Appendices to "Specific indefinites and dynamic presuppositions"

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Contents

A Details of the intermediate language

This appendix contains the details of the intermediate formal language used in the main paper. There are three versions of it, corresponding to the three main sections of the paper, Sections 3–5, and their relative complexity is incremental. The first, simplest language, which we call $\mathcal{L}$, is essentially a 'lifted' version of Dynamic Predicate Logic (DPL) of Groenendijk & Stokhof 1991, and is used in Section 3. Section 4 assumes a more complex version $\mathcal{L}^+$, which is an extension of $\mathcal{L}$ augmented with plurality. Finally, $\mathcal{L}^+_f$ incorporates discourse referents for natural functions, which play a key role in Section 5.

A.1 $\mathcal{L}$

A.1.1 Syntax

$\mathcal{L}$ has the same syntax as DPL, which in turn has the standard syntax of Predicate Logic.

The vocabulary includes: (i) (and) (parentheses), (ii) a denumerably many variables $(x_1, x_2, \ldots)$ and constants, (iii) functions and (iv) n-ary predicates $(0 \leqslant n)$. Terms and formulas are defined in the standard manner.

(1) The set of terms $T_{\mathcal{L}}$ of $\mathcal{L}$ is the smallest set that satisfies the following.

a. Variables and constants are all members of $T_{\mathcal{L}}$.

b. If $\ulcorner\Phi\urcorner$ is a function and $\ulcorner\tau\urcorner \in T_{\mathcal{L}}$, then $\ulcorner\Phi(\tau)\urcorner \in T_{\mathcal{L}}$.

(2) The set of formulas $F_{\mathcal{L}}$ of $\mathcal{L}$ is the smallest set that satisfies the following.

a. If $\ulcorner\Pi\urcorner$ is a 0-ary predicate, then $\ulcorner\Pi\urcorner \in F_{\mathcal{L}}$.

b. If $\ulcorner\Pi\urcorner$ is an n-ary predicate and $\ulcorner\tau_1\urcorner, \ldots, \ulcorner\tau_n\urcorner$ are terms, then $\ulcorner\Pi\,\tau_1 \ldots \tau_n\urcorner \in F_{\mathcal{L}}$.

c. If $\ulcorner\tau_1\urcorner$ and $\ulcorner\tau_2\urcorner$ are terms, then $\ulcorner\tau_1 = \tau_2\urcorner \in F_{\mathcal{L}}$.

d. If $\ulcorner\phi\urcorner, \ulcorner\psi\urcorner \in \mathcal{L}$, then $\ulcorner\neg\phi\urcorner, \ulcorner(\phi \wedge \psi)\urcorner \in F_{\mathcal{L}}$.

e. If $\ulcorner\phi\urcorner \in \mathcal{L}$ and $\ulcorner\xi\urcorner$ is a variable, then $\ulcorner\exists\xi\phi\urcorner \in F_{\mathcal{L}}$.

We treat $\ulcorner(\phi \to \psi)\urcorner$ as a shorthand for $\ulcorner\neg(\phi \wedge \neg\psi)\urcorner$ and $\ulcorner\forall\xi\phi\urcorner$ for $\ulcorner\neg\exists\xi\neg\phi\urcorner$. We do not discuss $\ulcorner\vee\urcorner$, as its dynamic properties are contentious (see, e.g., Groenendijk & Stokhof 1991, Geurts 1999, Rothschild 2017 for discussion).

A.1.2 Semantics

Models for $\mathcal{L}$ consist of three components: (i) a non-empty set $\mathcal{D}$ of entities, (ii) a non-empty set $\mathcal{W}$ of possible worlds such that $\mathcal{D} \cap \mathcal{W} = \varnothing$, and (iii) an interpretation function $\mathcal{I}$ that assigns to each constant a member of $\mathcal{D}$, to each function, a function over $\mathcal{D}$, and to each n-ary predicate a function from possible worlds to functions from n-tuple of individuals to truth-values (0-ary predicates are mapped to properties, functions from truth-values).

Given a model $\mathcal{M} = \langle\mathcal{D}, \mathcal{W}, \mathcal{I}\rangle$, a partial function from variables to $\mathcal{D}$ is called an assignment in $\mathcal{M}$. We denote the set of all assignments by $\mathcal{A}$ and $\mathcal{W} \times \mathcal{A}$ by $\mathcal{C}$, the set of all (Heimian) contexts. We write $a \leqslant b$ if for all variables $\ulcorner\xi\urcorner \in \mathrm{dom}(a)$, $a(\xi) = b(\xi)$.

Let $\mathcal{M} = \langle\mathcal{D}, \mathcal{W}, \mathcal{I}\rangle$ be a model for $\mathcal{L}$. Terms are interpreted as function from assignments to individuals, and formulas as functions over contexts. I use the notation for partial function in Heim & Kratzer 1998.

Firstly, terms are interpreted as follows.

(3) a. For any constant $\ulcorner\gamma\urcorner$,

$$[\![\gamma]\!]_{\mathcal{M}} = \lambda a \in \mathcal{A}.\, \mathcal{I}(\gamma)$$

b. For any variable $\ulcorner\xi\urcorner$,

$$[\![\xi]\!]_{\mathcal{M}} = \lambda a \in \mathcal{A}\colon\ v \in \mathrm{dom}(a).\ a(\xi)$$

c. For any function $\ulcorner\Phi\urcorner$ and $\ulcorner\tau\urcorner \in T_{\mathcal{L}}$,

$$[\![\Phi(\tau)]\!]_{\mathcal{M}} = \lambda a \in \mathcal{A}\colon\ a \in \mathrm{dom}([\![\tau]\!]_{\mathcal{M}}).\ \mathcal{I}(\Phi)([\![\tau]\!]_{\mathcal{M}}(a))$$

Secondly, atomic formulas are interpreted as in (4). To simplify, let's write $c \in \mathrm{dom}([\![\tau]\!]_{\mathcal{M}})$ iff for each $\langle w, a\rangle$, $a \in \mathrm{dom}([\![\tau]\!]_{\mathcal{M}})$ for any $\ulcorner\tau\urcorner \in T_{\mathcal{L}}$.

(4) a. For any 0-ary predicate $\ulcorner\Pi\urcorner$,

$$[\![\Pi]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}.\ \{\, \langle w, a\rangle \in c \mid \mathcal{I}(\Pi)(w) = 1 \,\}$$

b. For any n-ary predicate $\ulcorner\Pi\urcorner$ and any $\ulcorner\tau_1\urcorner, \ldots, \ulcorner\tau_n\urcorner \in T_{\mathcal{L}}$,

$$\begin{aligned}[\![\Pi\,\tau_1 \ldots \tau_n]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}\colon\ & c \in \mathrm{dom}([\![\tau_1]\!]_{\mathcal{M}}) \text{ and ... and } c \in \mathrm{dom}([\![\tau_n]\!]_{\mathcal{M}}).\\ & \{\, \langle w, a\rangle \in c \mid \mathcal{I}(\Pi)(w)(\langle [\![\tau_1]\!]_{\mathcal{M}}(a), \ldots, [\![\tau_n]\!]_{\mathcal{M}}(a)\rangle) = 1 \,\}\end{aligned}$$

c. For any $\ulcorner\tau_1\urcorner, \ulcorner\tau_2\urcorner \in T_{\mathcal{L}}$,

$$\begin{aligned}[\![\tau_1 = \tau_2]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}\colon\ & c \in \mathrm{dom}([\![\tau_1]\!]_{\mathcal{M}}) \text{ and } c \in \mathrm{dom}([\![\tau_2]\!]_{\mathcal{M}}).\\ & \{\, \langle w, a\rangle \in c \mid [\![\tau_1]\!]_{\mathcal{M}}(a) = [\![\tau_2]\!]_{\mathcal{M}}(a) \,\}\end{aligned}$$

Finally, complex formulas are interpreted as in (5). We define random assignment, which is used in the interpretation of the existential quantifier $\exists$, as the following operation: For any $c \in \mathcal{C}$ and for any variable $\ulcorner\xi\urcorner$:

$$c[\xi] \coloneqq \{\, \langle w, b\rangle \mid \text{for some } \langle w, a\rangle \in c,\ a \leqslant b \text{ and } \mathrm{dom}(a) \cup \{\,\xi\,\} = \mathrm{dom}(b) \,\}$$

(5) a. $[\![\neg\phi]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\phi]\!]_{\mathcal{M}}).$
$\{\, \langle w, a\rangle \in c \mid \text{for no } b \text{ such that } a \leqslant b \text{ and } \langle w, b\rangle \in [\![\phi]\!]_{\mathcal{M}}(c) \,\}$

b. $[\![(\phi \wedge \psi)]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\phi]\!]_{\mathcal{M}}) \text{ and } [\![\phi]\!]_{\mathcal{M}}(c) \in \mathrm{dom}([\![\psi]\!]_{\mathcal{M}}).$
$[\![\psi]\!]_{\mathcal{M}}([\![\phi]\!]_{\mathcal{M}}(c))$

c. $[\![\exists\xi\phi]\!]_{\mathcal{M}} = \lambda c \in \mathcal{C}\colon\ \text{for each } \langle w, a\rangle \in c,\ \xi \notin \mathrm{dom}(a) \text{ and } c[\xi] \in \mathrm{dom}([\![\phi]\!]_{\mathcal{M}}).$
$[\![\phi]\!]_{\mathcal{M}}(c[\xi])$

Note that $\ulcorner\exists\urcorner$ prevents reassignment of values to old variables by requiring $\ulcorner\xi\urcorner$ to be not in the domain.

We denote the 'tautologous' 0-ary predicate by $\ulcorner\top\urcorner$, which is interpreted as the total identity function over $\mathcal{C}$.

As usual, we will drop the model parameter whenever it is not relevant.

A.1.3 A compositional translation from a fragment of English to $\mathcal{L}$

To achieve a compositional semantics from a fragment of English to $\mathcal{L}$, we will enrich $\mathcal{L}$ with semantic types and the λ-operator. Let's call the resulting language $\mathcal{L}_\lambda$.

In addition to the standard semantic types, we will make use of bullet types $(\sigma \bullet \tau)$, which will be useful in the bi-dimensional theory of meaning we adopt.

(6) Semantic types

a. e and t are semantic types.
b. If σ and τ are semantic types, then (σ, τ) is a semantic type.
c. If σ and τ are semantic types, then $\sigma \bullet \tau$ is a semantic type.

