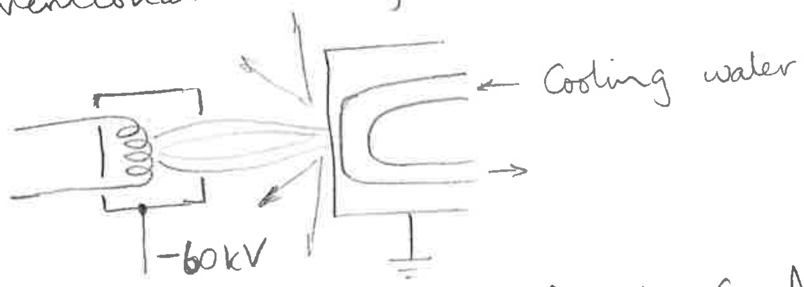
Lecture 6

Synchrotron Radiation

(Not in Warren: based on these references:
"Neutron and SR for condensed matter studies" ed J. Baruchel
Editions de Physique Springer Verlag (1993) vol 1
J. Schwinger, Phys Rev. 75 1912 (1949)
H. Winick "Sychrotron Radiation Research" (Plenum, 1980))

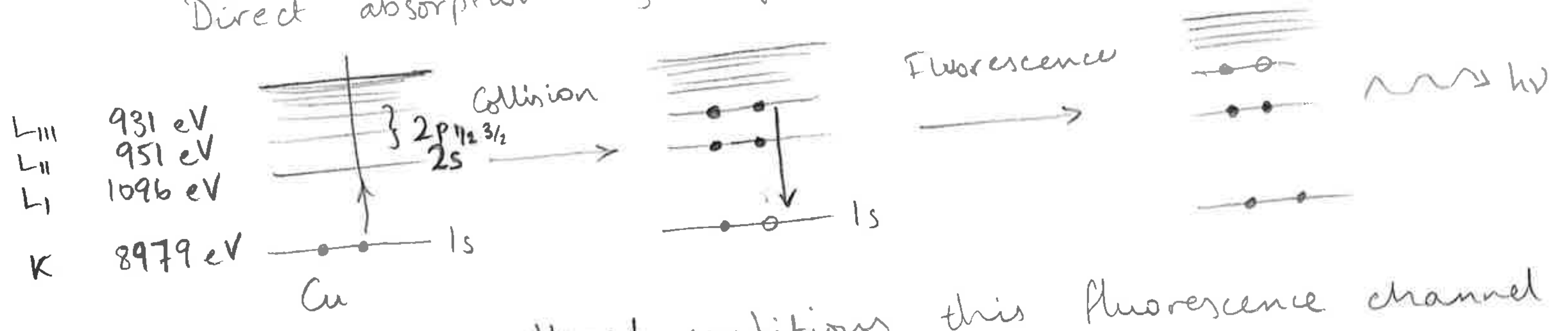
1. 'Conventional' X-ray Tube



Cathode   Anode Cu, Mo, Cr, Ag etc

Bremsstrahlung of electron in anode → local accelerations due to interactions with ions → continuous spectrum.

Direct absorption by target atom of electron.



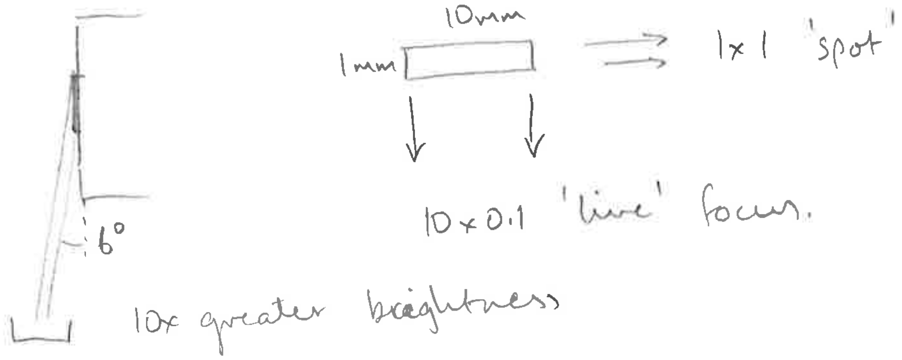
i) Even under optimal conditions this fluorescence channel is $\sim 10^{-6}$ of all electrons in beam.

