between my proposal and (a version of) the theory proposed by Van Geenhoven 1998, which I believe is the most prominent among the previous presuppositional theories of specific indefinites. I argue there that the theory proposed in this paper is both conceptually and empirically superior to it, especially with respect to the interaction between specificity and quantification, and a large chunk of the present paper (Section 4) is devoted to developing an account of it.

Beyond the comparison with Van Geenhoven 1998, however, no other existing theories of specific indefinites will be discussed in this paper. Given the exceptionally rich theoretical landscape, it would not be possible to develop a comprehensive set of comparisons with the proposed theory without sacrificing the readability.[1] I wish to conduct such comparisons in some other future occasion.

Having said that, I would like to briefly discuss one major motivation for adopting the presuppositional approach to specificity: It is an advantage of this approach it assigns different meanings to simple sentences containing a specific indefinite vs. a non-specific/plain indefinite. This is most acutely illustrated in languages that have morphosyntactic means of marking specificity. One of the most famous examples is Turkish 'Differential Object Marking', where by a direct indefinite object bearing an

[1]To name some (but not all!) major approaches and key refereces: the domain restriction approach Schwarzschild 2002, the choice function approach Reinhart 1997, Winter 1997, Kratzer 1998, Matthewson 1998, Chierchia 2001, Schwarz 2001, Schlenker 2006, Schwarz 2011, the (in)dependence-marking approach (Farkas 1994, 1997, 2002a,b, Brasoveanu & Farkas 2011), the alternative-based approach (Charlow 2014, 2020).

overt accusative suffix is obligatorily interpreted as specific (Enç 1991, among others).[2] Compare, for example, (3) with and without the case suffix *-ı*.

(3) Ali bir kitab(-**ı**) aldı.
Ali one book(-ACC) bought
'A book is such that Ali bought it.' (Enç 1991: p. 5)

For many (if not all) existing theories of specific indefinites, the interpretive effect of specificity marking is only observable when another scopal element is present in the sentence, and consequently, such theories assign the same meaning to the two versions of (3). Presuppositional theories of specific indefinites, on the other hand, start from the assumption that sentences with specific and non-specific indefinites differ in the presuppositional dimension of meaning, offering a way to understand the semantic contribution of the specificity marking (as well as of its absence) in a systematic way. For example, according to the theory proposed here, the specific version of (3) presupposes that there is a book, while the non-specific version of (3) does not. This is certainly not meant to be a knockdown argument against the other approaches, but I consider it to be a virtue of the presuppositional approach.

Furthermore, I will argue that the idea of dynamic presuppositions provides a novel way of understanding peculiar interpretive properties of specific indefinites containing the adjective *certain*. It has been widely acknowledged that *certain* marks specificity, but it is also known that specific indefinites with and without *certain* exhibit some intriguing interpretive differences (Hintikka 1986, Enç 1991, Schwarz 2001, Farkas 2002b, Schlenker 2006, Solomon 2011, Ionin 2015). I propose that the semantic properties of *certain* are explained in terms of a dynamic presupposition that introduces a discourse referent for a 'natural function'.

The organisation of the present paper is as follows. We will start by reviewing key facts about the scopal properties of specific indefinites in Section 2. I will provide a preliminary account of their exceptional wide scope in terms of dynamic presuppositions in Section 3. Section 4 is devoted to developing a theory of how specific indefinites are interpreted in the scope of quantifiers, an empirical domain where the present theory makes particularly desirable predictions. Section 5 presents an analysis of *certain*. Section 6 is the conclusion.

2 Indefinites with exceptional wide scope

The empirical observation we will aim to account for is that indefinites across languages often often (if not always) give rise to wide scope readings even when they occur in grammatical contexts where other quantificational noun phrases cannot. We will illustrate this observation with examples below. Since we do not have much to add to what is already known in this regard, if the reader is already familiar with

[2] I should however note that it is possible that the Differential Object Marking in Turkish does not always correspond to the type of specificity I am after. In particular, it might be able to signal 'domain specificity', which essentially amounts to the 'definiteness' (in some broad sense) of the domain of quantification. In fact, that is what Enç 1991 proposes for accusative marked indefinite objects in Turkish, although it should be noticed that domain specificity itself does not explain exceptional wide scope and Enç does not directly address this issue. My intention here is to illustrate the idea with the example in (3), leaving a more serious empirical investigation of specificity marking for another occasion.

the facts, it is safe to skip to the next section.

2.1 Restrictions on scope ambiguity

Natural language quantifiers exhibit scope ambiguity. There seems to be considerable cross-linguistic variation in terms of which scopal elements in what grammatical contexts give rise to scope ambiguity, but it is a consensus that an English sentence like (4) is scopally ambiguous. In the examples in this section, I will highlight the quantifier to exhibit (or not exhibit) wide scope in bold, and underscore the other relevant scopal element.

(4) a. Someone flunked **every introductory course in Linguistics**.
b. She showed me a picture of **every car she has owned**.

In what follows, we will call the reading where the linearly preceding element takes wide scope, the surface scope reading, and the reading where the linearly following quantifier takes wide scope, the inverse scope reading.

It should be noted that it is not always easy to raise convincing evidence for a given scopal interpretation, especially when it purportedly entails another available interpretation, but since I am simply reviewing the basic facts from the literature here, I will omit detailed empirical discussion. See Ruys 2001 for discussion on this point and relevant references.

Quantifier scope in natural language is known to be constrained in various ways. The primary way to detect such constraints is by observing the absence of scope ambiguity. Below we will illustrate the constraints with universal quantifiers.

The first class of constraints are structural in nature: Quantifiers occurring inside certain grammatical environments—known as 'scope islands'—do not exhibit scope ambiguity with respect to scopal elements that sit outside. The most notable class of scope islands is finite clauses, including complement and relative clauses, as illustrated by the examples below, which lack inverse scope readings.[3]

(5) a. Someone met with a student [who flunked **every introductory course in Linguistics**].
b. Someone told him [that I flunked **every introductory course in Linguistics**].

In the next section, I will discuss the antecedent of an *if*-clause as a representative example of scope islands, so let us also consider an example of this sort here.

(6) If [**each friend of mine from Texas** had died in the fire], I would have inherited a fortune. (adapted from Fodor & Sag 1982: p. 370)

This example does not have an inverse scope reading where the universal quantifier takes scope over the conditional construction.

Another class of structural constraints is discussed in terms of *scope freezing effects*. The most famous case of this is the double object construction in English (Larson 1990, Bruening 2001). Concretely, the example in (7) does not have an inverse

[3] Various exceptions to this generalization have been pointed out in the literature (e.g., see Reinhart 1997, Winter 1997, Fox 2003, Szabolcsi 2010, Syrett 2015). This complication does not concern us in this paper.

scope reading, unlike those in (8), which do not involve the double object construction.

(7) She showed a child **every picture with animals**.

(8) a. She showed a picture with animals to **every child**.
b. She showed every picture with animals to **a child**.

A third constraint on quantifier scope is semantic in nature: When negative quantifiers like *no NP* are involved, inverse scope readings become absent. There are two sub-cases. Firstly, negative quantifiers themselves cannot take wide scope over a linearly preceding quantifier in a simple sentence like (9), unlike positive quantifiers like *every NP* in (4a) above.

(9) A student flunked **no introductory course in Linguistics**.

Under the inverse scope reading, (9) would mean 'No student flunked any introductory course'; it clearly cannot mean it. Secondly, and more relevantly, a positive quantifier does not give rise to an inverse scope reading with respect to a negative quantifier in a sentence like (10).

(10) No student flunked **every introductory course in Linguistics**.

Under the inverse scope reading, this would also mean 'No student flunked any introductory course', but it cannot mean it, at least as not easily as the inverse scope reading of (4a).

2.2 Indefinites with exceptional wide scope

This paper has nothing to say about where the above constraints on quantifier scope come from, but what is crucial is the fact that indefinite noun phrases do not abide by any of the above constraints. To illustrate, the indefinites in the following sentences allow for wide scope interpretations over the underlined quantifiers.

(11) a. Everyone met with a student [who flunked **an introductory course in Linguistics**].
b. Everyone told him [that I flunked **an introductory course in Linguistics**].

(12) She showed every child **a picture with animals**.

(13) No student flunked **an introductory course in Linguistics**.

The main question that we will aim to answer is why indefinites do not abide by the restrictions on quantifier scope that seem to uniformly apply to other quantifiers. One possible reaction is to assume that different quantifiers are subject to different restrictions (as suggested by Barker 2022, for example). That is certainly an analytical possibility, especially given our poor understanding of why the restrictions on quantifier scope exist to begin with. I will, however, follow the majority here in pursuing a different approach that assumes a uniform set of constraints on quantifier scope, and attribute the exceptional scopal behavior of indefinites to their special anaphoric properties.[4]

[4] A potential advantage of my account over a theory like that of Barker 2022 that simply allows for

3 Exceptional wide scope via presupposition projection

This section presents the core formal proposal of the paper. For the purposes of exposition, I will use a version of Dynamic Predicate Logic (DPL) (Groenendijk & Stokhof 1991) as intermediate language, which is 'lifted' from the original version of DPL so that formulas are interpreted as functions over sets of world-assignment pairs, rather than relations between assignments. This change is necessary to implement the Stalnaker-Heim view of presuppositions as admittance conditions (which requires non-distributivity in the sense of Van Benthem 1986). I will introduce the crucial aspects of the intermediate language step-by-step in what follows. As usual, the formal intermediate language is eliminable (Montague 1970, 1973), but I believe it declutters the exposition and contributes toward readability.[5]

3.1 Plain indefinites

I start with the assumption that indefinites are generally ambiguous between a plain, non-specific reading and a specific reading (except in certain cases; see the end of this section). Following classical dynamic semantics, we assume that a plain indefinite introduces a discourse referent via *random assignment*, as illustrated in (14). We assume assignments to be a (partial) function from $\mathbb{N}$ to the set D of all entities in the model, called the domain, and represent indices that trigger random assignment as superscripts in the object language.[6]

(14) $$c[\textbf{a}^{8}_{\textbf{plain}}\ \textbf{train arrived}] = \left\{ \langle w, a[8 \mapsto t]\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, a\rangle \in c, \\ \quad t \in D \text{ is a train in } w \text{ and} \\ \quad t \text{ arrived in } w \end{array} \right\}$$

We express this function over contexts in our intermediate language simply as

$$\exists x_8(\mathsf{train}\, x_8 \wedge \mathsf{arrived}\, x_8).$$

Using $\rightsquigarrow$ to mean 'translates to (in the assertive dimension)', we will henceforth write (15) to mean the same thing as (14).

(15) $\textbf{a}^{8}_{\textbf{plain}}$ **train arrived** $\rightsquigarrow \exists x_8(\mathsf{train}\, x_8 \wedge \mathsf{arrived}\, x_8)$

This is informally paraphrased by 'There is something x_8 such that it$_{x_8}$ is a train and it$_{x_8}$ arrived'.

indefinites to take unlimited wide scope in Section 6. I would like to thank Chris Barker (p.c.) for asking me relevant questions.

[5]A version of DRT is another possible choice here, and it has some advantages. Most notably, compositional translations from natural language have been already worked out (e.g., Muskens 1996, Brasoveanu 2007). However, since we are not concerned with compositionality in the main paper and I suspect that DRT would be harder to understand for those who are not familiar with it, I decided to use a version of DPL, which is easily understandable for anyone who knows Predicate Logic. See Appendix A for the full details of the intermediate language used here, including a sub-sentential compositional translation from English.