L_λ has denumerably many variables of each semantic type. The variables of $\mathcal{L}$ are variables of type e.

Terms and formulas are inherited from $\mathcal{L}$, and they are meaningful expressions of types e and t, respectively. There are other meaningful expressions too. The set $U^\tau_{\mathcal{L}_\lambda}$ of meaningful expressions of a given type τ is defined as:

(7) a. $U^e_{\mathcal{L}_\lambda} = T_\mathcal{L}$.
b. $U^t_{\mathcal{L}_\lambda} = F_\mathcal{L}$.
c. If $\ulcorner\alpha\urcorner \in U^\tau_{\mathcal{L}_\lambda}$ and $\ulcorner\xi^\sigma\urcorner$ is a variable of type σ, then $\ulcorner\lambda\xi^\sigma.\,\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}_\lambda}$.
d. If $\ulcorner\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}_\lambda}$ and $\ulcorner\beta\urcorner \in U^\sigma_{\mathcal{L}_\lambda}$, then $\ulcorner\alpha(\beta)\urcorner \in U^\tau_{\mathcal{L}_\lambda}$.
e. If $\ulcorner\alpha\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}_\lambda}$, then $\ulcorner\pi_1(\alpha)\urcorner \in U^\sigma_{\mathcal{L}_\lambda}$ and $\ulcorner\pi_2(\alpha)\urcorner \in U^\tau_{\mathcal{L}_\lambda}$.
f. If $\ulcorner\alpha\urcorner \in U^\sigma_{\mathcal{L}_\lambda}$ and $\ulcorner\beta\urcorner \in U^\tau_{\mathcal{L}_\lambda}$, then $\ulcorner\langle\alpha, \beta\rangle\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}_\lambda}$.
g. Nothing else is in $U^\tau_{\mathcal{L}_\lambda}$ for any type τ.

We will now extent $[\![\cdot]\!]$ to assign meanings to the new expressions. Models are the same as before, but we speak of the domain of each semantic type, which is recursively defined as in (8). Let $\mathcal{M} = \langle\mathcal{D}, \mathcal{W}, \mathcal{I}\rangle$ be an arbitrary model for $\mathcal{L}$. Recall that $\mathcal{A}$ is the set of all assignments and $\mathcal{C} = \mathcal{W} \times \mathcal{A}$.

(8) For any semantic types σ and τ,
a. $D_e = \mathcal{D}^\mathcal{A}$
b. $D_t = \mathcal{C}^\mathcal{A}$
c. $D_{(\sigma,\tau)} = D_\tau^{D_\sigma}$
d. $D_{\sigma\bullet\tau} = D_\tau \times D_\sigma$

Note that e does not stand for individuals, but dynamic individuals, and t does not stand for truth-values, but functions over (Heimian) contexts (a.k.a. Context Change Potentials).

With these interpretations of types, we define $[\![\cdot]\!]$ as follows. Technically we need assignments to interpret variables: g is an λ-assignment if for any variable of type τ, $g(\tau) \in D_\tau$. We write $g[\xi^\sigma \mapsto s]$ for the λ-assignment that differs from g at most in that $g(\xi^\sigma) = s$.

(9) For any λ-assignment g,
a. For any $\ulcorner\tau\urcorner \in U^e_{\mathcal{L}_\lambda}$, $[\![\tau]\!]^g = [\![\tau]\!]$.
b. For any $\ulcorner\phi\urcorner \in U^t_{\mathcal{L}_\lambda}$, $[\![\phi]\!]^g = [\![\phi]\!]$.
c. For any $\ulcorner\lambda\xi^\sigma.\,\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}_\lambda}$, $[\![\lambda\xi^\sigma.\,\alpha]\!]^g$ is the total function that maps each $s \in D_\sigma$ to $[\![\alpha]\!]^{g[\xi^\sigma \mapsto s]}$.
d. For any $\ulcorner\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}_\lambda}$ and $\ulcorner\beta\urcorner \in U^\sigma_{\mathcal{L}_\lambda}$, $[\![\alpha(\beta)]\!]^g = [\![\alpha]\!]^g([\![\beta]\!]^g)$.
e. For any $\ulcorner\alpha\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}_\lambda}$,
(i) $[\![\pi_1(\beta)]\!]^g = \pi_1([\![\alpha]\!]^g)$ where π_1 is the first projection function; and
(ii) $[\![\pi_2(\beta)]\!]^g = \pi_2([\![\alpha]\!]^g)$ where π_2 is the second projection function.
f. For any $\ulcorner\alpha\urcorner \in U^\sigma_{\mathcal{L}_\lambda}$ and $\ulcorner\beta\urcorner \in U^\tau_{\mathcal{L}_\lambda}$, $[\![\langle\alpha, \beta\rangle]\!]^g = \langle[\![\alpha]\!]^g, [\![\beta]\!]^g\rangle$.

Note that the λ-terms of $\mathcal{L}_\lambda$ are all total functions, while the terms and formulas of $\mathcal{L}$ are all partial functions.

We can now give translations from English to $\mathcal{L}_\lambda$. We define the translation relation $\rightsquigarrow$ which maps each expression of the fragment into an expression of $\mathcal{L}_\lambda$.

(10) **train** $\mapsto \lambda x^e.\ \langle \top, \mathsf{train}\ x^e \rangle$

(11) **arrived** $\mapsto \lambda x^e.\ \langle \top, \mathsf{arrived}\ x^e \rangle$

Note that both the input and output are pairs. The first member of each pair represents the presupposition and the second member represents the assertive meaning. Note also that the inputs to predicates are non-pairs. Presuppositional DPs can be analyzed as quantifiers of type $((e, t \bullet t), t \bullet t)$, but we will not deal with such DPs.

In fact, in this fragment, we want to only consider one-place nouns and verbs that do not have presuppositions, so as to avoid complications that arise from presupposition projection though indefinites, which are orthogonal to our main interest. This simplification allows us to treat the plain indefinite article as a presupposition plug, although that is clearly wrong in the general case.

(12) $\mathbf{a}^n{}_{\mathbf{plain}} \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.\ \langle \top, \exists x_n^e(\pi_2(Q^{(e,t\bullet t)}(x_n^e)) \wedge \pi_2(P^{(e,t\bullet t)}(x_n^e))) \rangle$

The only mode of composition we need for this fragment is Functional Application.

(13) Let $\ulcorner\alpha\urcorner$ be a (binary) branching constituent with $\ulcorner\beta\urcorner$ and $\ulcorner\gamma\urcorner$ as its daughters such that $\beta \mapsto B$ such that $\ulcorner B\urcorner \in U_{\mathcal{L}_\lambda}^{(\sigma,\tau)}$ and $\gamma \mapsto C$ such that $\ulcorner C\urcorner \in U_{\mathcal{L}_\lambda}^{\sigma}$. Then $\alpha \mapsto B(C)$.

For instance, the translation of $\mathbf{a}^n{}_{\mathbf{plain}}$ **train** is equivalent to:

$$\lambda P^{(e,t\bullet t)}.\ \langle \top, \exists x_n^e(\mathsf{train}\ x_n^e \wedge \pi_2(P^{(e,t\bullet t)}(x_n^e))) \rangle$$

The specific version of the indefinite article is given the following denotation, with $\exists$ in the presupposition.

(14) $\mathbf{a}^n{}_{\mathbf{specific}} \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.\ \langle \exists x_n^e\ \pi_2(Q^{(e,t\bullet t)}(x_n^e)), \pi_2(P^{(e,t\bullet t)}(x_n^e))) \rangle$

The translation of $\mathbf{a}^n{}_{\mathbf{specific}}$ **train** is equivalent to:

$$\lambda P^{(e,t\bullet t)}.\ \langle \exists x_n^e\ \mathsf{train}\ x_n^e, P^{(e,t\bullet t)}(x_n^e)) \rangle$$

As promised in the main text, here is the denotation for VP negation:

(15) $\mathbf{not}_{\mathbf{VP}} \mapsto \lambda P^{(e,t\bullet t)} \lambda x^e.\ \langle \pi_1(P^{(e,t\bullet t)}(x^e)), \neg\pi_2(P^{(e,t\bullet t)}(x^e)) \rangle$

Similarly, we can give a material conditional analysis of conditionals as in (16).

(16) **If** $\mapsto \lambda S^{t\bullet t}.\lambda T^{t\bullet t}.\ \langle (\pi_1(S) \wedge (\pi_2(S) \rightarrow \pi_1(T))), (\pi_2(S) \rightarrow \pi_2(T)) \rangle$

A.2 $\mathcal{L}^+$

The intermediate language used in Section 4, $\mathcal{L}^+$, contains plurality.

A.2.1 Syntax

First, let us add the following rules to the syntax with new vocabulary items, namely: $\sqsubseteq$, δ, σ, [, and].

(17) The set $T_{\mathcal{L}^+}$ of all terms, and the set $F_{\mathcal{L}^+}$ of all formulas of $\mathcal{L}^+$ are the smallest sets that satisfy the following.

a. $T_{\mathcal{L}} \subseteq T_{\mathcal{L}^+}$.
b. $F_{\mathcal{L}} \subseteq F_{\mathcal{L}^+}$.
c. If $\ulcorner\tau_1\urcorner$ and $\ulcorner\tau_2\urcorner$ are terms, then $\ulcorner\tau_1 \sqsubseteq \tau_2\urcorner \in F_{\mathcal{L}^+}$.
d. If $\ulcorner\xi\urcorner$ is a variable, $\ulcorner\tau\urcorner \in T_{\mathcal{L}^+}$ and $\ulcorner\phi\urcorner \in F_{\mathcal{L}^+}$, then $\ulcorner\delta^{\xi}_{\tau}[\phi]\urcorner \in F_{\mathcal{L}^+}$.
e. If $\ulcorner\xi\urcorner$ is a variable and $\ulcorner\phi\urcorner \in F_{\mathcal{L}^+}$, then $\ulcorner\sigma\xi[\phi]\urcorner \in T_{\mathcal{L}^+}$.

A.2.2 Semantics

The model is now equipped with a partial order $\sqsubseteq$ among the members of the domain $\mathcal{D}$ such that $(\mathcal{D}, \sqsubseteq)$ is an atomic Boolean algebra. Let $\mathcal{E}$ be the minimal elements of $\mathcal{D}$.