ii) X-rays produced over 4π steradians. Even a well designed monochromator might accept $0.5^\circ \times 2^\circ = 0.002\%$

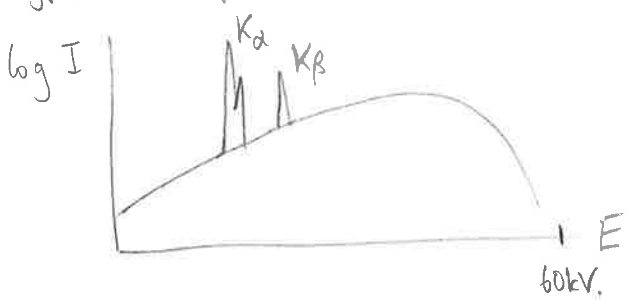
1 kW tube $\rightarrow \dfrac{1000}{10\,\text{kV} \times 1.6\times10^{-19}} \times 10^{-6} \sim 6\times10^{11}$ photons/sec $\rightarrow 1.5\times10^{7}$ usable photons/sec

iii) Limitation is power density in anode before melting.
Rotating anode wheel (6000 rpm) → 10× more power.
Biggest ever made = 90kW.

iv) Can increase effective brightness by take-off angle.



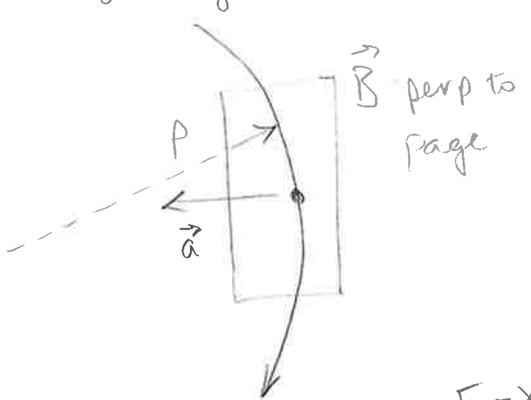
v) Typical spectrum



Kα split into $K_{\alpha 1}$ $K_{\alpha 2}$
Characteristic spectrum
I ~ 10 to 100 times above
Bremsstrahlung.

2. Synchrotron Radiation

To get more flux, need to increase energy density
much higher than any solid material can stand.
Accelerate free electrons → Bremsstrahlung.
Storage ring / accelerator with bending magnets.



electrons. $E = \gamma m_0 c^2$

$$F = e\vec{v} \times \vec{B}$$
$$= ma = m\frac{v^2}{\rho} \quad \text{(non-relativistic)}$$

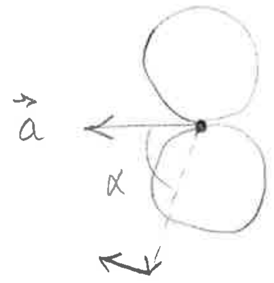
rest mass

$$\rho = \frac{mv}{eB} = \gamma \frac{m_0 c}{eB} \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$[m = \gamma m_0]$ relativistic

6.3

Radiation in rest-frame of electron

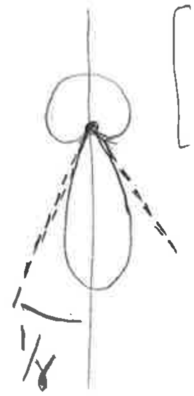


$$\vec{E} = \frac{e|a|}{c^2 R} \sin\alpha$$

Polarization as shown.

$$V = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} c$$

In Laboratory frame.



Lorentz transfor[m] of ‖el component not ⊥ comp.

opening angle ($\gamma \gg 1$)

Typical numbers

$E = 2.5\,\text{GeV}$ NSLS
$7\,\text{GeV}$ APS

$B \sim 1\,T$

$$\rho = 3.32 \frac{E\,[\text{GeV}]}{B\,[T]}$$

$\sim 10 - 20\,m$.