[6]For any assignment a, for any $n \in \mathbb{N}$, and for any $x \in D$ and for any $m \in \mathrm{dom}(a) \cup \{ n \}$,

$$a[n \mapsto x](m) := \begin{cases} x & \text{if } m = n \\ a(m) & \text{otherwise} \end{cases}$$

3.2 Specific indefinites and dynamic presuppositions

Turning now to specific indefinites, I propose that a specific indefinite performs random assignment in the presupposition, rather than in the assertive dimension of meaning. To formalize this idea, I will assume a bi-dimensional theory of presuppositions and represent the presuppositional and assertive content of an expression separately. In what follows, '$\dashrightarrow$' means 'translates in the presuppositional dimension', as opposed to '$\rightsquigarrow$', which means 'translates in the assertive dimension'. The proposal is illustrated in (16).

(16) a. $\mathbf{a}^{8}_{\mathbf{specific}}$ **train arrived** $\dashrightarrow \exists x_8\ \mathsf{train}\ x_8$ (presupposition)

b. $\mathbf{a}^{8}_{\mathbf{specific}}$ **train arrived** $\rightsquigarrow \mathsf{arrived}\ x_8$ (assertive meaning)

The two dimensions of meaning are interpreted as follows. As remarked in the introduction, we adopt the Stalnaker-Heim view of presuppositions that the presuppositional content π of a sentence is used to check if the input context is an admissible one by virtue of 'satisfying' π. Following in the standard formulation of this idea since Heim 1982, we define presupposition satisfaction with respect a context c as in (17) below, where $[\![\phi]\!]$ is the function over contexts denoted by formula ϕ in the intermediate language.

(17) c satisfies a presupposition π (written $c \rhd \pi$) iff for each $\langle w, a\rangle \in c$, there is $\langle w', a'\rangle \in c[\![\pi]\!]$ such that $w = w'$ and $a \leqslant a'$.

(18) $a \leqslant a'$ iff for any $n \in \mathrm{dom}(a)$, $a(n) = a'(n)$.

Note in particular that whenever $c \rhd \pi$, no possible worlds in c are eliminated. That is, if $c \rhd \pi$, then

$$\{\, w \mid \text{for some } a, \langle w, a\rangle \in c \,\} = \{\, w' \mid \text{for some } a', \langle w', a'\rangle \in c[\![\pi]\!] \,\}\,.$$

This means that presupposition satisfaction implies that the propositional content of π is already known to be true in the input context, and hence redundant. As will be important below, however, presupposition satisfaction does not require the anaphoric content of π to be redundant. That is, even when $c \rhd \pi$, we may have

$$\{\, a \mid \text{for some } w, \langle w, a\rangle \in c \,\} \neq \{\, a' \mid \text{for some } w', \langle w', a'\rangle \in c[\![\pi]\!] \,\}\,.$$

With this notion of presupposition satisfaction, we define the update rule as in (19). Crucially, the presuppositional content π is used, not only in checking satisfaction with respect to the input context c, but also to update c, before the update with the assertive content α.

(19) Let **S** $\dashrightarrow \pi$ and **S** $\rightsquigarrow \alpha$. Then:

$$c[\mathbf{S}] = \begin{cases} c[\![\pi]\!][\![\alpha]\!] & \text{if } c \rhd \pi \\ \# & \text{otherwise} \end{cases}$$

Applying this to the sentence with a specific indefinite in (16), the update proceeds as follows.

(20) $c[\mathbf{a}^{8}_{\mathbf{specific}}\ \mathbf{train\ arrived}]$

$$= \begin{cases} c[\![\exists x_8 \, \mathsf{train} \, x_8]\!][\![\mathsf{arrived} \, x_8]\!] & \text{if } c \rhd \exists x_8 \, \mathsf{train} \, x_8 \\ \# & \text{otherwise} \end{cases}$$

Notice in particular that $\exists x_8$ in the presuppositional content dynamically binds the variable x_8 in the assertive content, which is ensured by the following equivalence: For all c,

$$c[\![\exists x_8 \, \mathsf{train} \, x_8]\!][\![\mathsf{arrived} \, x_8]\!] = c[\![\exists x_8 \, (\mathsf{train} \, x_8 \wedge \mathsf{arrived} \, x_8)]\!]$$

This means that the sequential update with π and α in (20) has the exact same net effect as the example with a plain indefinite we saw above. For easier comparison, the analysis of plain indefinites is restated below in the same style. $\top$ is the formula in the intermediate language that denotes the identity function over contexts. (Since it is guaranteed that $c \rhd \top$ for any c, the update will never incur presupposition failure $\#$, but it is kept in (22) for consistency.)

(21) a. $\mathbf{a}^8_{\mathbf{plain}}$ **train arrived** $\dashrightarrow \top$

b. $\mathbf{a}^8_{\mathbf{plain}}$ **train arrived** $\rightsquigarrow \exists x_8 (\mathsf{train} \, x_8 \wedge \mathsf{arrived} \, x_8)$

(22) $c[\mathbf{a}^8_{\mathbf{plain}}$ **train arrived**$]$

$$= \begin{cases} c[\![\top]\!][\![\exists x_8 (\mathsf{train} \, x_8 \wedge \mathsf{arrived} \, x_8)]\!] & \text{if } c \rhd \top \\ \# & \text{otherwise} \end{cases}$$

Therefore, the sentences with plain and specific indefinites have the same update effect, and their difference is entirely in the presupposition: The sentence with a plain indefinite has no presupposition while the sentence with a specific indefinite presupposes that there is at least one train. This presupposition is propositionally very weak and the sentence is normally evaluated against a context that satisfies it, or at least one where it is easy to accommodate it. Consequently, it might not be easy to find direct evidence for this presupposition. To make the matter worse, the version of the sentence with a plain indefinite might also have a pragmatic—though according to the theory here, not a semantic—presupposition that trains exist, because if one wished to assert that a train arrived without presupposing that trains existed, one should perhaps first assert that trains exist and then say one arrived. If so, it is not too farfetched to assume that packing the two pieces of information in one sentence as in 'A train arrived' pragmatically favors a context that already presupposes there to be a train. Given such considerations, I suspect that convincing evidence for the proposed difference in semantic presupposition between the two simple sentences is very hard to come by. Thankfully, a more obvious difference between plain and specific indefinites will arise when we let the presupposition of the specific indefinite project through non-veridical operators. We will consider below two such operators, namely, negation and conditional.

3.3 Wide scope via presupposition projection

Let us start with negation. The meaning of **not** we assume is as in (23). For the sake of simplicity, I simplify the syntax and semantics of **not** by treating it as a sentential operator for expository purposes (see Appendix A for VP-level negation).

(23) If **S** $\dashrightarrow \pi$ and **S** $\rightsquigarrow \alpha$, then:

a. **not S** $\dashrightarrow \pi$

b. **not S** $\rightsquigarrow \neg\alpha$

As stated here, negation only negates the assertive content α of **S**, projecting out its presuppositional content π.[7] Here are some details about what $\neg$ does in the intermediate language: $\neg\phi$ eliminates all world-assignment pairs in the input context that would survive the update with ϕ, which may involve random assignment:[8]

(24) $c[\![\neg\phi]\!] = \{ \langle w, a\rangle \in c \mid \text{for no } b \text{ such that } a \leqslant b, \langle w, b\rangle \in c[\![\phi]\!] \}$

If a plain indefinite occurs under negation, $\exists$ in the meaning will take scope below $\neg$ in the meaning, as illustrated in (25). For the purposes of illustration, we place the indefinite in subject position, while the negation occurring above it.

(25) c[**not [a$^{8}_{\text{plain}}$ train arrived**]]

$$= \begin{cases} c[\![\top]\!][\![\neg\exists x_8(\textsf{train}\, x_8 \wedge \textsf{arrived}\, x_8)]\!] & \text{if } c \rhd \top \\ \# & \text{otherwise} \end{cases}$$

This does not entail that there is a train. If the indefinite is a specific indefinite, on the other hand, the existential quantifier will take scope over the negation, as the presupposition containing it will project, as in (26).

(26) c[**not [a$^{8}_{\text{specific}}$ train arrived**]]

$$= \begin{cases} c[\![\exists x_8\, \textsf{train}\, x_8]\!][\![\neg\textsf{arrived}\, x_8]\!] & \text{if } c \rhd \exists x_8\, \textsf{train}\, x_8 \\ \# & \text{otherwise} \end{cases}$$

In particular, the following equivalence holds for any c:

$$c[\![\exists x_8\, \textsf{train}\, x_8]\!][\![\neg\textsf{arrived}\, x_8]\!] = c[\![\exists x_8\, (\textsf{train}\, x_8 \wedge \neg\textsf{arrived}\, x_8)]\!]$$

Therefore, the meaning corresponds to the wide scope reading of the sentence, and as a consequence, the sentence entails that there is a train, unlike the version of the sentence with a plain indefinite.

For simple negative sentences like *A train did not arrive* and *Alex did not take a train*, it is not necessary to account for the wide scope reading of the indefinite as a specific indefinite, since a standard scope shifting mechanism (such as Quantifier Raising; or for a subject indefinite, potentially just A-movement to the surface position) should be enough to derive a wide scope reading with a plain indefinite. That reading would differ from (26) in that it should have no existence presupposition,

[7] As mentioned before Beaver 1992 also proposes a theory where presuppositions have update effects, but his negation only projects the propositional content of its argument without projecting its anaphoric content. I thank Pavel Astafiev (p.c.) for pointing this out to me.

[8] A technical note is in order here: It is necessary to guarantee no information loss, or 'downdate', to be possible, in order to properly define $\neg$ (or $\forall$), an issue that doesn't arise in a distributive system like the original version of DPL. The only device that could lead to a downdate in the present system is random assignment, and we prevent it from causing downdate by making use of the partiality of assignments as follows.

(i) a. $c \in \text{dom}([\![\exists x_n\phi]\!])$ iff for no $\langle w, a\rangle \in c$, $n \in \text{dom}(a)$.
b. Whenever $c \in \text{dom}([\![\exists x_n\phi]\!])$, $c[\![\exists x_n\phi]\!] = \{ \langle w, a[n \mapsto d]\rangle \mid \langle w, a\rangle \in c \wedge d \in D \}[\phi]$

We will however try to gloss over this complication in the main text (see Appendix A for some more discussion).

but as remarked above, it is not empirically easy to detect this presupposition itself, as its propositional content is very weak. Let us therefore consider a case of a non-veridical operator that involves a scope island, namely, the conditional antecedent.

Developing a serious analysis of conditionals would arguably require intensionalization of the entire theory, which I would like to avoid here, so I will adopt the material implication analysis of conditionals for the sake of illustration.

(27) If $\mathbf{S}_1 \dashrightarrow \pi_1$, $\mathbf{S}_1 \rightsquigarrow \alpha_1$, $\mathbf{S}_2 \dashrightarrow \pi_2$, and $\mathbf{S}_2 \rightsquigarrow \alpha_2$, then:
  a. **if S$_1$, then S$_2$** $\dashrightarrow (\pi_1 \wedge (\alpha_1 \rightarrow \pi_2))$
  b. **if S$_1$, then S$_2$** $\rightsquigarrow (\alpha_1 \rightarrow \alpha_2)$

$\rightarrow$ in the intermediate language defined in terms of $\neg$ and $\wedge$:

$$[\![(\phi \rightarrow \psi)]\!] = [\![\neg(\phi \wedge \neg\psi)]\!]$$

The material implication analysis of conditionals is known to suffer from a number of empirical issues, but that does not matter much for our purposes, because what is crucial is how presuppositions project from conditional antecedents, which I believe is less controversial than their assertive meaning. In particular, it is widely accepted that the presuppositional content π_1 of the antecedent simply projects out, as in the first conjunct of (27a).[9]

With the above analysis of conditionals, let us first analyze an example involving a plain indefinite in the antecedent. The predicted meaning is illustrated in (28). We abstract away from the consequent by assuming it to be an arbitrary sentence **S** such that **S** $\dashrightarrow \pi$ and **S** $\rightsquigarrow \alpha$.