(18) a. We say $x \in \mathcal{D}$ is an *upper bound* of $S \subseteq \mathcal{D}$ iff for each $s \in S$, $s \sqsubseteq x$.
b. For any $S \subseteq \mathcal{D}$, $\sup S$ is the *least upper bound* of S, if it exists.
c. $(\mathcal{D}, \sqsubseteq)$ is an atomic Boolean algebra iff each $x \in \mathcal{D}$ is $\sup S$ for some $S \subseteq \mathcal{E}$.
d. The join-operation corresponding to $\sqsubseteq$ is denoted by $\sqcup$ and defined by: for any $x, y \in \mathcal{D}$, $x \sqsubseteq y$ iff $x \sqcup y$.

In order to enable quantificational subordination, we follow Van den Berg 1996 and make use of sets of assignments. Now a context is a set of pairs each of which consists of a possible world and a set of assignments. Let $\mathcal{C} \coloneqq \mathcal{W} \times \wp(\mathcal{A})$.

For technical reasons, terms are interpreted as functions from $\mathcal{C}$ to $\mathcal{D}$. We adopt the following convention: For any $A \in \wp(\mathcal{A})$, $\xi \in \mathrm{dom}(A)$ iff for each $a \in A$, $\xi \in \mathrm{dom}(a)$ and $A(\xi) \coloneqq \sup\{\, a(x) \mid a \in A \,\}$. Note that assignments in the present system may assign plural individuals, as well as singular individuals (like in Brasoveanu 2007, 2008, 2010b, but unlike in Van den Berg 1996, Brasoveanu 2010a).

Random assignment is redefined as follows: For any $c \in \mathcal{C}$ and for any variable ξ,

$$c[\xi] = \left\{ \langle w, B \rangle \;\middle|\; \begin{array}{l} B \neq \varnothing \text{ and for some } \langle w, A \rangle \in c, \\ \text{for each } a \in A, \text{ for some } b \in B, \\ \quad a \leqslant b \text{ and } \mathrm{dom}(a) \cup \{\, \xi \,\} = \mathrm{dom}(b), \text{ and} \\ \text{for each } b \in B, \text{ for some } a \in A, a \leqslant b \\ \quad \mathrm{dom}(a) \cup \{\, \xi \,\} = \mathrm{dom}(b) \end{array} \right\}$$

(19) a. For any constant $\ulcorner\gamma\urcorner$,

$$[\![c]\!] = \lambda \langle w, A \rangle \in \mathcal{C}.\, I(\gamma)$$

b. For any variable $\ulcorner\xi\urcorner$,

$$[\![\xi]\!] = \lambda \langle w, A \rangle \in \mathcal{C}\colon\ \xi \in \mathrm{dom}(A).\, A(\xi)$$

c. For any function $\ulcorner\Phi\urcorner$ and any $\ulcorner\tau\urcorner \in T_{\mathcal{L}^+}$,

$$[\![\Phi(\tau)]\!] = \lambda \langle w, A \rangle \in \mathcal{C}\colon\ a \in \mathrm{dom}([\![\tau]\!]).\, I(\Phi)([\![\tau]\!](\langle w, A \rangle))$$

d. For any $\ulcorner\phi\urcorner \in F_{\mathcal{L}^+}$ and any variable $\ulcorner\xi\urcorner$,

$$\begin{aligned} [\![\sigma\xi[\phi]]\!] = \lambda \langle w, A \rangle \in \mathcal{C}\colon\ & \{\, \langle w, A \rangle \,\}[\xi] \in \mathrm{dom}([\![\phi]\!]). \\ & \sup\{\, B(\xi) \mid \langle w, B \rangle \in [\![\phi]\!](\{\, \langle w, A \rangle \,\}[\xi]) \,\} \end{aligned}$$

The σ-operator denotes a plural individual corresponding to the sum of all individuals x that satisfy ϕ relative to $\langle w, a\rangle$. In its meaning, ϕ is evaluated against $\{\langle w, A\rangle\}[\xi]$, which is the set of all ξ-variants of $\langle w, a\rangle$. This causes no issue because $\mathcal{L}^+$ (as well as $\mathcal{L}$) is distributive in the sense of Van Benthem 1986, i.e., for each formula $\phi \in F_{\langle \mathcal{L}^+\rangle}$ and for any $c \in \mathcal{C}$, $[\![\phi]\!](c) = \bigcup_{\langle w,A\rangle \in c} [\![\phi]\!](\{\langle w, A\rangle\})$ (proof omitted).

Atomic formulas are interpreted as follows.

(20) a. For any 0-ary predicate $\ulcorner\Pi\urcorner$,

$$[\![\Pi]\!] = \lambda c \in \mathcal{C}.\ \{\langle w, A\rangle \in c \mid \mathcal{I}(\Pi)(w) = 1\}$$

b. For any n-ary predicate $\ulcorner\Pi\urcorner$ and $\ulcorner\tau_1\urcorner, \dots, \ulcorner\tau_n\urcorner \in T_{\mathcal{L}^+}$,

$$[\![\Pi\,\tau_1 \dots \tau_n]\!] = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\tau_1]\!]) \text{ and ... and } c \in \mathrm{dom}([\![\tau_n]\!]).$$
$$\{\langle w, A\rangle \in c \mid \mathcal{I}(\Pi)(w)(\langle [\![\tau_1]\!](\langle w, A\rangle), \dots, [\![\tau_n]\!](\langle w, A\rangle)\rangle) = 1\}$$

c. For any $\ulcorner\tau_1\urcorner, \ulcorner\tau_2\urcorner \in T_{\mathcal{L}^+}$,

$$[\![\tau_1 = \tau_2]\!] = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\tau_1]\!]) \text{ and } c \in \mathrm{dom}([\![\tau_2]\!]).$$
$$\{\langle w, A\rangle \in c \mid [\![\tau_1]\!](\langle w, A\rangle) = [\![\tau_2]\!](\langle w, A\rangle)\}$$

d. For any $\ulcorner\tau_1\urcorner, \ulcorner\tau_2\urcorner \in T_{\mathcal{L}^+}$,

$$[\![\tau_1 \sqsubseteq \tau_2]\!] = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\tau_1]\!]) \text{ and } c \in \mathrm{dom}([\![\tau_2]\!]).$$
$$\{\langle w, A\rangle \in c \mid [\![\tau_1]\!](\langle w, A\rangle) \sqsubseteq [\![\tau_2]\!](\langle w, A\rangle)\}$$

We are mostly interested in distributive predicates, but the system is compatible with collective predication (see Van den Berg 1996, Nouwen 2003 for relevant discussion). Finally, complex formulas are interpreted as follows. Here are some abbreviations:

- $A \leqslant B$ iff for each $a \in A$, there is $b \in B$, $a \leqslant b$ and for each $b \in B$, there is $a \in A$ such that $a \leqslant b$.
- $A{\restriction}_{\xi=d} := \{a \in A \mid a(\xi) = d\}$.

(21) a. $[\![\neg\phi]\!] = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\phi]\!]_{\mathcal{M}}).$
$\{\langle w, A\rangle \in c \mid \text{for no } B \text{ such that } A \leqslant B \text{ and } \langle w, B\rangle \in [\![\phi]\!](c)\}$

b. $[\![(\phi \wedge \psi)]\!] = \lambda c \in \mathcal{C}\colon\ c \in \mathrm{dom}([\![\phi]\!]) \text{ and } [\![\phi]\!](c) \in \mathrm{dom}([\![\psi]\!]).\ [\![\psi]\!]([\![\phi]\!](c))$

c. $[\![\exists\xi\phi]\!] = \lambda c \in \mathcal{C}\colon$ for each $\langle w, A\rangle \in c$, for each $a \in A$, $\xi \notin \mathrm{dom}(a)$ and $c[\xi] \in \mathrm{dom}([\![\phi]\!]).\ [\![\phi]\!](c[\xi])$

d. $[\![\delta^{\xi}_{\tau}[\phi]]\!] = \lambda c \in \mathcal{C}\colon\ \{\langle w, A'{\restriction}_{\xi=d}\rangle \mid \langle w, A'\rangle \in \{\langle w, A\rangle\}[\xi]\} \in \mathrm{dom}([\![\phi]\!]).$

$$\left\{ \langle w, B\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, A\rangle \in c, \\ [\![\tau]\!](\langle w, A\rangle) = B(\xi) \text{ and} \\ \text{for each } d \in \mathcal{E} \text{ such that } d \sqsubseteq [\![\tau]\!](\langle w, A\rangle), \\ \langle w, B{\restriction}_{\xi=d}\rangle \in [\![\phi]\!](\{\langle w, A'{\restriction}_{\xi=d}\rangle \mid \langle w, A'\rangle \in \{\langle w, A\rangle\}[\xi]\}) \end{array} \right\}$$

The δ-operator is the crucial operator that enables quantificational subordination across two formulas each containing δ. As I have just transposed the original definition from Van den Berg 1996 with a fix from Nouwen 2003, and as a number of explanations have been offers by them as well as other works (e.g., Brasoveanu

2007, 2008, 2010b,a), I will not offer a detailed explanation of how quantificational subordination happens with δ.

It should be noted that the delimiters $[\cdot]$, which occur together with σ and δ, mark a domain that allows for rebinding of variables in the present language. Related to this, this may lead to information loss, or downdate. As Nouwen 2003, 2007 discusses in detail, it causes issues in certain cases. He proposes a stack-based system which circumvents this issue. In the present formulation $\mathcal{L}^+$ inherits this issue, but it could in theory be reformulated using Nouwen's stack-based dynamic semantics, although that is left for another occasion.

A.2.3 A compositional translation from a fragment of English to $\mathcal{L}^+$

As in the case of $\mathcal{L}$, we will build a translation system from a fragment of English to $\mathcal{L}^+$. As before, we start by enriching the language with the λ-operator, and call the resulting language $\mathcal{L}^+_\lambda$. The way this is done is identical to what we did above with $\mathcal{L}$.

(22) Semantic types

a. e and t are semantic types.

b. If σ and τ are semantic types, then (σ, τ) is a semantic type.