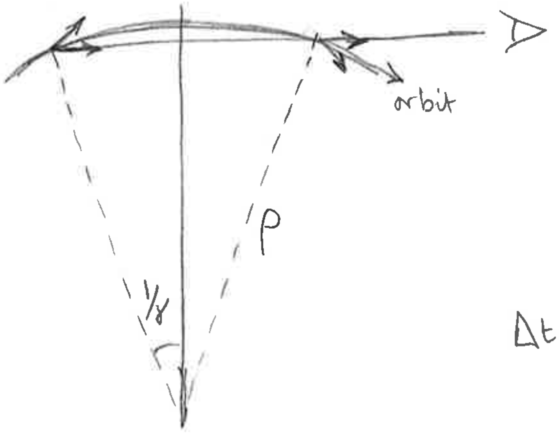
$mc^2 = 0.5\,\text{MeV}$

$\Rightarrow \gamma = 5000$ or $14\,000$

$1/\gamma \sim 0.01^\circ$ or 0.004°

$0.2\,\text{mrad}$ $0.07\,\text{mrad}$

3. Spectrum of S.R.



Electrons travel at speed

$$V = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} c \doteq \left(1 - \frac{1}{2\gamma^2}\right) c$$

Light travels at speed c

Duration of pulse:

$$\Delta t = \frac{2\rho/\gamma}{V} - \frac{2\rho \sin(1/\gamma)}{c}$$

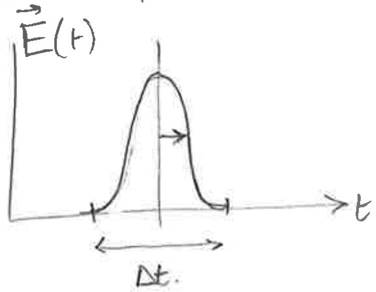
$$= \frac{2\rho}{c}\left(\frac{1}{\gamma}\left(1 + \frac{1}{2\gamma^2}\right) - \frac{1}{\gamma} + \frac{1}{6\gamma^3}\right)$$

$$= \frac{4}{3}\frac{\rho}{c}\frac{1}{\gamma^3} \text{ to leading order in } 1/\gamma$$

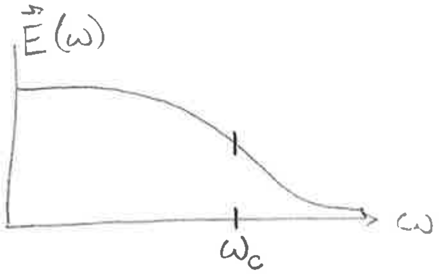
This is a <u>very</u> short pulse because speeds are almost equal!

$$\Delta t \sim \frac{10}{3\times10^8} \times 10^{-12} = 3\times10^{-20}\,s \;!$$

Pulse is shorter than the period of x-ray oscillator.
Obtain spectrum by Fourier transform:



$$\vec{E}(\omega) = \int \vec{E}(t)\, e^{i\omega t}\, dt$$



Period of a sine wave $\simeq$ Gaussian of half-width $\frac{\Delta t}{4}$ $\rightarrow$ Gaussian of half-width $\frac{4}{\Delta t}$

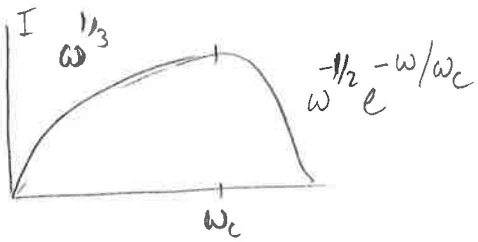
$$\omega_c = \frac{4}{\Delta t} = \frac{3c\gamma^3}{\rho} = \frac{3B\gamma^2}{m_0}$$

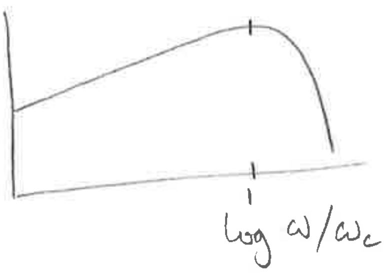
$$\lambda_c = \frac{2\pi c}{\omega_c} = \frac{2\pi\rho}{3\gamma^3} = \frac{2\pi m_0 c}{3B\gamma^2}$$

Ring	E	ρ	λ_c
NSLS	2.5 GeV	~10 m	1.8 Å
APS	7 GeV	~15 m	0.12 Å

More than one electron in ring: add up incoherently to obtain same spectrum. Each electron independent.