(28) a. **if [a$^8_{\text{plain}}$ train arrives], S** $\dashrightarrow (\top \wedge (\exists x_8(\text{train}\, x_8 \wedge \text{arrives}\, x_8) \rightarrow \pi))$
  b. **if [a$^8_{\text{plain}}$ train arrives], S** $\rightsquigarrow (\exists x_8(\text{train}\, x_8 \wedge \text{arrives}\, x_8) \rightarrow \alpha)$

Importantly, $\exists x_8$ takes scope below $\rightarrow$ in both the presupposition and assertion. Consequently, neither dimension of meaning entails the existence of a train and the sequential update with them in (29) will not result in a context that entails the existence of a train.

(29) c[**if [a$^8_{\text{plain}}$ train arrives], S**] $= \begin{cases} c[\![\texttt{P}]\!][\![\texttt{A}]\!] & \text{if } c \rhd \texttt{P} \\ \# & \text{otherwise} \end{cases}$

  where
  a. $\texttt{P} \equiv (\top \wedge (\exists x_8(\text{train}\, x_8 \wedge \text{arrives}\, x_8) \rightarrow \pi))$; and
  b. $\texttt{A} \equiv (\exists x_8(\text{train}\, x_8 \wedge \text{arrives}\, x_8) \rightarrow \alpha)$

If the indefinite is specific, on the other hand, the existential quantifier, being part of the presupposition, will project out of $\rightarrow$, as in (30).

(30) a. **if [a$^8_{\text{specific}}$ train arrives], S** $\dashrightarrow (\exists x_8\, \text{train}\, x_8 \wedge (\text{arrives}\, x_8 \rightarrow \pi))$
  b. **if [a$^8_{\text{specific}}$ train arrives], S** $\rightsquigarrow (\text{arrives}\, x_8 \rightarrow \alpha)$

(31) c[**if [a$^8_{\text{specific}}$ train arrives], S**] $= \begin{cases} c[\![\texttt{P}]\!][\![\texttt{A}]\!] & \text{if } c \rhd \texttt{P} \\ \# & \text{otherwise} \end{cases}$

[9]The second conjunct is the conditional presupposition that Geurts 1996, 1999 argues to be problematic, the so-called 'proviso problem'). This issue is orthogonal and we will stay away from it. Therefore, I will not discuss conditional examples with specific indefinties in the consequent.

where

a. $\mathtt{P} \equiv (\exists x_8\, \mathsf{train}\, x_8 \wedge (\mathsf{arrives}\, x_8 \to \pi))$
b. $\mathtt{A} \equiv (\mathsf{arrives}\, x_8 \to \alpha)$

Thanks to the projected presupposition '$\exists x_8\, \mathsf{train}\, x_8$', therefore, the update with the presuppositional meaning in (30a) will give rise to a context with a discourse referent for a train, unlike the version of the sentence with a plain indefinite. Notice also that the propositional content of the sequential update amounts to that there is a train and if that train arrives, the consequent holds, which is the wide scope reading of the indefinite.

Crucially, furthermore, the conditional antecedent being a finite clause, a standard scope-shifting mechanism, by assumption, cannot bring about a reading like this with a plain indefinite. Thus, this example nicely demonstrates that the mechanism of presupposition projection yields a wide scope reading that is otherwise unavailable.

3.4 Interim summary

To summarize the most important aspect of the proposal, a plain and a specific indefinite differ with respect to whether random assignment is triggered in the assertive meaning or in the presupposition, as illustrated by the following examples repeated from above.

(22) $c[\mathbf{a}^{8}_{\mathbf{plain}}\ \mathbf{train\ arrived}]$

$$= \begin{cases} c[\![\top]\!][\![\exists x_8(\mathsf{train}\, x_8 \wedge \mathsf{arrived}\, x_8)]\!] & \text{if } c \rhd \top \\ \# & \text{otherwise} \end{cases}$$

(20) $c[\mathbf{a}^{8}_{\mathbf{specific}}\ \mathbf{train\ arrived}]$

$$= \begin{cases} c[\![\exists x_8\, \mathsf{train}\, x_8]\!][\![\mathsf{arrived}\, x_8]\!] & \text{if } c \rhd \exists x_8\, \mathsf{train}\, x_8 \\ \# & \text{otherwise} \end{cases}$$

Note that this analysis can be seen as characterizing the interpretive effect of specificity marking (such as Differential Object Marking in Turkish): Specificity marking corresponds to $\exists$ in the presupposition. We remarked above that it is generally not easy to find convincing evidence for the difference between the above two updates, namely, the existence presupposition of the specific indefinite, but it appears that speakers generally feel some interpretive difference between a plain and a specific indefinite when the language offers an overt way of marking the latter, which could be understood under the current theory in terms of the dynamic existence presupposition it encodes. Furthermore, as explained above, the difference between plain and specific indefinites becomes more palpable when the sentences are embedded under non-veridical operators like negation and conditionals: The specific indefinite gives rise to wide scope readings via presupposition projection.

The proposed theory is also flexible enough to account for indefinites that lack specific readings. For instance, bare plurals in English (under the existential reading) and NPs with modified numerals (as opposed to bare numerals) are considered to be such indefinites that lack specific readings (see Carlson 1977, Van Geenhoven 1998, Dayal 2011 for bare plurals; Cresti 1995, Reinhart 1997, Ebert 2021, Ruys & Spector 2017 for modified numerals). Similarly, when used as pivots in *there* sentences, indefinites do not seem to give rise to specific readings.[10] Assuming that these empirical claims are correct, we can analyze them simply as indefinites that are unambiguously plain. We certainly ultimately wish to understand why these indefinites lack specific readings, especially those that cannot be analyzed in terms of lexical stipulations, but I leave it open for now, with a remark that according to the present theory, the reason should have to do with dynamic presuppositions.

3.5 Remarks on dynamic presuppositions

What crucially enables the proposed analysis of specific indefinites is dynamic presuppositions. As remarked before, whenever a presupposition is satisfied with respect to a context c, its propositional content is guaranteed to be inert with respect to c, but it can still have non-trivial anaphoric effects. Most previous research on presupposition focused exclusively on the propositional content of presuppositions, but for the current theory, they are assumed to have anaphoric content as well, and it is used to update the context of utterance according to the update rule in (19).

(19) Let $\mathbf{S} \dashrightarrow \pi$ and $\mathbf{S} \rightsquigarrow \alpha$. Then:

$$c[\mathbf{S}] = \begin{cases} c[\![\pi]\!][\![\alpha]\!] & \text{if } c \rhd \pi \\ \# & \text{otherwise} \end{cases}$$

This view might strike one as odd in that the presuppositional content π is used twice, once for satisfaction check and once for updating c, whereas the assertive content α is used only once. This asymmetry, however, would disappear if we adopted the full Stalnakerian view, according to which presuppositions and assertive meanings both impose felicity conditions on the context of use. Stalnaker's core intuition, as I understand it, is that presuppositions are backgrounded information and must be redundant (or more generally unremarkable), with respect to the context of utterance, while assertive meanings are foregrounded information and must *not* be redundant (or unremarkable) with respect to the context (see Stalnaker 1973, 1974, 1978, among others). We can incorporate this idea in the update rule as follows.[11]

(32) Let $\mathbf{S} \dashrightarrow \pi$ and $\mathbf{S} \rightsquigarrow \alpha$. Then:

$$c[\mathbf{S}] = \begin{cases} c[\![\pi]\!][\![\alpha]\!] & \text{if } c \rhd \pi \text{ and } c \not\rhd \alpha \\ \# & \text{otherwise} \end{cases}$$

According to this update rule, both the assertive meaning and presupposition impose felicity conditions, and when the felicity conditions are met, both are used to

[10] A more subtle case is that *a* indefinites with light NPs in English seem to favor plain readings (e.g., Fodor & Sag 1982, Geurts 2002, Endriss 2009). However, it is not clear if this is a grammatically encoded restriction or merely an extra-grammatical preference. See Ionin 2010 for relevant experimental data and discussion.

[11] Recall that $\rhd$ amounts to propositional redundancy. See Sudo 2017 for a different non-redundancy condition on assertive meaning that allows it to be propositionally redundant, if it is not anaphorically redundant.

update the context. I think this is conceptually more favorable, but for the sake of simplicity, I will ignore the felicity condition on assertive meanings in this paper.[12]

Dynamic presuppositions are not only conceptually natural, but also empirically motivated independently of specific indefinites. Elliott & Sudo 2021 argue that the contrast of the following sort provides evidence for them.

(33) a. Daniel isn't aware that there is **a mistake** in his proof. It is major.
b. Daniel isn't certain that there is **a mistake** in his proof. ??It is major.

Elliott & Sudo interprets the contrast in the felicity of pronominal anaphora here as follows. That the indefinite in bold cannot antecede the underlined pronoun in (33b) is as expected, given the negation in the matrix clause, which generally disrupts anaphora. What is surprising is that the anaphoric relation can be established in the factive version of the example in (33a). This is explained, if the factive presupposition, projected through the negation, is a dynamic presupposition that introduces a discourse referent.[13]

Elliott & Sudo also point out that dynamic presuppositions trigger (weak) crossover effects. For instance, the pronouns in (34) cannot be bound by the indefinite.

(34) ??Her co-author isn't aware that there is **a female philosopher** in the audience.

If the factive presupposition projects and introduce a discourse referent, it is expected that *her* should be able to be anaphoric to it, contrary to fact. We will not go here into the details of how crossover effects could be accounted for, but the general takeaway from (34) is that we ultimately need a general theory of crossover that not only targets quantifiers but also presuppositions. This is relevant for the proposed theory of specific indefinites, because specific indefinites also trigger crossover effects. To see this, consider the following examples.

(35) a. **A student in my class** met with her supervisor this afternoon.
b. ??Her supervisor met with **a student in my class** this afternoon.

The possessive pronoun *her* can be bound by the indefinite in (35a), which is unsurprising, regardless of whether the indefinite is plain or specific. (35b), on the other hand, does not allow for a bound reading of the pronoun. It is assumed that the standard binding mechanism cannot establish an anaphoric relation in a crossover configuration like this (see, e.g., Büring 2004, Shan & Barker 2006, Barker & Shan 2014, Chierchia 2020 for theories), but the theory of specific indefinites proposed above introduces a backdoor—presupposition projection. That is, assuming that the indefinite in (35b) is specific, the sentence should presuppose that there is a student in my class; if this dynamic presupposition is processed first, it should feed anaphora.

[12] I should note that the resulting theory is not fully Stalnakerian in that we see contexts to be sets of world-assignment pairs, rather than simply sets of possible worlds. Stalnaker 1998 suggests a way to eliminate assignments in the theory of anaphora, but does not offer a fully worked out theory.

[13] It appears that other inferences may also introduce discourse referents. For instance, the anaphora in the following example does not seem to be impossible, despite the fact that it lacks factivity.

(i) Daniel doesn't seem to think that there is **a mistake** in his proof. But it is major.

This is likely to be because it is inferable from the first sentence that Daniel is wrong and there is in fact a mistake in his proof, and this inference introduces a discourse referent.

To prevent it, we would need a theory of crossover that applies to dynamic presuppositions as well as to quantifiers.