(23) a. $U^e_{\mathcal{L}^+_\lambda} = T_\mathcal{L}$.

b. $U^t_{\mathcal{L}^+_\lambda} = F_\mathcal{L}$.

c. If $\ulcorner\alpha\urcorner \in U^\tau_{\mathcal{L}^+_\lambda}$ and $\ulcorner\xi^\sigma\urcorner$ is a variable of type σ, then $\ulcorner\lambda\xi^\sigma.\, \alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}^+_\lambda}$.

d. If $\ulcorner\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}^+_\lambda}$ and $\ulcorner\beta\urcorner \in U^\sigma_{\mathcal{L}^+_\lambda}$, then $\ulcorner\alpha(\beta)\urcorner \in U^\tau_{\mathcal{L}^+_\lambda}$.

e. If $\ulcorner\alpha\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}^+_\lambda}$, then $\ulcorner\pi_1(\alpha)\urcorner \in U^\sigma_{\mathcal{L}^+_\lambda}$ and $\ulcorner\pi_2(\alpha)\urcorner \in U^\tau_{\mathcal{L}^+_\lambda}$.

f. If $\ulcorner\alpha\urcorner \in U^\sigma_{\mathcal{L}^+_\lambda}$ and $\ulcorner\beta\urcorner \in U^\tau_{\mathcal{L}^+_\lambda}$, then $\ulcorner\langle\alpha,\beta\rangle\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}^+_\lambda}$.

g. Nothing else is in $U^\tau_{\mathcal{L}^+_\lambda}$ for any type τ.

The domains of the types are the same as before, except that $\mathcal{C} = \mathcal{W} \times \wp(\mathcal{A})$.

$[\![\cdot]\!]$ works in the same way as well.

(24) a. For any $\ulcorner\tau\urcorner \in U^e_{\mathcal{L}^+_\lambda}$, $[\![\tau]\!]^g = [\![\tau]\!]$.

b. For any $\ulcorner\phi\urcorner \in U^t_{\mathcal{L}^+_\lambda}$, $[\![\phi]\!]^g = [\![\phi]\!]$.

c. For any $\ulcorner\lambda\xi^\sigma.\, \alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}^+_\lambda}$, $[\![\lambda\xi^\sigma.\, \alpha]\!]^g$ is the total function that maps each $s \in D_\sigma$ to $[\![\alpha]\!]^{g[\xi^\sigma \mapsto s]}$.

d. For any $\ulcorner\alpha\urcorner \in U^{(\sigma,\tau)}_{\mathcal{L}^+_\lambda}$ and $\ulcorner\beta\urcorner \in U^\sigma_{\mathcal{L}^+_\lambda}$, $[\![\alpha(\beta)]\!]^g = [\![\alpha]\!]^g([\![\beta]\!]^g)$.

e. For any $\ulcorner\alpha\urcorner \in U^{\sigma\bullet\tau}_{\mathcal{L}^+_\lambda}$,

(i) $[\![\pi_1(\alpha)]\!]^g = \pi_1([\![\alpha]\!]^g)$ where π_1 is the first projection function; and

(ii) $[\![\pi_2(\alpha)]\!]^g = \pi_2([\![\alpha]\!]^g)$ where π_2 is the second projection function.

f. For any $\ulcorner\alpha\urcorner \in U^\sigma_{\mathcal{L}^+_\lambda}$ and $\ulcorner\beta\urcorner \in U^\tau_{\mathcal{L}^+_\lambda}$, $[\![\langle\alpha,\beta\rangle]\!]^g = \langle[\![\alpha]\!]^g, [\![\beta]\!]^g\rangle$.

Here are the translations of some example nouns and verbs, some of which have non-trivial presuppositions.

(25) a. **cat** $\mapsto \lambda x^e.\, \langle\top, \mathsf{cat}\, x^e\rangle$

b. **mouse** $\mapsto \lambda x^e.\, \langle\top, \mathsf{mouse}\, x^e\rangle$

(26) a. **arrived** $\mapsto \lambda x^e.\ \langle \top, \textsf{arrived}\ x^e \rangle$
b. **woke up** $\mapsto \lambda x^e.\ \langle \textsf{asleep}\ x^e, \textsf{woke.up}\ x^e \rangle$
c. **saw** $\mapsto \lambda x^e.\lambda y^e.\ \langle \top, \textsf{saw}\ x^e y^e \rangle$

As discussed in the main text, the problem of presupposition projection through (weak) existential quantifiers remains unsolved. Just for the sake of concreteness, let us simply assume existential projection. Note that this runs into the infamous binding problem (see Karttunen & Peters 1979, Beaver 2001, Sudo 2012, among others). To prevent the presupposition from having an anaphoric content, we make use of double negation.

(27) $\mathbf{a}^n{}_{\textbf{weak.plain}} \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.$
$$\left\langle \begin{array}{l} \neg\neg\exists x^e_n(\pi_1(Q^{(e,t\bullet t)}(x^e_n)) \wedge \pi_2(Q^{(e,t\bullet t)}(x^e_n)) \wedge (\pi_2(Q^{(e,t\bullet t)}(x^e_n)) \wedge \pi_1(P^{(e,t\bullet t)}(x^e_n)))), \\ \exists x^e_n(\pi_2(Q^{(e,t\bullet t)}(x^e_n)) \wedge \pi_2(P^{(e,t\bullet t)}(x^e_n))) \end{array} \right\rangle$$

For the strong uses of the indefinite determiner, we assume universal projection, as explained in Section 4.6 of the main text.

(28) $\mathbf{a}^{n,m}{}_{\textbf{strong.plain}} \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.$
$$\left\langle \begin{array}{l} \exists x^e_n((\pi_1(Q^{(e,t\bullet t)}(x^e_n)) \wedge x_n = \sigma x_n[\pi_2(Q^{(e,t\bullet t)}(x^e_n))]) \wedge \pi_1(P^{(e,t\bullet t)}(x^e_n))), \\ \exists x^e_m(x^e_m \sqsubseteq x^e_n \wedge \pi_2(Q^{(e,t\bullet t)}(x^e_n)) \wedge \pi_2(P^{(e,t\bullet t)}(x^e_m))) \end{array} \right\rangle$$

(29) $\mathbf{a}^{n,m}{}_{\textbf{strong.specific}} \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.$
$$\left\langle \begin{array}{l} (\exists x^e_n((\pi_1(Q^{(e,t\bullet t)}(x^e_n)) \wedge x_n = \sigma x_n[\pi_2(Q^{(e,t\bullet t)}(x^e_n))]) \wedge \pi_1(P^{(e,t\bullet t)}(x^e_n))) \\ \wedge\exists x^e_m(x^e_m \sqsubseteq x^e_n \wedge \pi_2(Q^{(e,t\bullet t)}(x^e_n)))), \\ \pi_2(P^{(e,t\bullet t)}(x^e_m))) \end{array} \right\rangle$$

Universal quantifier is analyzed as in (30).

(30) $\mathbf{every}^n \mapsto \lambda Q^{(e,t\bullet t)}.\lambda P^{(e,t\bullet t)}.$
$$\left\langle \begin{array}{l} \exists x^e_n((\pi_1(Q^{(e,t\bullet t)}(x^e_n)) \wedge x_n = \sigma x_n[\pi_2(Q^{(e,t\bullet t)}(x^e_n))]) \wedge \pi_1(P^{(e,t\bullet t)}(x^e_n))), \\ \delta^{x^e_n}_{x^e_n}[\pi_2(P^{(e,t\bullet t)}(x^e_n))] \end{array} \right\rangle$$

We assume the primary compositional rule is Functional Application, as introduced above. For instance, the translation of $\mathbf{every}^n$ **cat** is equivalent to:

$$\lambda P^{(e,t\bullet t)}.\ \left\langle \exists x^e_n(x_n = \sigma x^e_n[\textsf{cat}\ x^e_n] \wedge \pi_1(P^{(e,t\bullet t)}(x^e_n))), \delta^{x^e_n}_{x^e_n}[\pi_2(P^{(e,t\bullet t)}(x^e_n))] \right\rangle$$

Let us combining this with a VP containing a specific indefinite, e.g., **saw** $\mathbf{a}^{m,l}{}_{\textbf{strong.specific}}$. In order to combine a transitive verb and a quantifier as its object, something must be done. For now, let us adopt a tentative solution, which is to introduce a special compositional rule.

(31) Let $\ulcorner\alpha\urcorner$ be a (binary) branching constituent with $\ulcorner\beta\urcorner$ and $\ulcorner\gamma\urcorner$ as its daughter constituents such that $\beta \mapsto B$ such that $\ulcorner B\urcorner \in U^{(e,(e,t\bullet t))}_{\mathcal{L}^+_\lambda}$ and $\gamma \mapsto C$ such that $\ulcorner C\urcorner \in U^{((e,t\bullet t),t\bullet t)}_{\mathcal{L}^+_\lambda}$. Then $\alpha \mapsto \lambda y^e.C(\lambda x^e.\ B(x^e)(y^e))$.

Then the translation of **saw** $\mathbf{a}^{m,l}{}_{\textbf{strong.specific}}$ would be equivalent to:

$$\lambda y^e.\ \langle \exists x^e_m(x^e_m = \sigma x^e_m[\textsf{mouse}\ x^e_m] \wedge \exists x^e_l\ x^e_l \sqsubseteq x^e_m), \textsf{saw}\ x^e_l y^e \rangle$$

Combining this with **every**^{n} **cat**, we get:

$$\left\langle \begin{array}{l} \exists x^e_n(x^e_n = \sigma x^e_n[\mathsf{cat}\, x^e_n] \wedge \exists x^e_m(x^e_m = \sigma x^e_m[\mathsf{mouse}\, x^e_m] \wedge \exists x^e_l\, x^e_l \sqsubseteq x^e_m)), \\ \delta^{x^e_n}_{x^e_n}[\pi_2(P^{(e,t\bullet t)}(\mathsf{saw}\, x^e_l x^e_n))] \end{array} \right\rangle$$

This represents the wide scope reading. This is thanks to the fact that the presupposition of **saw a**^{m,l}**strong.specific** does not mention the subject variable y^e.

Recall that we postulate the presuppositional distributivity operator Δ_P. This is given the following translation.