Real spectrum (calculation involves Bessel functions) is slightly different from this, but same idea.





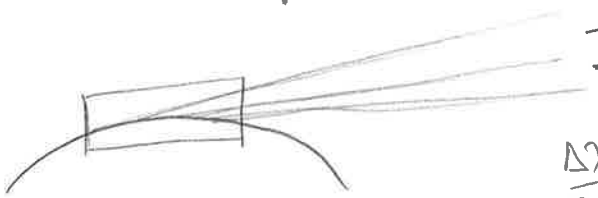
Important conclusion is that spectrum is continuous and has a cutoff at a 'critical energy' $\hbar\omega_c$

4. Flux, Brightness and Brilliance.

Standard universal functions for calculating spectrum.

Quantitative derivation of above:

Units define quantity for practical application:



$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta E}{E} = 0.1\% = \text{'typical' acceptance of experiment}$$

$$F(\text{ph/sec/mrad}/0.1\%) = 2.46 \times 10^{13}\, I[A]\, E[\text{GeV}]\, G_1(\omega/\omega_c)$$

λ_c/λ.

Flux is not a strong function of I or E.

Integrated value in direction perp. to plane.

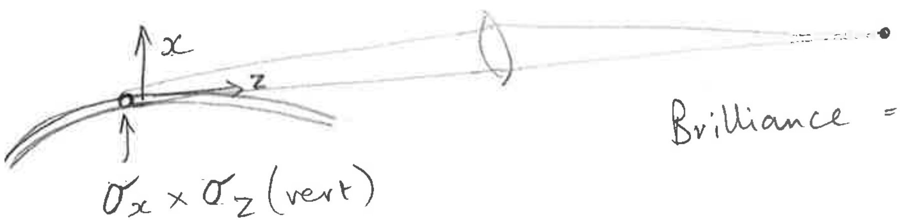
$$B(\text{ph/sec/mrad}^2/0.1\%) = 1.33 \times 10^{13}\, I[A]\, E^2[\text{GeV}]\, H_2(\omega/\omega_c)$$

Brightness is stronger function of E

Includes solid-angle out of plane, so accounts for narrowing w/E.

If expt has a small aperture brightness is what matters

If expt uses focussing optics on small sample, the spatial extent of the source matters as well:



$$\text{Brilliance} = \frac{\text{Brightness}}{\sigma_x \sigma_z}$$

F & B can be less than the ideal value because of divergence of electron beam in storage ring.

Need convolution of σ_x' and σ_z' with opening angle $1/\gamma$.

	σ_x	σ_z	σ_x'	σ_z'
APS	0.33	0.086	0.023	0.009
ESRF	0.4mm	0.08mm	0.015 mrad	0.007 mrad

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5. Insertion Devices

BMs are necessary component of storage ring. Design is fixed by their params B, ρ etc.

ID is some (optional) added component, usually in an otherwise unused 'straight section' of ring

Modern rings have many long straights for this.

i) Wavelength shifter

$B = 1T$ chosen conservatively for BMs because needs to be i) very stable ii) uniform iii) mass produced.

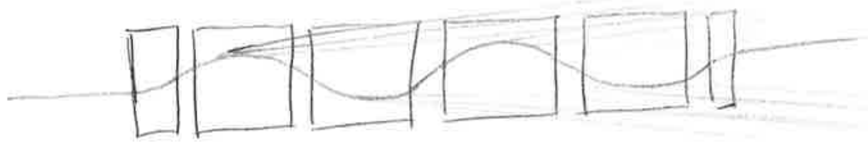
$B = 10T$ possible with superconductor; hence can raise ω_c by order of mag.

Spectrum as for much higher energy ring.

eg  can vary B without changing beam direction.

ii) Wiggler.

Same idea, repeated many times. Incoherent addition of light from each pole. → more flux.



Incoherent situation when deviation between poles is $\gg 1/\gamma$. Otherwise we have 'undulator'

iii) Undulator.

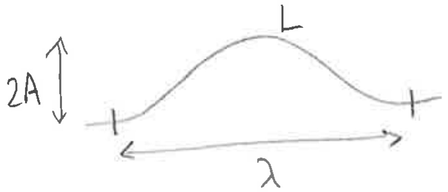
Coherent addition of radiation from each pole.