4 Quantifiers

Having presented the core of the proposal, we will now turn to the interactions between specific indefinites and quantifiers. Sentences containing quantifiers with specific indefinites underneath are known to pose recalcitrant issues for previous attempts to formally flesh out the presuppositional approach to specific indefinites (see Appendix B), and as we will discuss, they do pose some challenges for the current theory as well. Crucially, however, I will show that the challenges can be addressed with some additional theoretical machinery that has to do with plurality and distributivity that has been given independent motivation from plural anaphora.

We will mainly discuss universal quantifiers, and introduce the additional formal machinery step by step. Other quantifiers are mentioned in Section 4.6.

4.1 Nuclear scope

Let us first consider what will happen when an indefinite occurs in the nuclear scope of a universal quantifier as in (36).

(36) Every cat saw a mouse.

What the current theory predicts for a sentence like this depends on how a presupposition triggered in the nuclear scope of a universal quantifier projects. The literature on presupposition projection is more or less in agreement on this question: The presuppositions project universally with respect to the domain of quantification (Heim 1982, 1983, Beaver 2001, Chemla 2009, Sudo 2012). To see this concretely, consider the following example with the VP *woke up*, which presupposes that the subject was asleep up to the event time.

(37) Every cat woke up.

The entailment of this sentence that every cat was asleep survives embedding under various non-veridical operators, as in (38), suggesting that it is a presupposition of the sentence.

(38) a. It's not true that every cat woke up.
b. Perhaps every cat woke up.
c. If every cat woke up, I should feed them.
d. Did every cat wake up?

The following semantics for *every cat* encodes this projection profile in the second conjunct of the presupposition in (39a).

(39) If **VP** $\dashrightarrow \pi$, **VP** $\rightsquigarrow \alpha$,

a. **every**^{n} **cat VP** $\dashrightarrow (!\exists x_n\, \mathsf{cat}\, x_n \wedge \forall x_n(\mathsf{cat}\, x_n \to \pi\, x_n))$
b. **every**^{n} **cat VP** $\rightsquigarrow \forall x_n(\mathsf{cat}\, x_n \to \alpha\, x_n)$

The first conjunct of (39a) is the existence presupposition. As a matter of fact, the existence presupposition does not seem to feed anaphora, so it is stated with the closure operator ! (formally defined as double negation $\neg\neg$) in front of $\exists x_n$, which stativizes the meaning and blocks dynamic anaphora. When we discuss how the restrictor NP is interpreted below, this existential quantifier will be replaced by something else, so let us not dwell on it.

Applying this analysis to the example in (39), we obtain (40).

(40) **Every[3] cat woke up**

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \forall x_3(\mathsf{cat}\, x_3 \to \mathsf{asleep}\, x_3))$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \to \mathsf{woke.up}\, x_3)$

The predicted presupposition here is the conjunction of the existence presupposition that there is at least one cat and the projected presupposition that every cat was asleep. The assertion on the other hand says every cat woke up. This captures the intuitive meaning of the sentence.

With the above analysis of *every* at hand, let us now consider what will happen if the VP contains an indefinite. First, suppose that the indefinite is a plain indefinite. Assuming the proposal from the previous section that plain indefinites have no presuppositions (or equivalently their presuppositions are tautologous), we will end up with:

(41) **Every[3] cat saw $\mathbf{a}^{7}_{\mathbf{plain}}$ mouse**

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \forall x_3(\mathsf{cat}\, x_3 \to \top))$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \to \exists x_7(\mathsf{mouse}\, x_7 \wedge \mathsf{saw}\, x_3 x_7))$

Since the second clause of the presupposition in (41a) is vacuously true, the presupposition of the whole sentence is simply the existence presupposition that there is a cat. The assertion in (41b) says that every cat saw at least one mouse.

Suppose now that the indefinite is specific. In that case, we would like to have a wide scope reading of the indefinite. However, the analysis so far fails to derive it. Specifically, given the proposal from the previous section, the predicted meaning will be as follows, where $\exists$ appears in the presupposition.

(42) **Every[3] cat saw $\mathbf{a}^{7}_{\mathbf{specific}}$ mouse**

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \forall x_3(\mathsf{cat}\, x_3 \to \exists x_7\, \mathsf{mouse}\, x_7))$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \to \mathsf{saw}\, x_3 x_7)$

In Predicate Logic, we have the following equivalence (with $! \equiv \neg\neg$).

$$(!\exists x_3\, \mathsf{cat}\, x_3 \wedge \forall x_3(\mathsf{cat}\, x_3 \to \exists x_7\, \mathsf{mouse}\, x_7)) \Leftrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \exists x_7\, \mathsf{mouse}\, x_7)$$

However, in our dynamic logic, this equivalence does not hold, because $\forall x_3$ is 'externally static'—i.e., $\forall x_3 \phi$ makes the discourse referents introduced in ϕ inaccessible from outside—while $\exists x_7\, \mathsf{mouse}\, x_7$ is externally dynamic—i.e., $\exists x_7$ can dynamically bind x_7. More specifically, $\forall$ is defined as the dual of $\exists$: For any c, $c[\![\forall x_n \phi]\!] = c[\![\neg\exists x_n \neg\phi]\!]$

Since $\neg$ blocks anaphora from outside, $\forall x_n \phi$ is externally static. This is problematic, not only because we are not accounting for the wide scope reading, but also because x_7 in the assertive content of (41) will be unbound in the update. Concretely, with the meaning in (42), the update will proceed as follows.

(43) $c[\textbf{Every}^3 \textbf{ cat saw } \textbf{a}^7_{\textbf{specific}} \textbf{ mouse}] = \begin{cases} c[\![\mathtt{P}]\!][\![\mathtt{A}]\!] & \text{if } c \rhd \mathtt{P} \\ \# & \text{otherwise} \end{cases}$

where

a. $\mathtt{P} \equiv (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \forall x_3(\mathsf{cat}\, x_3 \rightarrow \exists x_7\, \mathsf{mouse}\, x_7))$

b. $\mathtt{A} \equiv \forall x_3(\mathsf{cat}\, x_3 \rightarrow \mathsf{saw}\, x_3 x_7)$

The sequential update with the presupposition and assertive meaning will leave the last occurrence of x_7 in the latter unbound, while we want it to be somehow bound by $\exists x_7$. In order to address this issue, I will make use of plural predication.

4.2 Plural predication

I propose that the presupposition of *every* does not involve $\forall$, but plural predication as in (44). We use the supremum operator $\sigma x_n[\phi]$. Roughly, it denotes the sum of individuals x_n that make ϕ true, e.g., $\sigma x_3[\mathsf{cat}\, x_3]$ represents the plurality consisting of all cats (in a given world).

(44) If **VP** $\dashrightarrow \pi$, **VP** $\rightsquigarrow \alpha$,

a. **every**^{n} **cat VP** $\dashrightarrow (!\exists x_n\, \mathsf{cat}\, x_n \wedge \pi\sigma x_3[\mathsf{cat}\, x_3])$

b. **every**^{n} **cat VP** $\rightsquigarrow \forall x_n(\mathsf{cat}\, x_n \rightarrow \alpha\, x_n)$

This accounts for universal projection with the help of the distributivity operator δ. Concretely, we assume that the presupposition of the VP *woke up* is inherently distributive, as guaranteed by δ in (45).

(45) **Every**^{3} **cat woke up**

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \delta^{x_3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\mathsf{asleep}\, x_3])$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \rightarrow \mathsf{woke.up}\, x_3)$

δ is essentially a universal quantifier whose domain is indicated by the subscript—$\sigma x_3[\mathsf{cat}\, x_3]$ in this case—so there is no obvious difference from the previous analysis in (40). But this change makes a crucial difference in our example with a specific indefinite in the nuclear scope of *every cat*. In this case, the presupposition of the VP contains no distributivity operator, simply because the presupposition is non-distributive, and denotes a constant function from (dynamic) individuals to $[\![\exists x_7\, \mathsf{mouse}\, x_7]\!]$. Thus, feeding the (dynamic) plural individual $[\![\sigma x_3[\mathsf{cat} x_3]]\!]$ to this constant function yields $[\![\exists x_7\, \mathsf{mouse}\, x_7]\!]$. Therefore, with the revised semantics for *every cat*, we will arrive at the following meaning.

(46) **Every**^{3} **cat saw a**$^7_{\textbf{specific}}$ **mouse**

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \exists x_7\, \mathsf{mouse}\, x_7)$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \rightarrow \mathsf{saw}\, x_3 x_7)$

Importantly, $\exists x_7$ in the presupposition will successfully dynamically bind x_7 in the assertive meaning as a result of sequential update with the presupposition and assertive meaning, as in (47).

(47) $c[\textbf{Every}^3 \textbf{ cat saw } \textbf{a}^7_{\textbf{specific}} \textbf{ mouse}] = \begin{cases} c[\![\mathtt{P}]\!][\![\mathtt{A}]\!] & \text{if } c \rhd \mathtt{P} \\ \# & \text{otherwise} \end{cases}$

where

a. $\mathtt{P} \equiv (!\exists x_3 \,\mathsf{cat}\, x_3 \wedge \exists x_7 \,\mathsf{mouse}\, x_7)$

b. $\mathtt{A} \equiv \forall x_3(\mathsf{cat}\, x_3 \rightarrow \mathsf{saw}\, x_3 x_7)$

In words, the presupposition is satisfied in a context where there is a cat (the existence presupposition of *every*) and there is a mouse (the vacuously projected presupposition of the specific indefinite). The update effect amounts to that there is a cat and there is a mouse and every cat saw the mouse. This is the wide scope reading of the indefinite, as desired.

4.3 Distributivity and quantificational subordination

In the above analysis we achieved what we wanted to achieve, thanks to the absence of a distributivity operator in the presupposition of a specific indefinite. There are, however, cases where a distributive operator must be present. Consider (48), with the bound reading of he pronoun *it*.

(48) Every cat ate a mouse it saw.

This particular sentence in English does not specify whether the indefinite is plain or specific, so let us consider both options. The plain reading poses no particular issue. It is analyzed as (49).

(49) **Every^3 cat ate a$^7_{\text{plain}}$ mouse it$_3$ saw**

a. $\dashrightarrow (!\exists x_3 \,\mathsf{cat}\, x_3 \wedge \top)$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \rightarrow \exists x_7(\mathsf{mouse}\, x_7 \wedge \mathsf{saw}\, x_3 x_7 \wedge \mathsf{ate}\, x_3, x_7))$

The presupposition is simply the existence presupposition that there is a cat and the assertive meaning represents the narrow scope reading (which is the only scopal possibility of the example at hand due to the bound pronoun).

What will happen if the indefinite is read specific? According to the analysis so far, the predicted meaning will look as follows.

(50) **Every^3 cat ate a$^7_{\text{specific}}$ mouse it$_3$ saw**

a. $\dashrightarrow (!\exists x_3 \,\mathsf{cat}\, x_3 \wedge \delta^{x_3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\exists x_7(\mathsf{mouse}\, x_7 \wedge \mathsf{saw}\, x_3 x_7)])$

b. $\rightsquigarrow \forall x_3(\mathsf{cat}\, x_3 \rightarrow \mathsf{ate}\, x_3 x_7)$

There is an issue here: The variable x_7 in the assertive meaning (50b) is not bound again!

For this example in English, one could potentially maintain that the unbound variable x_7 causes a crash in the interpretation and consequently this reading is unavailable. However, in a language with specificity marking, specificity marking seems to be compatible with a configuration like this. For instance, the following Turkish sentence with accusative marking seems to be acceptable.