(32) $\Delta_P \mapsto \lambda P^{(e,t\bullet t)}.\lambda x^e.\ \langle \delta^{x^e}_{x^e}[\pi_1(P^{(e,t\bullet t)}(x^e)], \pi_2(P^{(e,t\bullet t)}(x^e)\rangle$

With this operator applied to the VP before the subject combines, we will derive the following denotation.

$$\left\langle \begin{array}{l} \exists x^e_n(x^e_n = \sigma x^e_n[\mathsf{cat}\, x^e_n] \wedge \delta^{x^e_n}_{x^e_n}(\exists x^e_m(x^e_m = \sigma x^e_m[\mathsf{mouse}\, x^e_m] \wedge \exists x^e_l\, x^e_l \sqsubseteq x^e_m))), \\ \delta^{x^e_n}_{x^e_n}[\pi_2(P^{(e,t\bullet t)}(\mathsf{saw}\, x^e_l x^e_n))] \end{array} \right\rangle$$

Thanks to modal subordination mediated by δ, dynamic binding of x^e_l in the assertive meaning succeeds, and the resulting meaning is a narrow scope reading for the specific indefinite.

The assertive distributivity operator Δ_A is just like Δ_P, except that it introduces δ in the assertive meaning.

(33) $\Delta_A \mapsto \lambda P^{(e,t\bullet t)}.\lambda x^e.\ \langle \pi_1(P^{(e,t\bullet t)}(x^e), \delta^{x^e}_{x^e}[\pi_2(P^{(e,t\bullet t)}(x^e)]\rangle$

A.3 $\mathcal{L}^+_f$

Finally, we will add functional variables $(f_1, f_2, \dots)$.

(34) The set $T_{\mathcal{L}^+_f}$ of all terms, and the set $F_{\mathcal{L}^+_f}$ of all formulas of $\mathcal{L}^+_f$ are the smallest sets that satisfy the following.

a. $T_{\mathcal{L}^+} \subseteq T_{\mathcal{L}^+_f}$.
b. $F_{\mathcal{L}^+} \subseteq F_{\mathcal{L}^+_f}$.
c. If $\ulcorner\Phi\urcorner$ is a functional variable and $\ulcorner\tau\urcorner \in T_{\mathcal{L}^+_f}$, then $\ulcorner\Phi(\tau)\urcorner \in T_{\mathcal{L}^+_f}$.
d. If $\ulcorner\Phi\urcorner$ is a functional variable and $\ulcorner\psi\urcorner \in F_{\mathcal{L}^+_f}$, then $\ulcorner\exists\Phi\psi\urcorner \in F_{\mathcal{L}^+_f}$.

One crucial change in the semantics is that each assignment now maps variables $(x_1, x_2, \dots)$ to members of D_e and functional variables $(f_1, f_2, \dots)$ to members of $D_{(e,e)}(= D_e^{D_e})$. Since we are only interested in natural functions, we could require that the range of a given assignment is always a natural function, but since we do not have a general characterization of natural functions, we cannot state this restriction explicitly here.

The additional expressions are interpreted as follows. The other expressions are interpreted in the same way as in $\mathcal{L}^+$.

(35) a. For any functional variable $\ulcorner\Phi\ulcorner$ and any $\ulcorner\tau\urcorner \in T_{\mathcal{L}^+_f}$,

$$[\![\Phi(\tau)]\!] = \lambda\langle w, A\rangle \in \mathcal{C}\colon\ \Phi \in \mathrm{dom}(A).\ A(\Phi)([\![\tau]\!](\langle w, A\rangle))$$

b. For any functional variable ⌜Φ⌝ and any ⌜ψ⌝ $\in F_{\mathcal{L}_f^+}$,

$$[\![\exists\Phi\psi]\!] = \lambda c \in \mathcal{C}\colon \text{ for each } \langle w, A\rangle \in c, \text{ for each } a \in A, \Phi \notin \text{dom}(a) \text{ and } c[\Phi] \in \text{dom}([\![\phi]\!]).\ [\![\phi]\!](c[\Phi])$$

Recall at this point the issue of Δ_P interacting with the existential quantification over natural functions in the dynamic presupposition of *certain* discussed at the end of Section 5. This issue resolves by adopting the following definition of δ, which differs from the earlier definition in having the final line that requires that B assigns at most one value to each functional variable.

(36) $$\left[\!\left[\delta^{\xi}_{\tau}[\phi]\right]\!\right] = \lambda c \in \mathcal{C}\colon\ \{\langle w, A'\restriction_{\xi=d}\rangle \mid \langle w, A'\rangle \in \{\langle w, A\rangle\}[\xi]\} \in \text{dom}([\![\phi]\!]).$$
$$\left\{ \langle w, B\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, A\rangle \in c, \\ [\![\tau]\!](\langle w, A\rangle) = B(\xi) \text{ and} \\ \text{for each } d \in \mathcal{E} \text{ such that } d \sqsubseteq [\![\tau]\!](\langle w, A\rangle), \\ \langle w, B\restriction_{\xi=d}\rangle \in [\![\phi]\!](\{\langle w, A'\restriction_{\xi=d}\rangle \mid \langle w, A'\rangle \in \{\langle w, A\rangle\}[\xi]\}) \\ \text{for each functional variable } \ulcorner\Phi\urcorner \in \text{dom}(B), |B(\Phi)| = 1 \end{array} \right\}$$

This is rather uninsightful, but it has the effect of effectively scoping out any existential quantification over natural function over the distributivity operator.

A compositional translation can easily be constructed by extending $\mathcal{L}_\lambda^+$, so I will omit it here. In it the translation of *certain* will look as follows. (⌜$\mathcal{NF}\ f_n$⌝ is dispensable if we assume that functional variables always denote natural functions)

(37) $$\text{certain}^n_m \mapsto \lambda P^{(e,t\bullet t)}.\lambda x^e.$$
$$\langle(\exists f_n\, \text{NF}\, f_n \wedge \pi_1(P^{(e,t\bullet t)}(x^e))), (f_n(x_m) = x^e \wedge \pi_2(P^{(e,t\bullet t)}(x^e)))\rangle$$

Note that we cannot combine this with the weak plain indefinite article analyzed above, which is a presupposition plug and would disrupt the crossdimensional anaphora for f_n. However, as remarked above, this analysis of the weak plain indefinite article is simply problematic in several respects, and for reasons mentioned above, I cannot offer a more adequate analysis here.

B Comparison with Van Geenhoven 1998

In this appendix, we will review a previous presuppositional theory of specific indefinites proposed by Van Geenhoven 1998: Ch. 6, and compare it to the theory proposed in the main text. A crucial difference between these two theories lies in which theory of presupposition the idea is fleshed out. While the present paper adopts the Heim-Stalnaker view of presuppositions, Van Geenhoven 1998 uses the presupposition-as-anaphora theory (Van der Sandt 1992, Geurts 1999), which is standardly implemented in Discourse Representation Theory (DRT), so we will start by reviewing DRT first.

B.1 A DRT primer

DRT is a formal language for representing discourse. The original version was developed for pronominal anaphora, but has been extended to incorporate other aspects of information exchange (see Kamp 1981, Kamp & Reyle 1993, Geurts, Beaver & Maier 2020). Presupposition is one such extension.

B.1.1 Basics

Let us start with the simplest, first-order DRT for pronominal anaphora.

The main building blocks of DRT are *Discourse Representation Structures (DRSs)*, which represent discourses as well as sentence meanings.

(38) A DRS is a pair $\langle V, C \rangle$ such that:

a. V is a (possibly empty) set of variables; and
b. C is a set of DRS-conditions.

We adopt the linear notation (instead of boxes). DRS $\langle \{ x_1, \ldots, x_n \}, \{ c_1, \ldots, c_m \} \rangle$ is represented as $[x_1, \ldots, x_n \mid c_1, \ldots, c_n]$.

DRS-conditions are defined as:

(39) a. If P is an n-ary predicate and $x_1, \ldots x_n$ are all variables, then $P(x_1, \ldots, x_n)$ is a DRS-condition.
b. If x and y are variables, then $x = y$ is a DRS-condition.
c. If ϕ and ψ are DRSs, then $\neg\phi$, $\phi \vee \psi$, $\phi \Rightarrow \psi$, are DRS-conditions.
d. If ϕ and ψ are DRSs and ξ is a variable, then $\phi \langle \forall \xi \rangle \psi$ is a DRS-condition.
e. Nothing else is a DRS-condition.

It is assumed that a given pronoun needs to find a suitable antecedent in some DRS accessible to it. Anaphoric accessibility is defined in terms of a syntactic relation between DRSs:

(40) $\geq$ ('accessible to') is the smallest preorder (reflexive and transitive order) over DRSs such that for any DRSs $\phi = \langle V_\phi, C_\phi \rangle$, $\psi = \langle V_\psi, C_\psi \rangle$ and $\chi = \langle V_\chi, C_\chi \rangle$,

a. if $\neg\psi \in C_\phi$, then $\phi \geq \psi$;
b. if $(\psi \vee \chi) \in C_\phi$, then $\phi \geq \psi$ and $\phi \geq \chi$;
c. if $\psi \Rightarrow \chi \in C_\phi$, then $\phi \geq \psi \geq \chi$; and
d. if $\psi \langle \forall \xi \rangle \chi \in C_\phi$, then $\phi \geq \psi \geq \chi$.

Using $\geq$, we define the set of variables that are accessible from a given DRS ϕ:

(41) $\mathrm{Acc}(\phi) = \bigcup \{ V_\psi \mid \psi \geq \phi \}$

A DRS is said to be proper if all the pronouns in it are bound. Formally:

(42) The set of employed variables in DRS $\phi = \langle V, C \rangle$, $\mathrm{Emp}(\phi)$ is the smallest set such that:

a. if $P(x_1, \ldots, x_n) \in C$, then $x_1, \ldots, x_n \in \mathrm{Emp}(\phi)$; and
b. if $x = y \in C$, then $x, y \in \mathrm{Emp}(\phi)$.

(43) A DRS ϕ is proper iff for each ψ such that $\phi \geq \psi$, $\mathrm{Emp}(\psi) \subseteq \mathrm{Acc}(\psi)$.

For example, the following DRSs are proper.

