PHAS1102 – Physics of the Universe
Problem class 1 – Stellar astrophysics (Raman Prinja)
Notes for answers

(All questions cover the lecture material from first 2 weeks of the course)

Question 1

The spectra of the two stars can be approximated by blackbodies, for which the relation between temperature and wavelength of the emission peak can be written as:

$\lambda_{\max} \approx \dfrac{3\times 10^{-3}}{T}$ where $\lambda_{\max}$ is in m and T in K. Thus

$T \approx \dfrac{3\times 10^{-3}}{\lambda_{\max}} = \dfrac{3\times 10^{-3}}{6\times 10^{-8}}\,\mathrm{K} = 10^{5}\,\mathrm{K}$ (for $\lambda_{\max}$ = 60 nm) → early (O-) type star

$T \approx \dfrac{3\times 10^{-3}}{\lambda_{\max}} = \dfrac{3\times 10^{-3}}{1.5\times 10^{-6}}\,\mathrm{K} = 2\times 10^{3}\,\mathrm{K}$ (for $\lambda_{\max}$ = 1.5 μm) → late (M-) type star

Question 2

If the ionisation potential of the hydrogen atom is 13.6 eV, then $E(1) = -13.6$ eV and

$E(n) = -\dfrac{13.6}{n^2}\,\mathrm{eV}$

Thus $E(2) - E(3) = -13.6\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)\mathrm{eV} = -1.89\,\mathrm{eV}$ and

$$\lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{1.89\times 1.6\times 10^{-19}}\,\mathrm{m} = 6.577\times 10^{-7}\,\mathrm{m} = 657.7\,\mathrm{nm}$$

Transitions whose lower level has $n = 2$ are called the *Balmer series*.
The transition between $n = 2$ and $n = 3$ is called Hα.

Question 3

Set λ_{obs} = 588.965 nm and λ_{rest} = 588.995 nm. Then

$\dfrac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{rest}}}{\lambda_{\mathrm{rest}}} = \dfrac{v}{c} = -5.1\times 10^{-5}$ and $v = -5.1\times 10^{-5} c = -15.3\,\mathrm{km\,s^{-1}}$

The velocity is negative (the line is blueshifted), so the cloud is coming towards us.

The line at 588.995 nm falls in the middle of the visible spectral band.

$$\nu_{\text{rest}} = \frac{c}{\lambda_{\text{rest}}} = \frac{3\times10^{8}\times10^{9}}{588.995}\frac{\text{nm}\,\text{s}^{-1}}{\text{nm}} = 5.1\times10^{14}\,\text{Hz}$$

[1]

$$E_{\text{rest}} = h\nu_{\text{rest}} = 6.63\times10^{-34}\times5.1\times10^{14}\ \text{J s Hz} = 3.4\times10^{-19}\ \text{J}$$

Question 4

Applying the Stefan-Boltzmann law, $L \propto T^4R^2$ [1], $\frac{L_C}{L_A} = \frac{T_C^4R_C^2}{T_A^4R_A^2} = \frac{10000^4}{4000^4 2^2} = 9.7$

Question 5

The Sun, classified as a G2V star, is of spectral type G and a yellow star roughly in the middle of the Harvard sequence, which goes as O,B,A,F,G,K,M, from the hottest to the coolest (and/or from the bluest to the reddest) stars. All spectral types are divided into 10 sub-classes (8 sub-classes for O-types) by decreasing temperature, so G2 is towards the top temperature for G stars. V is the luminosity class, which goes from I to V in order to decreasing luminosity (V=dwarf stars). Star A is a K2II star, this is of a later spectral type (redder star). Its luminosity class II tells us that it is a (bright) giant star.

Question 6

The average temperature of the photosphere is 5800K, and so if the umbra of a sunspot is 1600K cooler than the surrounding photosphere, then it has an average temperature of 4200K.
Now recall the Stefan-Boltzmann law: Flux $\propto T^4$
and that this is the same as the intensity of the radiation. Therefore,
Flux from umbra /Flux from surrounding photosphere = $(4200/5800)^4 = 0.275$

Hence the energy flux from the umbra of the sunspot is only 27.5% of that from the surrounding photosphere. Therefore while a sunspot is very hot and thus very bright (compared to terrestrial light sources) it is dim in comparison to the surrounding photosphere and appears dark.

Question 7

Brightness, b, depends upon luminosity, L, and distance, d, according to the
$\propto L/d^2$
inverse square law: $b \propto L/d^2$
and so if two stars are at the same distance from Earth then the brightness is determined by the luminosity. The total luminosity is proportional to the energy flux and the surface area:
$L \propto$ (Energy Flux) x (Surface Area)
Since the stars have the same size, the relative luminosity is determined by the stars' respective energy flux, which is in turn, determined from the Stefan- Boltzmann law: Flux $\propto T^4$
Thus the brighter star will be the hotter one and a blue star is hotter than a red one. Hence the blue star will be brighter in the night sky.