(51) Her kedi *pro* gördüğü bir fare-yi yedi
every cat *pro* saw one mouse-ACC ate
'Every cat ate a mouse that it saw.'

It is entirely possible that the specificity marking here does not correspond to the notion of specificity I am after (but domain specificity, for example; see fn. 2), but I take it to suggest that it would be desirable to derive a specific reading in this example too.

In order to achieve that, I propose to revise the semantics of *every cat* with δ in the assertive meaning, in place of $\forall$, as in (52b).

(52) If **VP** $\dashrightarrow \pi$, **VP** $\rightsquigarrow \alpha$,

a. **every**^{n} **cat VP** $\dashrightarrow (!\exists x_n\,\mathsf{cat}\,x_n \wedge \pi\sigma x_3[\mathsf{cat}x_3])$

b. **every**^{n} **cat VP** $\rightsquigarrow \delta^{x_n}_{\sigma x_n[\mathsf{cat}\,x_n]}[\alpha\, x_n]$

Applying this analysis to (48) will yield the following meaning with $\delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}$ in both the presupposition and assertive meaning.

(53) **Every**^{3} **cat ate a**$^7_{\textbf{specific}}$ **mouse it**$_3$ **saw**

a. $\dashrightarrow (!\exists x_3\,\mathsf{cat}\,x_3 \wedge \delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}[\exists x_7(\mathsf{mouse}\,x_7 \wedge \mathsf{saw}\,x_3x_7)])$

b. $\rightsquigarrow \delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}[\mathsf{ate}\,x_3x_7]$

How does this solve the binding issue? I propose that it solves the issue thanks to so-called *quantificational subordination*. Quantificational subordination is a phenomenon where an anaphoric relation is mediated by distributive quantification. An example of this is given in (54).

(54) The cats all saw a mouse. Then they all ate it.

Importantly, even when *a mouse* in the first sentence is understood to take narrow scope below the distributivity operator *all*, *it* in the second sentence can be read as bound by it. It is considered that within the scope of distributivity operators that distribute over the same plurality, as in this example, pronominal anaphora happens as if it is not embedded. Based on this observation, Van den Berg 1996 proposes a formal theory of quantificational subordination where the distributivity operator δ has the function of mediating anaphora within its scope by recording and dynamicaly passing on the necessary anaphoric information. For instance, the sequence of sentences in (54) is analyzed as performing the sequential update in (55). To simplify, I will focus on the assertive meaning of this example.

(55) $c\left[\!\left[\delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}(\exists x_7(\mathsf{mouse}\,x_7 \wedge \mathsf{saw}\,x_3x_7))\right]\!\right]\left[\!\left[\delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}(\mathsf{ate}\,x_3x_7)\right]\!\right]$

I need to put aside the formal details aside (see Appendix A), but in this representation, x_7 is dynamically bound by $\exists x_7$ thanks to $\delta^{x_3}_{\sigma x_3[\mathsf{cat}\,x_3]}$, which commonly occurs in the two updates.[14]

By the same token, x_7 in (53b) will also be bound by $\exists x_7$ in (53a) in the sequential update with the presupposition followed by the assertive meaning, as in (56), which is entirely parallel to (55).

[14] Nouwen 2003, 2007 points out that the formulation in Van den Berg 1996, which we adopt here, undergenerates, because within the scope of δ, it is predicted that it is impossible to refer to the original value of the variable it operates on—x_3 in the following example—or to the totality of the mice, but such anaphora is possible. He offers a theory that circumvents these undergeneration issues by ensuring that no information gets lost. It would be simply technical routine to graft this theory onto our proposal, but we will stick to the current version.

(56) $c[\textbf{Every}^3 \textbf{ cat ate a}^7_{\textbf{specific}} \textbf{ mouse it}_3 \textbf{ saw}] = \begin{cases} c[\![\mathtt{P}]\!][\![\mathtt{A}]\!] & \text{if } c \rhd \mathtt{P} \\ \# & \text{otherwise} \end{cases}$

where

a. $\mathtt{P} \equiv (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \delta^{x3}_{\sigma x_3[\mathsf{cat}\, x_3]}(\exists x_7(\mathsf{mouse}\, x_7 \wedge \mathsf{saw}\, x_3 x_7)))$

b. $\mathtt{A} \equiv \delta^{x3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\mathsf{ate}\, x_3 x_7]$

In the resulting reading, the presupposition is satisfied in a context that entails that there is a cat and every cat saw a mouse, and the sequential update amounts to that the cats all saw a mouse and they all ate it. Notice that this sequential update is identical to what would happen with a plain indefinite, but there is an additional presupposition this time, similarly to the case of unembedded smile sentences.[15]

4.4 The distribution of δ

Recall that we derived the wide scope reading of (36), repeated below, on the assumption that there is no δ in the presupposition.

(36) Every cat saw a mouse.

With the revised analysis of *every cat* with δ in the assertive meaning, the wide scope reading of this example is now represented as (57).

(57) $\textbf{Every}^3 \textbf{ cat saw a}^7_{\textbf{specific}} \textbf{ mouse}$

a. $\dashrightarrow (!\exists x_3\, \mathsf{cat}\, x_3 \wedge \exists x_7\, \mathsf{mouse}\, x_7)$

b. $\rightsquigarrow \delta^{x3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\mathsf{saw}\, x_3 x_7]$

Notice that had there been a δ in the presupposition, as in (58), we would derive a narrow scope reading via the mechanism of quantificational subordination:

(58) a. $(!\exists x_3(\mathsf{cat}\, x_3) \wedge \delta^{x3}_{\sigma x_e[\mathsf{cat}\, x_3]}[\exists x_7\, \mathsf{mouse}\, x_7])$

b. $\delta^{x3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\mathsf{saw}\, x_3 x_7]$

This narrow scope reading differs from the reading with a plain indefinite in that its presupposition entails that there is a mouse. While it appears difficult to find direct evidence for this reading of the example, it seems desirable to have this ambiguity, for reasons independent of specific indefinites. To see this, consider the following sentence, where the indefinite does not allow for a specific reading due to the *there* construction.

(59) Every cat is unaware that there is a mouse.

This example has a factive presupposition that there is a mouse. Since we assume that it is a dynamic presupposition, we should also ask about its anaphoric potential. In particular does it simply introduce a discourse referent representing a mouse, or does it introduce a potentially different mouse for each cat, licensing quantifica-

[15] The sentence is judged as infelicitous, if it is commonly known that the cats all saw exactly one mouse, but the analysis offered here, on its own, fails to account for it. however, this is arguably part of a general constraint on the use of an indefinite that blocks it whenever its definite counterpart could be used (Hawkins 1978, 1991, Heim 1991). I will not offer an explicit account of it here.

tional subordination in a subsequent sentence? It seems that both anaphoric possibilities are available, as evidenced by (60).

(60) a. Right now, every cat is unaware that there is a mouse. <u>It</u> is hiding behind the sofa.
b. Right now, every cat is unaware that there is a mouse. But sooner or later they will all catch <u>it</u>.

To capture both anaphoric possibilities, let us assume that δ can be optionally present in the presupposition. When it is present, it licenses quantificational subordination, as in (60b), while when it is absent, the presupposition introduces a single discourse reference representing one mouse.

This is reminiscent of optional distributivity for sentences like (61): It can be read as each boy eating one pizza, which is the distributive reading, or as the boys sharing one pizza, which is the collective reading.

(61) The boys ate one pizza.

This ambiguity is standardly captured by positing a covert distributivity operator Δ in the object language. The version of the sentence with Δ corresponds to the distributive reading and the version without corresponds to the collective reading.[16]

(62) a. The boys Δ ate one pizza. (Distributive)
b. The boys ate one pizza. (Collective)

Extending this analysis to the presuppositional case, we distinguish two versions of Δ: Δ_A introduces distributivity in the assertive dimension of meaning (in examples like (62)), while Δ_P introduces distributivity in the presuppositional dimension of meaning, as in (63).[17] In this representation, δ in the presupposition (63a) comes from Δ_P (recall that *every* itself is non-distributive in the presuppositional dimension), while δ in the assertive meaning (63b) is due to the inherent distributivity of *every*.

(63) **Every**^{3} Δ_P **cat saw a**$^7_{\textbf{specific}}$ **mouse**
a. $\dashrightarrow$ $(!\exists x_3\, \mathsf{cat}\, x_3 \wedge \delta^{x_3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\exists x_7\, \mathsf{mouse}\, x_7])$
b. $\rightsquigarrow$ $\delta^{x_3}_{\sigma x_3[\mathsf{cat}\, x_3]}[\mathsf{saw}\, x_3 x_7]$

This concludes our analysis of specific indefinites in the nuclear scope of universal quantifiers. Here is a summary: According to our analysis, there are three readings: (i) a plain reading which is a narrow scope reading without a presupposition, (64); (ii) a specific reading without δ, (57), which is a wide scope reading; and (iii) a specific reading with δ, (63), which is a narrow scope reading with a presupposition.

[16] Alternatively, we can assume that Δ is always present with this type of predicates but control the reading by relativising it to different covers: a total cover gives rise to full-on distributivity, a singleton minimal cover to (scope-less) collectivity, as proposed by Schwarzschild 1993. Δ_p proposed below could be given a similar analysis.

[17] Note that I am not encoding homogeneity, which would be appropriate for Δ_A (NB: δ in the intermediate language is simply a universal quantifier). To account for it, one can use one's favorite theory of homogeneity (e.g., Križ 2015, Bar-Lev 2021). As for Δ_P, it might also give rise to homogeneity effects, but because presuppositions do not get negated, it is not clear at this point if that is necessary.

(64) **Every^3 cat saw a$^7_{\text{plain}}$ mouse**

a. $\dashrightarrow !\exists x_3\, \textsf{cat}\, x_3$

b. $\rightsquigarrow \delta^{x_3}_{\sigma x_3[\textsf{cat}\, x_3]}[\exists x_7\, \textsf{mouse}\, x_7 \wedge \textsf{saw}\, x_3 x_7]$

(57) **Every^3 cat saw a$^7_{\text{specific}}$ mouse**

a. $\dashrightarrow (!\exists x_3\, \textsf{cat}\, x_3 \wedge \exists x_7\, \textsf{mouse}\, x_7)$

b. $\rightsquigarrow \delta^{x_3}_{\sigma x_3[\textsf{cat}\, x_3]}[\textsf{saw}\, x_3 x_7]$

(63) **Every^3 Δ_P cat saw a$^7_{\text{specific}}$ mouse**

a. $\dashrightarrow (!\exists x_3\, \textsf{cat}\, x_3 \wedge \delta^{x_3}_{\sigma x_3[\textsf{cat}\, x_3]}[\exists x_7\, \textsf{mouse}\, x_7])$

b. $\rightsquigarrow \delta^{x_3}_{\sigma x_3[\textsf{cat}\, x_3]}[\textsf{saw}\, x_3 x_7]$

When the presupposition needs to be distributive as in 'Every cat ate a mouse it saw', the second option becomes unavailable.

4.5 Restrictor

Having accounted for indefinites in the nuclear scope of universal quantifiers, let us now turn to indefinites in the restrictors of universal quantifiers. Again, we need to know the general pattern of how presuppositions project from the restrictors of universal quantifiers, but unfortunately, there is not much discussion in the literature on this point. Rather, most existing discussion seems to be about how they should *not* project, in light of blatantly wrong predictions made by some existing theories (e.g., Heim 1982 predicts that everything in the domain of the model satisfies the presupposition). In order to evaluate the present theory of specific indefinites, however, we need to make some commitments. While I cannot offer a comprehensive account, but in order to develop a reasonable analysis, let us start with the following example.