(44) a. There is a^x cat. It$_x$ is not chasing a^y mouse. It$_x$ is sleeping.
b. $$\left[x \;\middle|\; \begin{array}{l} \text{cat}(x), \\ \neg \left[y \;\middle|\; \begin{array}{l} \text{mouse}(y), \\ \text{chase}(x, y) \end{array} \right], \text{sleeping}(x) \end{array} \right]$$

(45) a. If a^x farmer owns a^y donkey, he$_x$ vaccinates it$_y$.

b. $\left[\;\middle|\;\left[x, y \;\middle|\; \begin{array}{l}\text{farmer}(x),\\ \text{donkey}(y),\\ \text{own}(x, y)\end{array}\right] \Rightarrow [\,|\,\text{vaccinate}(x, y)]\right]$

(46) a. Every^x farmer that owns a^y donkey vaccinates it$_y$.

b. $\left[\;\middle|\;\left[y \;\middle|\; \begin{array}{l}\text{donkey}(y),\\ \text{own}(x, y)\end{array}\right] \langle\forall x\rangle\, [\,|\,\text{vaccinate}(x, y)]\right]$

On the other hand, the following DRSs are not proper.

(47) a. There isn't a^x cat. It$_x$ is sleeping.

b. $[\,|\,\neg[x\,|\,\text{cat}(x)], \text{sleeping}(x)]$

(48) a. Every^x farmer that owns it$_y$ vaccinates a^y donkey.

b. $\left[\;\middle|\;[\,|\,\text{own}(x, y)]\, \langle\forall x\rangle \left[y \;\middle|\; \begin{array}{l}\text{donkey}(y),\\ \text{vaccinate}(x, y)\end{array}\right]\right]$

Improper DRSs contain free pronouns that are yet to be resolved. They may become proper when embedded in larger DRSs.

A DRS is evaluated with respect to a first-order model $M = \langle D, I\rangle$ and an embedding (= assignment function) f, which is a partial function from variables to D.

(49) For any DRS $\phi = \langle V, C\rangle$, any model $M = \langle D, I\rangle$, and any embedding f, $[\![\phi]\!]^f_M = 1$ iff $V \subseteq \text{dom}(f)$ and for each $\psi \in C$, $[\![\psi]\!]^f_M = 1$.

DRS-conditions are interpreted as follows:

$$\begin{aligned}
[\![P(x_1, \ldots, x_n)]\!]^f_M &= 1 \text{ iff } \langle f(x_1), \ldots, f(x_n)\rangle \in I(P)\\
[\![x = y]\!]^f_M &= 1 \text{ iff } f(x) = f(y)\\
[\![\neg\phi]\!]^f_M &= 1 \text{ iff for no } g \geqslant f, [\![\phi]\!]^g_M = 1\\
[\![\phi \vee \psi]\!]^f_M &= 1 \text{ iff for some } g \geqslant f, [\![\phi]\!]^g_M \text{ or } [\![\psi]\!]^g_M = 1\\
[\![\phi \Rightarrow \psi]\!]^f_M &= 1 \text{ iff for each } g \geqslant f, \text{ such that } [\![\phi]\!]^g_M = 1, \text{ for some } h \geqslant g, [\![\psi]\!]^h_M = 1\\
[\![\phi\, \langle\forall x\rangle\, \psi]\!]^f_M &= 1 \text{ iff for each } a \in D \text{ and for each } g \geqslant f[x \mapsto a] \text{ such that } [\![\phi]\!]^g_M = 1,\\
&\qquad \text{for some } h \geqslant g, [\![\psi]\!]^h_M = 1
\end{aligned}$$

(50) a. g is an extension of f ($g \geqslant f$) iff for each $x \in \text{dom}(f)$, $f(x) = g(x)$.

b. $f[x \mapsto a]$ is the unique embedding such that

(i) for each $y \in \text{dom}(f)$, $f(y) = g(y)$,

(ii) $g(x) = a$,

(iii) $\text{dom}(g) = \text{dom}(f) \cup \{x\}$.

(51) DRS ϕ is true with respect to M iff for some embedding f, $[\![\phi]\!]^f_M = 1$.

Existential Closure is part of the definition of DRS-truth, (51) and the meanings of the connectives: In $[x_1, \ldots, x_n \mid \ldots]$, the variables $x_1, \ldots, x_n$ are interpreted existentially.

Groenendijk & Stokhof 1991 give a proof for the equivalence between first-order Dynamic Predicate Logic and first-order DRT. See Muskens 1996 for a compositional translation from a fragment of English into DRT (see also Brasoveanu 2007).

B.1.2 Two-stage model

A two-stage interpretive model is often assumed, especially in the literature on presuppositions, so let us introduce it.

1. A sentence/utterance is translated into a DRS. Variables that need to be anaphorically resolved are underlined.
2. This DRS is merged with the DRS that represents the conversational background of the utterance, and the anaphoric dependencies are resolved with respect to it.

To illustrate, in (52) is the translation of a simple sentence with a free pronoun.

(52) It$_y$ is sleeping. $\rightsquigarrow$ $\left[\underline{y} \,\middle|\, \text{sleeping}(y)\right]$

Suppose that this sentence was uttered against the background context represented by the DRS in (53).

(53) $[x, a \mid \text{Alice}(a), \text{cat}(x), \text{owns}(a, x), \text{fat}(x), \neg[\ \mid \text{young}(x)]]$

Then (52) gets merged with this DRS, yielding:

(54) $\left[x, a, \underline{y} \,\middle|\, \text{Alice}(a), \text{cat}(x), \text{owns}(a, x), \text{fat}(x), \neg[\ \mid \text{young}(x)], \text{sleeping}(y)\right]$

Formally, DRS merger is point-wise union:

(55) For any DRSs $\phi = \langle V_\phi, C_\phi \rangle$ and $\psi = \langle V_\psi, C_\psi \rangle$,
$\phi \sqcup \psi = \langle V_\phi \cup V_\psi,\ \ C_\phi \cup C_\psi \rangle$

Finally, the anaphoric dependency is resolved by syntactically transforming (54) by inserting an equality statement (e.g., $x = y$ in this case):

(56) $[x, a, y \mid \text{Alice}(a), \text{cat}(x), \text{owns}(a, x), \text{fat}(x), \neg[\ \mid \text{young}(x)], \text{sleeping}(y), x = y]$

This is semantically equivalent to:

(57) $[x, a \mid \text{Alice}(a), \text{cat}(x), \text{owns}(a, x), \text{fat}(x), \neg[\ \mid \text{young}(x)], \text{sleeping}(x)]$

In DRT, anaphora resolution is a syntactic transformation, which follows the following two steps, for each underlined element x in ϕ: (a) insert an equality statement $x = y$ for some $y \in \text{Acc}(\phi)$ and (b) remove the underline.

B.2 Presuppositions as anaphora

Van der Sandt 1992 proposes that presuppositions can be seen as propositional anaphora (an earlier implementation of this idea was proposed in Van der Sandt 1988), and offers a formalization of the theory in DRT (also see Geurts 1999, as well as discussion on the view that reduces pronominal anaphora to presupposition, rather than the other way around). This view is distinct from the Heim-Stalnaker view of presuppositions, where presuppositions are pre-conditions for successful dynamic update. Since Van Geenhoven 1998 adopts the presupposition-as-anaphora view, we will review it here in detail

B.2.1 Presupposition binding

The core idea of Van der Sandt 1992 is to treat presuppositions as 'propositional anaphors', formalized as DRSs that need to be resolved. Let us start with a somewhat sloppier version for illustrative purposes (see Section B.2.5 for a more formally precise explanation).

Presuppositions are represented as underlined DRSs. For example, the existence presupposition of a definite article is analyzed as in (58) (we ignore the uniqueness presupposition for the sake of simplicity).

(58) The cat is sleeping. $\rightsquigarrow \left[\ \middle|\ \underline{[x \mid \text{cat}(x)]}, \text{sleeping}(x)\right]$

Suppose that the background context looks like:

(59) $[a, y, z \mid \text{Alice}(a), \text{cat}(y), \text{dog}(z), \text{owns}(a, y), \text{owns}(a, z), \text{outside}(z)]$

First, merge the two DRSs:

(60) $$\left[a, y, z \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{cat}(y), \text{dog}(z), \text{owns}(a, y), \text{owns}(a, z), \text{outside}(z), \\ \underline{[x \mid \text{cat}(x)]}, \text{sleeping}(x), \end{array}\right]$$

Then, we can resolve the presupposition by 'binding' the DRS to the main DRS. This amounts to inserting $x = y$ and merging the presuppositional DRS and the main DRS:

(61) $$\left[a, x, y, z \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{cat}(y), \text{dog}(z), \text{owns}(a, y), \text{owns}(a, z), \text{outside}(z), \\ \text{cat}(x), \text{sleeping}(x), x = y \end{array}\right]$$

(61) is equivalent to (62), and this represents the state of the discourse after the utterance + presupposition resolution.

(62) $$\left[a, y, z \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{cat}(y), \text{dog}(z), \text{owns}(a, y), \text{owns}(a, z), \text{outside}(z), \\ \text{sleeping}(y) \end{array}\right]$$

Presupposition can be bound to a proposition in a different but accessible DRS.

(63) If there is a cat and a dog, then the cat is upstairs.
$$\rightsquigarrow \left[\ \middle|\ [y, z \mid \text{cat}(y), \text{dog}(z)] \Rightarrow \left[\ \middle|\ \underline{[x \mid \text{cat}(x)]}, \text{upstairs}(x)\right]\right]$$

No matter what the background discourse, the presupposition can be bound to the conditional antecedent DRS in this example. Without loss of generality, let's consider an empty background. Then, after presupposition binding:

(64) $[\ \mid [y, z, x \mid \text{cat}(y), \text{dog}(z), \text{cat}(x), x = y] \Rightarrow [\ \mid \text{upstairs}(x)]]$

And this can be simplified to:

(65) $[\ \mid [y, z \mid \text{cat}(y), \text{dog}(z)] \Rightarrow [\ \mid \text{upstairs}(y)]]$

Thus, the presupposition is resolved within the sentence, and as a consequence, (63) feels like there's no presupposition overall.

B.2.2 Presupposition accommodation

There's one important difference between pronominal anaphora and presuppositions: In case there's no suitable antecedent, pronominal anaphora is simply infelicitous, while presuppositions can be *accommodated*, at least in some cases.

Van der Sandt 1992 and Geurts 1999 consider this difference to be a matter of degree: Pronouns carry very little descriptive information, so it's harder to guess who/what the intended referents are. Unlike pronouns, 'heavy' definite descriptions *the guy sitting next to me on the tube* and demonstratives like *that guy* carry more clues as to the identity of the intended referent, so easier to accommodate. Lighter definite descriptions like *the tall guy* lie somewhere in between.

