PHAS1102 – Physics of the Universe

Practice Problem Sheet 3 - solutions

1.

Let be $m_1 = 3$ and $m_2 = 4$; then $\log_{10}(f_1/f_2) = (m_2 - m_1) \log_{10} 2.512$
$= \log_{10} 2.512$ 1

i.e. $f_1/f_2 = 2.512$, and $f_1 = 2.512\ f_2$ 1

Thus the combination of the two stars will produce a flux $f_{SUM} = f_1 + f_2$
$= 3.512\ f_2$ 1

So $m_{SUM} - m_2 = \log_{10}(f_2/f_{SUM})/\log_{10} 2.512 = -1.36$, and $m_{SUM} = 2.6$ 2

[5 marks total]

2.

Sirius A luminosity is then

$$L = 23.5 \times 4 \times 10^{26} \text{ Watt} = 9.4 \times 10^{27} \text{ Watt}$$ 1

Since fraction of mass liberated into energy by the fusion of
H to He = 0.007 (= efficiency), $E_{total} = 0.007\ mc^2$ 1

The H mass burnt into He per sec will be

$$\frac{L}{\text{efficiency} \times c^2} = \frac{9.4 \times 10^{27}}{0.007 \times (3 \times 10^8)^2} = 1.5 \times 10^{13} \text{ kg s}^{-1}$$ 1

If we assume that 10% of its H mass burns into He, then, since Sirius A mass is

$$M = 2.3 \text{ solar mass} = 2.3 \times 2 \times 10^{30} \text{ kg} = 4.6 \times 10^{30} \text{ kg}$$ 1

Sirius A lifetime will be

$$\frac{0.1 \times M}{\text{Rate of burning}} = \frac{4.6 \times 10^{29}}{1.5 \times 10^{13}} \text{s} = 3 \times 10^{16} \text{ s} = \frac{3 \times 10^{16}}{3 \times 10^{7}} \text{years} = 10^{9} \text{ years}$$ 1

while the Sun's lifetime is expected to be 10^{10} years. 1

[6 marks total]

3.

Using Wien's displacement law

$$\lambda_{\max} = \frac{3\times10^{-3}}{T}\text{ metre}$$

where T in Kelvin, we find

$$T_{wd} = \frac{3\times10^{-3}}{\lambda_{wd}}\text{ K} = \frac{3\times10^{-3}}{3\times10^{-8}}\text{ K} = 10^{5}\text{ K}$$ 1

and

$$T_{rg} = \frac{3\times10^{-3}}{\lambda_{rg}}\text{ K} = \frac{3\times10^{-3}}{1.5\times10^{-6}}\text{ K} = 2\times10^{3}\text{ K}$$ 1

Since the luminosity $L \propto R^2T^4$ where R is the stellar radius and T the temperature, then the ratio of the luminosities will be

$$\frac{L_{rg}}{L_{wd}} = \frac{R_{rg}^2T_{rg}^4}{R_{wd}^2T_{wd}^4} = \frac{\left(10^{11}\right)^2\times\left(2\times10^{3}\right)^4}{\left(10^{6}\right)^2\times\left(10^{5}\right)^4} = 1600$$ 1

[3 marks total]