(65) Every cat that woke up yawned.

What does this presuppose? My tentative answer is the following analysis.

(66) If **NP** $\dashrightarrow \pi$, **NP** $\rightsquigarrow \alpha$,

a. **every^n NP yawned** $\dashrightarrow (\exists x_n\, \pi\, x_n \wedge x_n = \sigma x_n[\alpha\, x_n])$

b. **every^n NP yawned** $\rightsquigarrow \delta^{x_n}_{x_n}[\textsf{yawned}\, x_n]$

In words, the presupposition (66a) says that there is some individual x_n (usually a plurality) for which the presupposition π of the NP holds, and that individual is identical to the plurality consisting of all the individuals for which the assertion α of the NP holds. Note that this entails the existence presupposition that there is at least one individual that satisfies α, given that variables must denote. Therefore, we need not state the existence presupposition separately anymore.[18]

Applying (66) to the example at hand, we obtain. Recall that we assume that the presupposition of *woke up* is inherently distributive.

(67) **Every^6 cat that woke up yawned**

a. $\dashrightarrow (\exists x_6\, \delta^{x^6}_{x_6}[\textsf{asleep}\, x_6] \wedge x_6 = \sigma x_6[\delta^{x_6}_{x_6}[\textsf{cat}]\, x_6 \wedge \textsf{woke.up}\, x_6)])$

[18] In fact, we cannot state it separately in this intermediate language, because $\exists$ does not allow rebinding (unlike σ and δ), as discussed in fn. 8.

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\textsf{yawned}\, x_6]$

The presupposition states that there is some individual that is made up of individuals that were asleep and it is identical to the sum of all cats that woke up. The assertion states that each of these individuals yawned. This captures the intuitive meaning of the sentence. Notice that the presupposition of the restrictor NP existentially projects in the sense that whenever the presupposition is satisfied, there must be at least one cat that was asleep (and woke up).

Now, what if the restrictor contains an indefinite? If the definite is plain, the predicted meaning is as follows.

(68) **Every**^{6} **cat that saw a**$^3_{\textbf{plain}}$ **mouse yawned**

a. $\dashrightarrow (\exists x_6 \top \wedge x_6 = \sigma x_6[\delta^{x_6}_{x_6}[\textsf{cat}\, x_6 \wedge \exists x_3(\textsf{mouse}\, x_3 \wedge \textsf{saw}\, x_6 x_3)]])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\textsf{yawned}\, x_6]$

The presupposition simply amounts to an existence presupposition that there is at least one cat that saw a mouse.

More interestingly, if the indefinite is specific, we obtain the following meaning.

(69) **Every**^{6} **cat that saw a**$^3_{\textbf{specific}}$ **mouse yawned**

$\dashrightarrow (\exists x_6 \exists x_3\, \textsf{mouse}\, x_3 \wedge x_6 = \sigma x_6[\delta^{x_6}_{x_6}[\textsf{cat}\, x_6 \wedge \textsf{saw}\, x_6 x_3]])$

$\rightsquigarrow \delta^{x_6}_{x_6}[\textsf{yawned}\, x_6]$

This is a wide scope reading: The presupposition states that there is an individual x_6 and there is a mouse x_3 and x_6 is identical to the sum of all cats that saw x_3.

With the optional Δ_P, we can also derive a narrow scope reading.[19]

(70) **Every**^{6} **cat that** Δ_P **saw a**$^3_{\textbf{specific}}$ **mouse yawned**

$\dashrightarrow (\exists x_6 \delta^{x_6}_{x_6}[\exists x_3\, \textsf{mouse}\, x_3] \wedge x_6 = \sigma x_6[\delta^{x_6}_{x_6}[\textsf{cat}\, x_6 \wedge \textsf{saw}\, x_6 x_3]])$

$\rightsquigarrow \delta^{x_6}_{x_6}(\textsf{yawned}\, x_6)$

The presupposition does the following. It says that there is an individual x_6, and for each of its atomic component, it introduces a mouse x_3; then it says that x_6 is identical to the sum of all individuals each of whom is a cat and saw the mouse x_3 that it is paired with. This is a narrow scope reading.

As in the case of the nuclear scope, the possibility of optionally inserting Δ_P is independently motivated by the felicity of both types of anaphora in (71).

(71) Every cat that is unaware that there is a mouse yawned.

a. It is hinding behind the sofa.

b. They will all eventually catch it.

Putting together what we developed in the preceding subsections, our final analysis of **every** looks like this:

(72) If **NP** $\dashrightarrow \pi_R$, **NP** $\rightsquigarrow \alpha_R$, **VP** $\dashrightarrow \pi_S$, **VP** $\rightsquigarrow \alpha_S$,

a. **every**^{n} **NP VP** $\dashrightarrow ((\exists x_n\, \pi_R\, x_n \wedge x_n = \sigma x_n[\alpha_R\, x_n]) \wedge \pi_S\, x_n)$

b. **every**^{n} **NP VP** $\rightsquigarrow \delta^{x_n}_{x_n}[\alpha_S\, x_n]$

[19] This enables donkey anaphora (in the existential reading), thanks to quantificational subordination.

The analysis offered here is a major improvement over previous incarnations of the presuppositional approach to specific indefinites, for which quantificational sentences are known to pose a particularly difficult problem, so much so that they have sometimes been regarded as a major shortcoming of the presuppositional approach as a whole. I have just shown that they are not a problem.

The rest of this section discusses two further issues that have to do with quantification, namely, other quantifiers and intermediate scope readings. The discussion will be open-ended in certain respects, and if one wishes, one could jump to the next section without losing the main thread.

4.6 Other quantifiers

The analysis of *every* we arrived at above can easily be adapted for other quantifiers that trigger existence presuppositions and project the presupposition of the nuclear scope universally, simply by changing the assertive meaning appropriately. For instance, (the strong use of) *no*, which is considered to have this presupposition profile, can be given the following analysis with δ—a universal quantifier—taking scope over negation.

(73) If **NP** $\dashrightarrow \pi_R$, **NP** $\rightsquigarrow \alpha_R$, **VP** $\dashrightarrow \pi_S$, **VP** $\rightsquigarrow \alpha_S$,

a. **no**^{n} **NP VP** $\dashrightarrow ((\exists x_n\, \pi_R\, x_n \wedge x_n = \sigma x_n[\alpha_R\, x_n]) \wedge \pi_S\, x_n)$

b. **no**^{n} **NP VP** $\rightsquigarrow \delta^{x_n}_{x_n}[\neg \alpha_S\, x_n]$

Let us apply this analysis to some concrete examples. (74a) presupposes that every cat was asleep, as predicted by this analysis. Furthermore, we predict the same scope possibilities for (74b) as the example with a universal quantifier analyzed above, which seems to be a reasonable prediction.

(74) a. No cat woke up.

b. No cat saw a mouse.

There are, however, reasons to be unsatisfied with the above analysis of *no*. Firstly, a decompositional analysis of *no* in terms of negation taking scope over an existential quantifier, rather than δ over $\neg$, would arguably be more desirable (Rullmann 1995, Abels & Martí 2010, Penka 2011). I have to leave open how such an account could be achieved in the present framework. Secondly, *no* has a 'weak' use, as in (75) (Milsark 1977, McNally 2019).

(75) a. There was no cat that fell asleep.

b. There was no cat that woke up.

It is fair to say that not much attention has so far been paid to how the weak-strong distinction affects presupposition projection, but an issue that seems particularly difficult to tackle is that in the general case, weak *no* does not have that the restrictor is non-empty, e.g., (75a) does not entail the existence of a cat, but with a presupposition trigger, as in (75b), it seems that the sentence does entail the existence of a cat (that was asleep). The analysis above clearly cannot capture the weak use. In particular, it always has an existence presupposition, although that is arguably appropriate for the 'strong' use of *no*. I will not attempt to capture the weak use here.

I would also like to discuss *some*. Similarly to *no*, *some* has strong and weak uses.

Besides the weak-strong distinction, furthermore, it gives rise to an additional complication: As an indefinite, *some* has plain and specific readings. Importantly, the plain-specific distinction is to be distinguished from the weak-strong distinction. For one, strong *some* does not always give rise to exceptional wide scope, so it has a plain and specific reading. Concretely, it is considered that only strong quantifiers can be subjects of individual-level predicates, but narrow scope readings are still possible with such predicates in complex sentences, as illustrated by (76).

(76) a. If some of John's relatives are Chinese, I'll be surprised.
b. Mary doubts that some of John's relatives are tall.

In addition, those existential quantifiers that do not seem to have specific readings also have weak and strong readings. For instance, modified numerals do not seem to give rise to exceptional wide scope readings (Cresti 1995, Reinhart 1997, Ebert 2021, Ruys & Spector 2017), suggesting that they lack specific readings, but they can perfectly well function as the subjects of individual-level predicates—so they have strong readings—and can appear in *there*-sentences—so they have weak readings.

One implicational relation that seems to hold between the strong-weak and plain-specific distinctions is that weak indefinites are always plain and do not give rise to exceptional wide scope readings. Therefore, for *some*, we postulate a three-way distinction: weak-plain, strong-plain, and strong-specific. As in the case of *no*, I cannot offer a concrete analysis of the weak reading here, but the following is a tentative analysis for the plain and specific readings of the strong use of *some*.

I tentatively assume, without argument, that the presupposition of the nuclear scope projects universally through strong *some*, similarly to *every* and *no*. It is often assumed that presuppositions project existentially, rather than universally, through *some*, but I assume that is due to the weak reading of *some*, which is left unaccounted for here. See Sudo et al. 2011 and Fox 2012 for arguments that presuppositions can project either existentially or universally through *some*.

With this assumption, the plain strong reading of *some* will look as follows. Note that we postulate two indices (cf. Van den Berg 1996, Brasoveanu 2007, 2008), as (strong) *some* generally gives rise to two anaphoric possibilities, anaphora to the entire domain of quantification ('maxset anaphora') and to the intersection of the restrictor and nuclear scope ('refset anaphora'). Here, x_n accounts for the former, and x_m accounts for the latter.

(77) If **NP** $\dashrightarrow \pi_R$, **NP** $\rightsquigarrow \alpha_R$, **VP** $\dashrightarrow \pi_S$, **VP** $\rightsquigarrow \alpha_S$,

a. $\textbf{some}^{n,m}_{\textbf{strong.plain}}$ **NP VP** $\dashrightarrow ((\exists x_n\, \pi_R\, x_n \wedge x_n = \sigma x_n[\alpha_R\, x_n]) \wedge \pi_S\, x_n)$

b. $\textbf{some}^{n}_{\textbf{strong.plain}}$ **NP VP** $\rightsquigarrow \exists x_m(x_m \sqsubseteq x_n \wedge \alpha_S\, x_m)$

Note that $\exists x_m$ occurs in the assertive meaning here. We obtain the specific counterpart of this by moving $\exists x_m$ in the presupposition so that it will project and extend its scope.

(78) If **NP** $\dashrightarrow \pi_R$, **NP** $\rightsquigarrow \alpha_R$, **VP** $\dashrightarrow \pi_S$, **VP** $\rightsquigarrow \alpha_S$,

a. $\textbf{some}^{n,m}_{\textbf{strong.specific}}$ **NP VP**
$\dashrightarrow (((\exists x_n\, \pi_R\, x_n \wedge x_n = \sigma x_n[\alpha_R\, x_n]) \wedge \pi_S\, x_n) \wedge \exists x_m\, x_m \sqsubseteq x_n)$

b. $\textbf{some}^{n}_{\textbf{strong.specific}}$ **NP VP**
$\rightsquigarrow \alpha_S\, x_m$

Another 'quantifier' that is worth mentioning here is *the*. It has a lexically encoded existence presupposition and a maximality presupposition, the latter of which can be expressed by requiring that the supremum of those individuals that satisfy the assertive meaning of the NP satisfy the assertive meaning too, as expressed by the third conjunct of (79a). If the NP is singular, therefore, the supremum must be an atomic individual, which amounts to a uniqueness presupposition.