The presupposition-as-anaphora theory analyses presupposition accommodation as simple merger of a presuppositional DRS with an accessible DRS. For example, if (66) is uttered out of the blue:

(66) a. The king of Bhutan is young.
b. $\left[\ \middle|\ \underline{[x \mid \text{KoB}(x)]}, \text{young}(x)\right]$

When (66) is merged with the background DRS and the presupposition is accommoted, we will obtain the following (if there's no inconsistency):

(67) $[\ldots, x \mid \ldots, \text{KoB}(x), \text{young}(x)]$

Formally, the difference between binding and accommodation is that the former involves insertion of an equality statement $x = y$, the latter does not.

It is assumed that binding is preferred to accommodation, whenever possible. See Van der Sandt 1992, Geurts 1999, Beaver & Zeevat 2007 for more discussion on this.

B.2.3 Local accommodation

Accommodation can target non-global DRSs as well.

(68) It didn't start raining. It never rained!

The first sentence of (68) is interpreted as:

(69) $\left[\ \middle|\ \neg\left[\ \middle|\ \underline{[\ \mid \text{not\_raining\_before}]}, \text{raining\_now}\right]\right]$

If we accommodate the presupposition globally, we'll get (70), but this won't be consistent with the second sentence.

(70) $[\ \mid [\ \mid \text{not\_raining\_before}], \neg[\ \mid \text{raining\_now}]]$

To obtain the coherent interpretation of the discourse, the presupposition needs to be accommodated under negation, as in (71).

(71) $[\ \mid \neg[\ \mid \text{not\_raining\_before}, \text{raining\_now}]]$

B.2.4 Trapping

Trapping is a hard constraint on accommodation (and binding) in this theory. To illustrate, consider the following example.

(72) Every cat looks down on its owner.

This is interpreted as the DRS in (73) (we will omit the non-human presupposition of *it*).

(73) $\left[\ \middle|\ [\ \mid \mathrm{cat}(x)\,]\,\langle\forall x\rangle \left[\ \middle|\ \underline{[y \mid \mathrm{owner\_of}(y,z)\,]}, \mathrm{look\_down}(x,y)\right]\right]$

After resolving the pronoun to x, we obtain (74).

(74) $\left[\ \middle|\ [\ \mid \mathrm{cat}(x)\,]\,\langle\forall x\rangle \left[\ \middle|\ \underline{[y \mid \mathrm{owner\_of}(y,x)\,]}, \mathrm{look\_down}(x,y)\right]\right]$

The possessive presupposition cannot be accommodated globally, because that'll leave x unbound, making the DRS improper.

(75) $\left[y \,\middle|\, \begin{array}{l} \mathrm{owner\_of}(y,x), \\ [\ \mid \mathrm{cat}(x)\,]\,\langle\forall x\rangle\,[\ \mid \mathrm{look\_down}(x,y)\,] \end{array} \right]$

The most natural interpretation is the local accommodation reading (we will discuss below another theoretical option, intermediate accommodation).

(76) $[\ \mid [\ \mid \mathrm{cat}(x)\,]\,\langle\forall x\rangle\,[y \mid \mathrm{owner\_of}(y,x), \mathrm{look\_down}(x,y)\,]\,]$

In a case like this, we say the presupposition is trapped by the quantifier (due to the variable it binds in the presupposition).

B.2.5 Formal details of presupposition binding and accommodation

Here are some more formal details from Van der Sandt 1992. DRSs are redefined as triples, $\langle V, C, A\rangle$, rather than pairs, in order to incorporate presuppositions.

- V is a set of variables.
- C is a set of DRS-conditions.
- A is a set of DRSs, representing presuppositions.

Presupposition binding amounts to merging the DRSs in A to an accessible DRS. Formally, it happens as follows.

Let $K = \langle V_K, C_K, A_K\rangle$ be a DRS with non-empty A_K.

1. Find a DRS $\phi = \langle V_\phi, C_\phi, A_\phi\rangle$ in A_K such that $A_\phi = \varnothing$.
2. Take some $\psi = \langle V_\psi, C_\psi, A_\psi\rangle$ such that $\psi \geq K$.
3. Binding ϕ to ψ amounts to the following transformation:
    - Let f be a function from V_ϕ to $\mathrm{Acc}(\psi)$.
    - Delete ϕ.

- Change ψ to $\langle V_\phi \cup V_\psi,\ C_\phi \cup C_\psi \cup \{\, x = f(x) \mid x \in V_\phi \,\},\ A_\psi \rangle$.

Accommodating ϕ to ψ amounts to the following transformation:

- Let f be a function from V_ϕ to $\mathrm{Acc}(\psi)$.
- Delete ϕ.
- Change ψ to $\langle V_\phi \cup V_\psi,\ C_\phi \cup C_\psi,\ A_\psi \rangle$.

4. Call the resulting DRS K and repeat, unless A_K is empty.

When there are multiple presuppositional DRSs, it makes sense to work on them top-down, but see Geurts 1999: pp. 54–55 for an alternative idea.

B.3 Exceptional wide scope via presupposition projection

Van Geenhoven 1998 uses the above framework of presuppositions to account for specific indefinites. The idea is that indefinites can be presuppositional, and when they are presuppositional, they project like other presuppositions, and give rise to exceptional wide scope.

In addition to this idea for specific indefinites, Van Geenhoven 1998 also proposes that non-presuppositional indefinites are all 'incorporated', but this part of her theory is independent from her analysis of specific indefinites, so we will ignore it below and simply assume that indefinites can have presuppositional and non-presuppositional readings.

B.3.1 Exceptional wide scope via presupposition projection

Suppose that the indefinite *a relative of John's* is presuppositional. What this means for Van Geenhoven 1998 is that it introduces a presuppositional DRS that gets resolved, in the following manner.

(77) If a relative of John's dies, he will be rich.

1. $\left[\ \middle|\ \left[\ \middle|\ \underline{\left[x \ \middle|\ \underline{[j \mid \text{John}(j)]}, \text{relative}(x,j)\right]}, \text{dies}(x)\right] \Rightarrow \left[\ \middle|\ \underline{[y \mid \text{male}(y)]}, \text{rich}(y)\right]\right]$

2. Accommodate $\underline{[j \mid \text{John}(j)]}$ in the main DRS:

$$\left[\, j \ \middle|\ \begin{array}{l} \text{John}(j), \\ \left[\ \middle|\ \underline{[x \mid \text{relative}(x,j)]}, \text{dies}(x)\right] \Rightarrow \left[\ \middle|\ \underline{[y \mid \text{male}(y)]}, \text{rich}(y)\right] \end{array} \right]$$

3. Accommodate $\underline{[x \mid \text{relative}(x,j)]}$ in the main DRS:

$$\left[\, j, x \ \middle|\ \begin{array}{l} \text{John}(j), \text{relative}(x,j), \\ [\ \mid \text{dies}(x)] \Rightarrow \left[\ \middle|\ \underline{[y \mid \text{male}(y)]}, \text{rich}(y)\right] \end{array} \right]$$

4. Bind $\underline{[y \mid \text{male}(y)]}$ to the main DRS via $y = j$:

$$\left[\, j, x, y \ \middle|\ \begin{array}{l} \text{John}(j), \text{relative}(x,j), \text{male}(y), y = j, \\ [\ \mid \text{dies}(x)] \Rightarrow [\ \mid \text{rich}(y)] \end{array} \right]$$

5. Simplify:
$$\left[\, j, x \;\middle|\; \begin{array}{l}\text{John}(j), \text{relative}(x, j), \text{male}(j), \\ [\;|\; \text{dies}(x)\,] \Rightarrow [\;|\; \text{rich}(j)\,]\end{array} \right]$$

The final DRS represents a exceptional wide scope reading.

B.3.2 Intermediate scope readings

Intermediate scope readings of specific indefinites are derived via non-global accommodation. Consider, (78), for which global accommodation is not available due to trapping.

(78) Every professor rewarded every student who read a book he had recommended. (Abusch 1993: p. 90)

This is interpreted as the DRS in (79) (we omit the gender presupposition of *he*).

(79) $$\left[\;\middle|\; [\;|\;\text{professor}(x)\,]\,\langle\forall x\rangle \left[\;\middle|\; \left[\;\middle|\; \begin{array}{l}\text{student}(y), \text{read}(z), \\ \underline{\left[\, z \;\middle|\; \begin{array}{l}\text{book}(z), \\ \text{recommended}(w, z)\end{array}\right]}\end{array}\right] \langle\forall y\rangle\, [\;|\;\text{rewarded}(x, y)\,] \right]\right]$$

Resolving the pronoun to x yields (80).

(80) $$\left[\;\middle|\; [\;|\;\text{professor}(x)\,]\,\langle\forall x\rangle \left[\;\middle|\; \left[\;\middle|\; \begin{array}{l}\text{student}(y), \text{read}(z), \\ \underline{\left[\, z \;\middle|\; \begin{array}{l}\text{book}(z), \\ \text{recommended}(x, z)\end{array}\right]}\end{array}\right] \langle\forall y\rangle\, [\;|\;\text{rewarded}(x, y)\,] \right]\right]$$

The presuppositional indefinite cannot be accommodated globally, due to trapping. If it is accommodated locally, we obtain (81), which represents the intermediate scope reading of the example.

(81) $$\left[\;\middle|\; [\;|\;\text{professor}(x)\,]\,\langle\forall x\rangle \left[\, z \;\middle|\; \begin{array}{l}\text{book}(z), \text{recommended}(x, z), \\ \left[\;\middle|\; \begin{array}{l}\text{student}(y), \\ \text{read}(z)\end{array}\right] \langle\forall y\rangle\, [\;|\;\text{rewarded}(x, y)\,]\end{array}\right]\right]$$

B.4 Critical discussion and comparison

Critiques of the theory of specific indefinites we have just reviewed can already be found in the literature (e.g., Endriss 2009: §4.6, Geurts 2010, Onea 2015, Ebert 2021). I will discuss two particularly major issues below, which seem to be regarded as significant issues. I largely agree with this conclusion in the literature, but I would also like to stress that the presuppositional theory proposed in the main part of the present paper do not run into the same issues, which means that they are not issues inherent to the core idea of specific indefinites as presuppositional indefinites, but rather of the particular implementation of the idea offered in Van Geenhoven 1998.