(79) If **NP** $\dashrightarrow \pi_R$, **NP** $\rightsquigarrow \alpha_R$, **VP** $\dashrightarrow \pi_S$, **VP** $\rightsquigarrow \alpha_S$,

a. **the**^{n} **NP VP**

$\dashrightarrow ((((\exists x_n\, \pi_R\, x_n \wedge x_n = \sigma x_n[\alpha_R\, x_n]) \wedge \alpha_R\, x_n) \wedge \pi_S\, x_n))$

b. **the**^{n} **NP VP**

$\rightsquigarrow \alpha_S\, x_n$

4.7 Intermediate scope readings

Lastly, I would like to show that the current theory can deal with intermediate scope readings of specific indefinites. Contrary to the surmise of Fodor & Sag 1982 that specific indefinites always take maximal scope, later research found a plethora of examples where a specific indefinite takes scope outside the immediate scope island but still below some other operator higher up. Concretely, here is one such example.

(80) Every professor rewarded every student who read a book he had recommended. (Abusch 1993: p. 90)

With the pronoun *he* understood as being bound by *every professor*, the indefinite *a book he had recommended* cannot take scope over *every professor*. Crucially, it can take scope between the two universal quantifiers. The following situation makes this intermediate scope reading true, and the narrow scope readings false.

(81) a. Professor A recommended Book 1 and Book 2 and rewarded every student who read Book 1, but didn't reward students who read Book B or any other book.

b. Professor B recommended Book 2 and Book 3 and rewarded every student who read Book 2, but didn't reward students who read Book 3 or any other book.

c. Professor C recommended Book 3 and Book 4 and rewarded every student who read Book 3, but didn't reward students who read Book 4 or any other book.

This observation might look problematic for the theory proposed here, because presuppositions normally project to the top most level. However, the intermediate reading of (81) can be generated with the covert operator Δ_P, as in (82). As the example at hand contains a bound pronoun, its presence is actually forced.

(82) **Every**^{2} **professor** Δ_P **rewarded every**^{1} **student who read a**$^3_{\textbf{specific}}$ **book he**$_2$ **had recommended**

a. $\dashrightarrow ((\exists x_2 \delta^{x_2}_{x_2}[\exists x_1 \exists x_3(\mathsf{book}\, x_3 \wedge \mathsf{recommended}\, x_2 x_3)] \wedge x_2 = \sigma x_2[\delta^{x_2}_{x_2}[\mathsf{professor}\, x_2]]) \wedge \delta^{x_2}_{x_2}[x_1 = \sigma x_1[\mathsf{student}\, x_1 \wedge \mathsf{read}\, x_1 x_3]])$

b. $\rightsquigarrow \delta^{x_2}_{x_2}[\delta^{x_1}_{x_1}[\mathsf{rewarded}\, x_2 x_1]]$

In words, x_2 represents the plurality consisting of all professors, and the first clause of the presupposition introduces for each of them a book x_3 that x_2 recommended and a plurality x_1 consisting of all the students that read x_3. The assertive meaning simply states that each of x_2 rewarded each of x_1, where the value of x_1 varies across the values of x_2 thanks to quantificational subordination mediated by $\delta^{x_2}_{x_2}$.

The above example contains a bound pronoun, but it is not necessary to have bound pronouns for intermediate scope readings to be arise, as shown in the following examples. This is as predicted by the proposed theory, as insertion of Δ_P is assumed to be freely available, whenever there's an appropriate plural term in the presupposition.

(83) a. Each student has to come up with three arguments which show that some condition proposed by Chomsky is wrong.
b. Everybody told several stories that involved some member of the Royal family. (Farkas 1981: p. 64)

(84) a. Most linguists have looked at every analysis that solves some problem.
b. Each student has to find all arguments in the literature which showed that some condition proposed by Chomsky is wrong. (Reinhart 1997: p. 346)

Having said this, however, intermediate scope readings involving certain operators cannot be analyzed in the same way. In particular, it is known that a specific indefinite can take below negation, as in (85).

(85) John wasn't examined by every professor who is competent on some problem. (Ruys & Spector 2017: p. 32)

In order to account for the relevant reading, we have to resort to 'local accommodation' of the presupposition below *wasn't* but above *every*. One potential concern here is that presupposition accommodation below negation is normally highly marked. For example, it is not impossible to understand (86) as not entailing the (unique) existence of a Japanese professor, but seems to require a marked intonation and/or a context where someone is publicly committed to the existence of a Japanese professor.

(86) John wasn't examined by the Japanese professor.

It is not very clear to me if we can maintain that the intermediate scope reading of (85) is similarly marked. To be fair, it is not easy to make a fair comparison between sentences with different semantic and pragmatic properties, when it comes to subtle judgments like this. However, I would like to stress that the theory at least has the mechanism of presupposition accommodation that can be used to account for the intermediate scope reading of (85).

5 *Certain*

We have just seen that the issue of quantification, which has so far been considered to be a major dent in the presuppositional approach to specificity, can be addressed by combining the idea of dynamic presuppositions with the Heim-Stalnaker view of presuppositions. In this section, we will turn to another advantage of the proposed theory: Dynamic presuppositions enables a reasonable account of peculiar interpretive properties of the specificity marker *certain*, which have hitherto eluded formal analyses. I will propose that *certain* triggers a dynamic presupposition that references a 'natural function'.

Certain is widely recognized as a specificity marker in English, but at the same time, empirical facts suggest that its semantic contribution goes beyond merely marking an indefinite to be specific (Hintikka 1986, Enç 1991, Schwarz 2001, Farkas 2002a, Schlenker 2006, Endriss 2009, Solomon 2011, Ionin 2015). To see this, let us take the following pair of examples due to Schwarz 2001.

(87) a. No boy tried every dish that a female relative of his had made.
b. No boy tried every dish that a certain female relative of his had made.
(Schwarz 2001: p. 890)

Consider first the example without *certain* in (87a). Under the bound interpretation of *his*, *a female relative of his* cannot take maximally wide scope, and naturally (87a) receives either a narrow scope reading or a intermediate scope reading (which are truth-conditionally hard to distinguish for this particular example). As the reader can verify, the theory proposed above can account for these readings. The important observation here is that the version of the example with *certain* in (87b) allows for a reading that (87a) does not. More specifically, (87b) is judged as true in a scenario like the following where (87a) is judged as false.

(88) No boy tried every dish that his grandmother had made, and every boy tried every dish that his mother had made.

The relevant reading of (87b) entails that for each boy, there is a female relative of his such that he did not try every dish she had made, but importantly, the intuition tells us that in order for the sentence to be true, the boys' female relatives need to be the same type of relative for all boys, e.g., their grandmothers, their mothers, their sisters, etc., and cannot be random like John's grandmother, Bill's mother, and Chris's sister, etc. Following previous authors, we simply characterize this intuition in terms of 'natural functions' (cf. Groenendijk & Stokhof 1984, Endriss 2009, Szabolcsi 2010), although this notion largely remains a blackbox at this point. The reading of (87b) that we are after is, therefore, analyzed as follows.

(89) There is a natural function $f \in D_{\langle e,e\rangle}$ from boys to their relatives such that no boy x tried every dish that $f(x)$ had made.

In order to derive this reading, I propose that *certain* has a dynamic presupposition that introduces a discourse referent for a natural function, and in its assertion the natural function so introduced is applied to a hidden variable. I assume that this variable can be either referential or bound by a quantifier, very much like in the case of modifiers like *different* (as suggested previously by Solomon 2011). As the analogy with *different* will be instructive, let us delve into it.

5.1 External and internal readings of *different*

Different gives rise to two interpretations, which are illustrated by (90).[20]

(90) a. I recommended a book to Patrick. But he read a different book.
b. Everyone read a different book.

Naturally, the interpretation of *different* in (90a) refers to the book mentioned in the first sentence. More specifically, *a different book* is an indefinite ranging over books that are different from *it*. Thus, in analyzing the meaning of *different* in this sentence, it makes sense to postulate a hidden variable referring to the book mentioned in the first sentence.

Turning to (90b), it admits a similar reading about a book different from some salient book but crucially it also has a reading that does not require a contextually salient antecedent. On this reading, the sentence simply means that there is a one-to-one relation between the people and the books they read. The common analysis of this reading is that the hidden variable of *different* is bound by *everyone*.

For the purposes of this paper, we need not delve into any more detail of the analysis of *different* (see, e.g., Carlson 1987, Moltmann 1992, Beck 2000, Dotlačil 2010, Brasoveanu 2011), but we follow the literature on *different* and call these readings external and internal readings, respectively. In the external reading, the hidden variable is syntactically free and refers to a contextually salient antecedent; in the internal reading, it is bound by a quantifier.[21]

I will claim below that *certain* also has a hidden variable and gives rise to external and internal readings.

5.2 External reading of *certain*

I propose that *cetain* triggers a dynamic presupposition that introduces a 'natural function' over individuals and in its assertive meaning the natural function is applied to a hidden variable. This is illustrated by the following example, where the hidden variable x_6 is left free. NF stands for 'natural function' in the intermediate language (see Appendix A for a way of encoding this restriction at the model level).

(91) $\mathbf{A}^{2}_{\mathbf{plain}}$ **certain**$^{9}_{6}$ **mouse entered**
a. $\dashrightarrow \exists f_9\, \mathsf{NF}\, f_9$
b. $\rightsquigarrow \exists x_2((f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2) \wedge \mathsf{entered}\, x_2)$

[20] There is a third interpretation, when the noun is plural, as in (i).

(i) Different books are on the desk.

In addition to the external reading, (i) has a reading that does not require a contextually salient antecedent and the noun phrase means something similar to *several/many types of*. We will not be concerned with this reading here.

[21] I would like to mention an observation due to Spector 2004 about the availability of internal readings of *certain*, and an interesting parallel to *different* noticed by Solomon 2011: Spector 2004 observe that some quantifiers like *many cats* and *most cats* do not give rise to internal readings of *certain* and Solomon 2011 points out that the same set of quantifiers fail to give rise to internal readings of *different*. While I cannot offer an explanation of why this is so (or critiques of previous attempts by Spector 2004 and Solomon 2011, but these observations suggest that there is a common restriction on the hidden variables in these two adjectives. A complication here is that the internal reading of *same* seems to restricted in the same way.

Note that this presupposition is propositionally very weak. In fact, it is safe to simply take for granted the existence of natural functions over individuals. However, the anaphoric content of the presupposition has a substantial effect: it is applied in the assertive meaning (91b) to a free variable x_6, which by assumption is resolved to a contextually salient individual, most typically the speaker. Assuming that x_6 refers to the speaker, the sequential update with (91a) and (91b) will amount to: For some natural function f_9, the mouse that f_9 maps the speaker to entered.[22] Consequently, the meaning of this sentence is slightly different from a plain indefinite sentence without *certain* in that it implies that the speaker stands in a non-arbitrary relation with the mouse in question.

One nice consequence of this analysis is that it captures the common, but what nebulous, intuition that *certain* tends to connote some sort of familiarity on the speaker's part, which is often informally paraphrased by something like 'a mouse that the speaker has in mind' in terms of a dynamic presupposition that introduces a natural function. Notice in particular that without the notion of dynamic presupposition, it is harder to develop an account like this. For one, the familiarity is clearly not asserted, so the quantification over natural functions should not be in the assertive meaning. Furthermore, the use of *certain* does not necessarily refer to a contextually salient natural function, so postulating a free variable for natural functions would not be desirable.