B.4.1 Obligatory accommodation

The first issue I would like to discuss is perhaps conceptual in nature, but the solution is not obvious. Recall that according to the presupposition-as-anaphora theory, presuppositions must be either bound or accommodated, and whenever binding is possible, it is preferred. The theory of Van Geenhoven 1998 must assume, however, that presuppositional indefinites are never bound and are always accommodated.[1]

If presuppositional indefinites could be bound, presuppositional indefinites should function as anaphoric expressions. It is rather clear that indefinites are never anaphoric, but to illustrate, consider the following example.[2]

(82) A tall man and a short man came in. A man smiled.

Clearly, *a man* in the second sentence cannot refer to either of the previously mentioned men. Theoretically, however, if this indefinite is read presuppositionally, and if its presupposition could be bound, it should give rise to an anaphoric reading. To prevent the anaphoric reading, therefore, it needs to be assumed that the presuppositions of specific indefinites are never bound.

Van Geenhoven 1998 herself acknowledges the importance of this assumption, as well as the tension it creates with the assumption that presupposition binding is generally preferred to presupposition accommodation. In fact, she even goes as far as suggesting to drop this latter assumption, but as far as I can see, that would cause overgeneration issues for regular presuppositions.[3]

Under the theory proposed in the present paper, this issue does not arise. Recall that the presuppositions of specific indefinites are simply existential statements that are propositionally very weak and are often simply satisfied. Furthermore, we assumed that indefinites, specific or not, are associated with the so-called Novelty Condition (Heim 1982) that they introduce novel discourse referents (which we simply hardwired in the above compositional implementation as part of the definedness condition for $\exists$ in the intermediate language). This forces indefinites to be non-anaphoric. Our implementation of the condition was a rather crude and I have no reason to exclude other ways of implementing it, including the possibility to derive it from a general principle like *Maximize Presupposition!* (Heim 1991), but the important consequence is that there is no need for an uncomfortable assumption like the presuppositions of specific indefinites are always accommodated.

[1] I should also mention that Cresti 1995 puts forward a related theory of specific indefinites that is couched in Heim's 1982 File Change Semantics. She runs into a very similar (and perhaps worse) issue that ndefinites with exceptional scope need to be always accommodated, although she doesn't explicitly discuss this issue.

[2] Note that the non-synonymy between indefinites and definites is a related but different issue, which Van Geenhoven 1998: pp. 221–222 discusses. She suggests that definites might have more presuppositions than indefinites, so they never mean exact the same thing, although she does not seem to propose anything concrete. One could probably say something about uniqueness, at least when the noun is singular count, and postulate a principle like Maximize Presupposition! (Heim 1991) to render indefinites unacceptable. This issue is separate from the non-bindability of indefinites, as the definite version of (82) is not felicitous, but that does not make (82) felicitous.

[3] Geurts 2010 puts forward a possible way out: He proposes that indefinites are never presuppositional, but can be backgrounded, and backgrounded information project like presuppositions. The crucial difference is that backgrounded information need not be 'accommodated'. But as far as I can tell, the proposal is just terminological. In fact, he seems to think that all presuppositions are just backgrounded information, in which case, the difference between binding and accommodation still arises. Abusch 1993 and Onea 2015 similarly call the relevant content of a specific indefinite something other than presupposition, but it ought to be explained why it does not give rise to anaphoric interpretation.

B.4.2 *Intermediate accommodation*

Independently of the theory of Van Geenhoven 1998, the presupposition-as-anaphora theory is known to be plagued by the issue of intermediate accommodation, and Van Geenhoven 1998 inherits it. In a nutshell, the issue is that there are configurations that give rise to binding possibilities, both for pronominal anaphora and presupposition binding, but accommodation does not seem to be possible, which include the configuration of classical donkey anaphora.

To illustrate, consider the following conditional statement.

(83) If Alice is married, then her child lives with her.

This is interpreted as:

(84) $$\left[\;\middle|\; \left[\;\middle|\; \underline{[a \mid \text{Alice}(a)]}, \text{married}(a)\right] \Rightarrow \left[\;\middle|\; \frac{\left[x \;\middle|\; \underline{[y \mid \text{female}(y)]}, \text{child\_of}(x,y)\right],}{\underline{[z \mid \text{female}(z)]}, \text{live\_with}(x,z)}\right]\right]$$

Resolve the proper name's presupposition and the pronominal anaphora, we obtain:

(85) $$\left[a \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{female}(a) \\ [\;\mid \text{married}(a)] \Rightarrow \left[\;\middle|\; \underline{[x \mid \text{child\_of}(x,a)]}, \text{live\_with}(x,a)\right] \end{array}\right]$$

We are left with the possessive presupposition. It can be accommodated in multiple places. The first option is to accommodate it globally, as in (86).

(86) $$\left[a, x \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{female}(a), \text{child\_of}(x,a), \\ [\;\mid \text{married}(a)] \Rightarrow [\;\mid \text{live\_with}(x,a)] \end{array}\right]$$

Or, it can be accommodated in the consequent DRS, as in (87).

(87) $$\left[a \;\middle|\; \begin{array}{l} \text{Alice}(a), \text{female}(a), \\ [\;\mid \text{married}(a)] \Rightarrow [x \mid \text{child\_of}(x,a), \text{live\_with}(x,a)] \end{array}\right]$$

A third option is to accommodate the presupposition in the antecedent DRS, as in (88).

(88) $$\left[a \;\middle|\; \begin{array}{l} \text{Alice}(a), \\ [x \mid \text{married}(a), \text{child\_of}(x,a)] \Rightarrow [x \mid \text{live\_with}(x,a)] \end{array}\right]$$

Note that the antecedent DRS should be accessible from the consequent DRS, as it is how donkey anaphora like (89a) is resolved and how presupposition filtering in a sentence like (89b) is accounted for in terms of presupposition binding.

(89) a. If a farmer owns a donkey, he vaccinates it.
b. If Alice is married, she lives with her husband.

The issue is that the third reading does not seem to be available, at least easily. This observation itself is not a particularly grave issue, because it is assumed that accommodation at a higher DRS is generally preferred to accommodation at a lower

DRS, whenever both are possible (see Van der Sandt 1992, Geurts 1999, Beaver & Zeevat 2007 for empirical arguments). In the case at hand, therefore, the most preferred option is the global accommodation, which could make the other options not easily perceivable.

One could, however, push the global accommodation reading out of the picture by making use of trapping. Consider the example in (72) again, which we used to illustrate the trapping condition.

(72) Every cat looks down on its owner.

Above, we pointed out that the global accommodation reading of the possessive presupposition is not possible, and the most natural reading seems to be the local accommodation reading, which is represented in (76).

(76) $[\ |\ [\ |\ \text{cat}(x)]\ \langle\forall x\rangle\ [y\ |\ \text{owner\_of}(y,x), \text{look\_down}(x,y)]]$

There is, however, another theoretical possibility, which is to accommodate the presupposition in the restrictor DRS. Note that the following examples with pronominal anaphora and presupposition binding, respectively, show that the restrictor DRS is accessible from the nuclear scope DRS.

(90) a. Every farmer who owns a donkey vaccinates it.
b. Every student that got an A knows that they got an A.

The intermediate accommodation reading of (72) is represented by (91).

(91) $[\ |\ [y\ |\ \text{cat}(x), \text{owner\_of}(y,x)]\ \langle\forall x\rangle\ [\ |\ \text{look\_down}(x,y)]]$

Given the general preference for accommodation in a higher DRS, this reading should actually be the more preferred reading for this example, rather than the local accommodation reading. But it appears that it is not easily available.

It is perhaps easier to evaluate this prediction with the following example taken from Beaver 2001.

(92) Every German woman drives her car to work. (Beaver 2001: p. 119)

This doesn't seem to mean 'Every German woman that has a car drives it to work', which is what the intermediate accommodation reading essentially means. Rather the sentence intuitively entails that every German woman has a car, an entailment predicted by the local accommodation reading.

Intermediate accommodation has been discussed, often as a major issue for the presuppositoin-as-anaphora theory (e.g., Beaver 2001: §5.6, Van Geenhoven 1998: pp. 200–201, Yeom 1998: pp. 219–221, Jäger 2007: pp. 134–135, Beaver & Zeevat 2007). At least for quantificational cases, Geurts & Van der Sandt 1999 offer a possible solution, but I believe it does not save the version of the issue for Van Geenhoven 1998, which seems to be more clearly problematic.

To illustrate the version of the problem for Van Geenhoven 1998, consider the following example.

(93) Every boy called a French relative of his.

We home in on the bound interpretation of the pronoun *his*, and on the reading where the object indefinite is read presuppositionally. This reading is translated into the following DRS.

(94) $\Big[\ \Big|\ [\ |\ \text{boy}(x)\,]\,\langle\forall x\rangle\Big[\ \Big|\ \underline{[y\ |\ \text{French}(y), \text{relative}(y,x)\,]}, \text{called}(y)\Big]\Big]$

The presupposition in the nuclear scope DRS cannot be globally accommodated due to trapping, and the two options left are to accommodate it in the restrictor DRS, or in the local nuclear scope DRS. The former option is higher, so is preferred, by assumption. This reading is represented by the DRS in (95).

(95) $[\ |\ [y\ |\ \text{boy}(x), \text{relative}(y,x)\,]\,\langle\forall x\rangle\,[\ |\ \text{called}(y)\,]\,]$

In plain English, this means 'Every boy who has a French relative of his called them', which is a donkey sentence. Clearly (93) mean that. Rather, it has a stronger entailment that every (relevant) boy has a French relative.

The theory proposed in the current paper does not run into this issue. The key difference from Van Geenhoven 1998 is the underlying theory of presupposition we assumed. The issue of intermediate accommodation is specific to the presupposition-as-anaphora theory, and does not arise in the Heim-Stalnaker view of presupposition, as intermediate accommodation is not even an option there. It seems fair to say that the Heim-Stalnaker view generally better explains presupposition projection through quantifiers than the presupposition-as-anaphora theory.

To complete the discussion, here is how (93) would be analyzed under the theory of the present paper. The presupposition of the specific indefinite projects universally through the domain of quantification, giving rise to a universal presupposition that the boys all have a French relative, and a similarly universal assertion that the boys all called them, where the anaphora is resolved via quantificational subordination. Theoretically, there is no particular reason to exclude the local accommodation reading where the presupposition is accommodated within the nuclear scope (although that would make the sentence indistinguishable from the plain indefinite reading), but importantly, there is no such thing as an intermediate accommodation reading in this theory.

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