In the above example, we analyzed the indefinite itself to be plain, but nothing in the theory prevents the parse of the sentence where it is read specific, in which case the predicted meaning will be as in (92).

(92) **$\mathbf{A}^{2}_{\text{specific}}$ certain$^{9}_{6}$ mouse entered**

a. $\dashrightarrow \exists x_2(\exists f_9\, \mathsf{NF}\, f_9 \wedge (f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2))$

b. $\rightsquigarrow \mathsf{entered}\, x_2$

The presupposition in this case entails the existence of a mouse that the speaker (or some other discourse salient individual) is related to via a natural function, but otherwise the update effect will be identical.

Whether both plain and specific readings of the indefinite are available or not is not immediately clear for this simple, unembedded example. Furthermore, embedding it in the scope of an operator will not be very informative in settling this question, because due to presupposition projection, the existential quantification over natural functions will project out, while the variable of the indefinite continuing to be asserted to be identical to the output of the natural function. Specifically, consider the following conditional example.

(93) If a certain mouse entered, she caught it.

Ignoring the presupposition of the consequent, this example, with a plain or specific *a*, should have the same presupposition as the corresponding reading of the unembedded example above. In particular, even if the indefinite is plain, the presupposition introduces a natural function, and the assertive meaning will be about a

[22] I need to leave it open why it is not *the certain mouse*, rather than *a certain mouse*, given the uniqueness is satisfied. It is possible that *a certain* is an idiomatic phrase and *a* is not an ordinary indefinite article, as suggested by the compatibility with plural noun phrases with numerals like *a certain two people* (but **a certain people*). Thanks for Patrick D. Elliott (p.c.) for relevant discussion.

particular mouse to which the function maps the speaker (or whatever contextually salient individual that the hidden variable refers to).

It seems to me to be harmless to have the plain-specific ambiguity here, so let us leave it at that.

5.3 Internal readings

Let us now turn to the internal reading of *certain*, where the hidden variable is bound by a quantifier. In such a case, the natural function may return different individuals for different values of the variable. Consider, for example, (94).

(94) **Every cat^6 saw a$^2_{\text{plain}}$ certain$^9_6$ mouse**

a. $\dashrightarrow (\exists x_6 \exists f_9\, \mathsf{NF}\, f_9 \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\exists x_2((f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2) \wedge \mathsf{saw}\, x_6 x_2)]$

Importantly the assertive meaning (94b) amounts to that every cat saw the mouse that the natural function maps it to. This captures the intuition that the sentence expressions that the cats and mice bear some unnamed non-arbitrary relation.

Just for the sake of completeness, let us consider other available parses of the same sentence as well. Firstly, nothing in the theory prevents *a* from receiving a specific reading, in which case, the predicted meaning looks as follows.

(95) **Every cat^6 saw a$^2_{\text{specific}}$ certain$^9_6$ mouse**

a. $\dashrightarrow (\exists x_6(\exists x_2(\exists f_9\, \mathsf{NF}\, f_9 \wedge (f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2))) \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\mathsf{saw}\, x_6 x_2]$

Notice that this is a wide scope reading, as it is about a single mouse x_2, which the natural function f_9 maps every cat to.

There are other wide scope readings, which obtain via an external reading. Firstly, with a plain indefinite:

(96) **Every cat^6 saw a$^2_{\text{plain}}$ certain$^9_5$ mouse**

a. $\dashrightarrow (\exists x_6 \exists f_9\, \mathsf{NF}\, f_9 \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\exists x_2((f_9(x_5) = x_2 \wedge \mathsf{mouse}\, x_2) \wedge \mathsf{saw}\, x_6 x_2)]$

This is a 'wide scope reading', not in the formal syntactic sense, but in the sense that $f_9(x_5)$, with a free variable x_5, does not co-vary with the quantifier. Another wide scope reading ensues when the external reading of *certain* is coupled with a specific reading of *a*:

(97) **Every cat^6 saw a$^2_{\text{specific}}$ certain$^9_5$ mouse**

a. $\dashrightarrow (\exists x_6 \exists x_2(\exists f_9\, \mathsf{NF}\, f_9 \wedge (f_9(x_5) = x_2 \wedge \mathsf{mouse}\, x_2)) \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\mathsf{saw}\, x_6 x_2]$

In this case, the existence of a mouse that bears some non-arbitrary relation to the value of the free variable x_5, typically the speaker, is presupposed.

Admittedly, it is not clear if empirical evidence for this exact set of wide scope readings could be found, but I see no particular empirical issue of having this ambiguity either.

Lastly, we should also consider the possibility of inserting Δ_P, which would introduce δ in the presupposition. When it is present while *certain* receives an internal reading and *a* receives a plain reading, the predicted meaning is (98).

(98) **Every cat^6 Δ_P saw a$^2_{\textbf{plain}}$ certain$^9_6$ mouse**

a. $\dashrightarrow (\exists x_6 \delta^{x_6}_{x_6}[\exists f_9 \, \mathsf{NF}\, f_9] \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\exists x_2((f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2) \wedge \mathsf{saw}\, x_6 x_2)]$

The cross-dimensional functional anaphora here succeeds thanks to quantificational subordination mediated by δ, and the overall reading amounts to a narrow scope reading of the existential quantification over natural functions: The presupposition says that each cat is paired with some natural function, and they each saw the mouse that the natural function they are paired with maps them to. We presumably would like to block this reading, however, because the sentence seems to require that the cats and mice stand in the same non-arbitrary relation.

In fact, a similar trick would overgenerate for Schwarz's example (87b). Concretely, when Δ_P is present, we allow the natural function to co-vary with the quantifier. Consequently, it could be that for John, the relevant female relative is his grandmother, while for Bill, she is his mother and for Chris, she is his sister.

(99) **No boy^6 Δ_P ate a$^7_{\textbf{plain}}$ dish that a$^2_{\textbf{plain}}$ certain$^9_6$ female relative of his$_6$ had made**

a. $\dashrightarrow (\exists x_6 \delta^{x_6}_{x_6}[\exists f_9 \, \mathsf{NF}\, f_9] \wedge x_6 = \sigma x_6[\mathsf{boy}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\neg\exists x_7(\mathsf{dish}\, x_7 \wedge (\exists x_2((f_9(x_6) = x_2 \wedge (\mathsf{female}\, x_2 \wedge \mathsf{relative}\, x_2 x_6) \wedge \mathsf{ate}\, x_6 x_3)))]$

Requiring the indefinite to be specific will not solve this issue. The following meaning still allows co-variation between the natural function and the quantifier.

(100) **Every cat^6 Δ_P saw a$^2_{\textbf{specific}}$ certain$^9_6$ mouse**

a. $\dashrightarrow (\exists x_6 \delta^{x_6}_{x_6}[\exists x_2(\exists f_9 \, \mathsf{NF}\, f_9 \wedge (f_9(x_6) = x_2 \wedge \mathsf{mouse}\, x_2))] \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\mathsf{saw}\, x_6 x_2]$

Furthermore, a similar issue arises with the external reading of *certain*. Concretely, the following is no longer a wide scope reading: Even though the hidden variable x_5 is free and has a fixed value, f_9 co-varies with the quantifier.

(101) **Every cat^6 Δ_P saw a$^2_{\textbf{plain}}$ certain$^9_5$ mouse**

a. $\dashrightarrow (\exists x_6 \delta^{x_6}_{x_6}[\exists f_9 \, \mathsf{NF}\, f_9] \wedge x_6 = \sigma x_6[\mathsf{cat}\, x_6])$

b. $\rightsquigarrow \delta^{x_6}_{x_6}[\exists x_2((f_9(x_5) = x_2 \wedge \mathsf{mouse}\, x_2) \wedge \mathsf{saw}\, x_6 x_2)]$

For a similar reason to above, if this reading is available, we will fail to capture the intuition that the relation between the cats and mice is somehow uniform. A similarly problematic reading obtains when the indefinite is read specific.

The above discussion leads us to conclude that we wish to prohibit Δ_P from scoping over $\exists f_9$. However, in the present theory, there is no way of syntactically blocking it. In particular in the example like Schwarz's example (87b), the bound pronoun requires the presence of Δ_P. Luckily for us, there is actually a technical, brute-force solution to this issue. I cannot discuss it here without introducing all the formal details of the theory, so I defer it to Appendix A.[23] However, it begs a question as to why existential quantification over natural functions works this way, and existential quantification over individuals does not. I have to leave it for future research.

6 Conclusion

In this paper, I put forward a novel presuppositional theory of specific indefinites. The key idea is dynamic presuppositions, presuppositions with both anaphoric and propositional content. I offered a formulation of the idea within the broad Heim-Stalnaker view of presuppositions as conditions on successful update, as schematized in the update rule in (19).

(19) Let **S** $\dashrightarrow \pi$ and **S** $\rightsquigarrow \alpha$. Then:

$$c[\mathbf{S}] = \begin{cases} c[\![\pi]\!][\![\alpha]\!] & \text{if } c \rhd \pi \\ \# & \text{otherwise} \end{cases}$$

I demonstrated that the theory not only accounts for specific indefinites occurring in contexts that project presuppositions, such negation and conditional antecedents, but also those occurring in contexts that involve quantificational sentences, where presuppositions are 'filtered'. This constitutes a major progress for the presuppositional approach to specific indefinites, as quantified sentences posed a major issue for previous incarnations (as discussed specifically for the theory of Van Geenhoven 1998 in Appendix B). It was also shown that the theory offers a way to understand the interpretative contributions of *certain*, including the intuition characterized informally as 'an individual that the speaker has in mind', in terms of dynamic presuppositions about natural functions.

While I confined my attention to extensional constructions in this paper, additional support for the present theory might come from attitude contexts. The presuppositional approach generally predicts a reading for specific indefinites in attitude contexts that would be harder to capture in other approaches to specific indefinites, so if there is evidence that such a reading is available, it will support the presuppositional approach.

Observe first that when a presupposition is triggered under *hope*, it generally gives rise to a presupposition that the attitude holder believe it to be true (and sometimes also projects out in addition; see Heim 1992, Geurts 1999, Sudo 2014). Concretely, (102) presupposes that the linguist believes/is aware that the cat is asleep now.

(102) The linguist hopes that the cat will wake up soon.

Given this projection pattern, we would expect a specific indefinite occurring under *hope* to give rise to a presupposition about the existence of an individual. For instance, if the indefinite is specific, the presupposition of the example should entail that the linguist *believes* that there is a Uyghur student, while if the indefinite is plain, its assertion should entail the speaker *hopes* there be a Uyghur student.

[23]The rough idea is to require in the semantics of δ that all function variables in the output have a uniform value (or be 'singular' if you like), preventing co-variation with other variables.

(103) The linguist hopes that a Uyghur student will take her course.

It seems to me that the specific reading as characterized above is indeed available, and if so, any simple scoping mechanism would not be able to derive it. Rather, it would strongly suggest that specific indefinites have to do with presuppositions.

To make this data point more concrete and argument theoretically sound, however, further work is needed. Empirically, data needs to be carefully assessed, especially because sentences like (103) are many-ways ambiguous. That is, in addition to the plain-specific ambiguity, there is also *de re*/*de dicto* ambiguity to take care of. Theoretically, furthermore, a proper analysis of this construction would require an intensional version of the formal theory. There is certainly no reason to suspect that that could not be done, but it is left for another occasion.

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