Will be different phase at different λ's

Device has modified wavelength spectrum.

6.8

Contour along sine wave



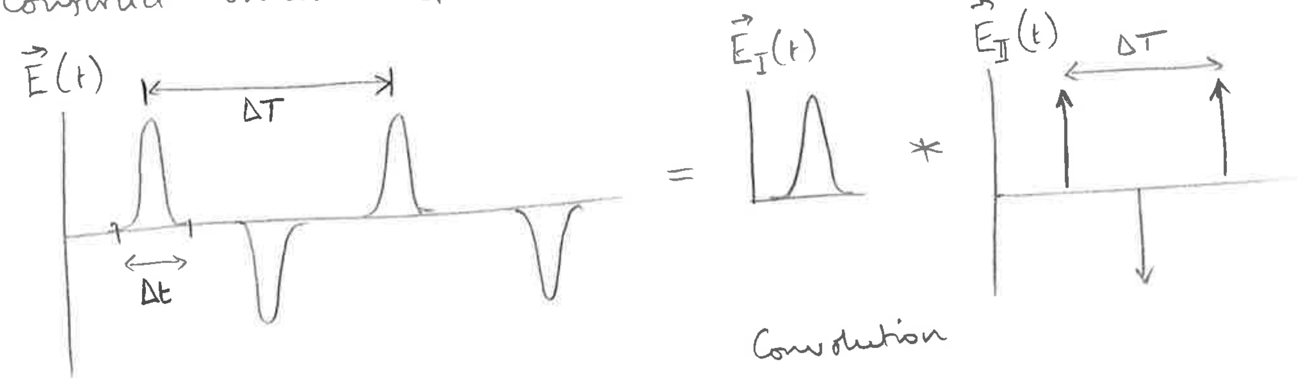
$$L = \lambda\left(1 + \frac{\pi^2 A^2}{\lambda^2}\right)$$

$$A = \frac{K}{\gamma}\frac{\lambda_u}{2\pi} \qquad L = \lambda_u\left(1 + \frac{K^2}{4\gamma^2}\right)$$

$$\Delta T = \underset{\text{electron}}{\frac{L}{V}} - \underset{\text{photon}}{\frac{\lambda_u}{c}} = \frac{\lambda_u}{c}\left(\left(1+\frac{K^2}{4\gamma^2}\right)\left(1+\frac{1}{2\gamma^2}\right) - 1\right)$$

$$= \frac{\lambda_u}{c}\left(\frac{1}{2} + \frac{K^2}{4}\right)\frac{1}{\gamma^2} = \frac{\lambda_u}{2c\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Construct on-axis spectrum:



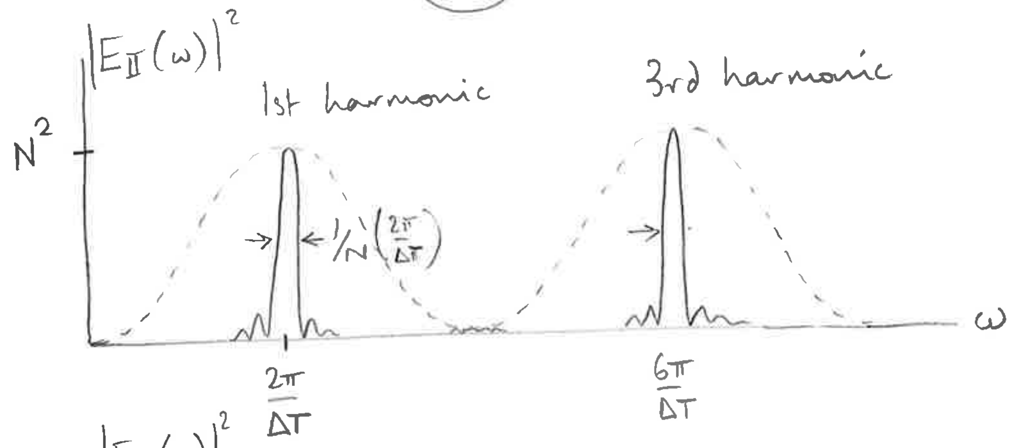
Convolution

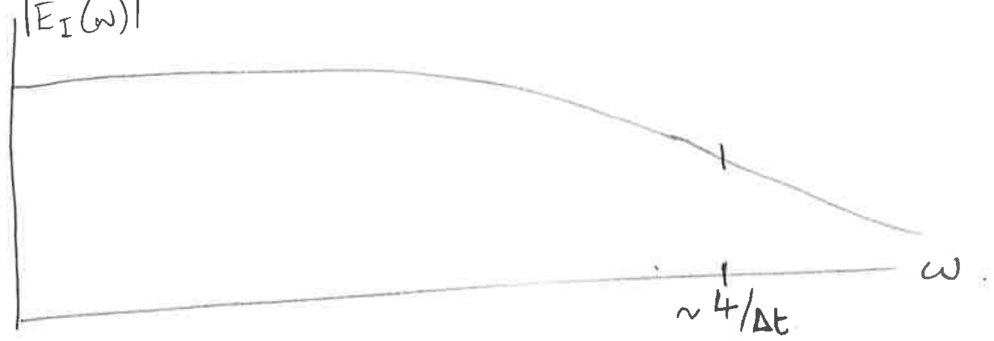
Fourier transform = product of two transforms.

$$E_{II}(\omega) = \sum_{n=1}^{2N} (-1)^n e^{i\omega n \Delta T/2} = \left(1 - e^{i\omega\Delta T/2}\right)\sum_{n=1}^{N} e^{i\omega n \Delta T}$$

$$= \left(1 - e^{i\omega\Delta T/2}\right)\frac{1 - e^{i\omega N \Delta T}}{1 - e^{i\omega\Delta T}}$$

$$|E_{II}(\omega)|^2 = \sin^2\frac{\omega\Delta T}{4}\;\sin^2\frac{N\omega\Delta T}{2} \Big/ \sin^2\frac{\omega\Delta T}{2}$$





j'th harmonic occurs at $\omega_j = 2\pi/\Delta T \cdot j$.

$$\lambda_j = \frac{2\pi c}{\omega_j} = \frac{c\Delta T}{j} = \frac{\lambda_u}{2\gamma^2 j}\left(1 + \frac{K^2}{2}\right)$$

Off-axis result, viewing at angle θ

$$\lambda_j = \frac{\lambda_u}{2\gamma^2 j}\left(1 + \frac{K^2}{2} + (\gamma\theta)^2\right)$$

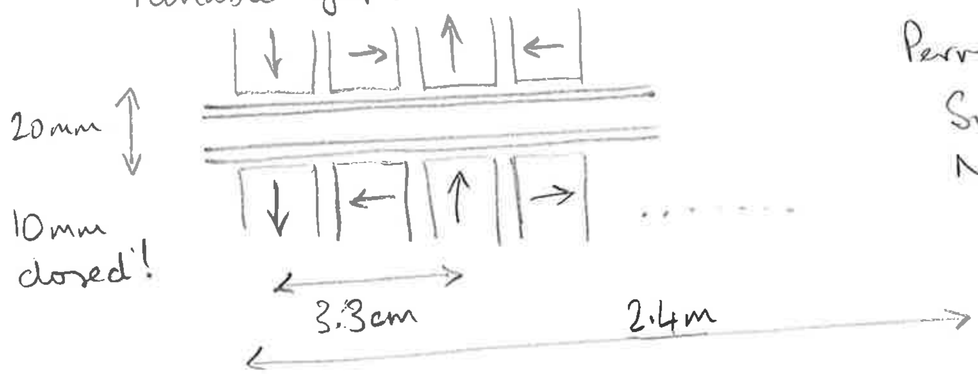
Here the + and − contributions are no longer exactly out of phase, so 2nd harmonic is allowed too.

Typical numbers $\lambda_1 = 1\,\text{Å}$, small K
$\gamma = 14{,}000$ (APS) $\Rightarrow \lambda_u = 4\,\text{cm}$ (3.3 actual)

→ With the constraints of building a vacuum chamber around the beam, this seriously limits the value of B_0 (on axis field) which can be attained.

→ Just won't work for machines smaller than APS!

Typical construction (major engineering exercise!)
Tunable gap:



Permanent magnets
$SmCo_6$
NdFeB

N = 72 poles.

Field errors will destroy undulator effect.
Must keep B_0 const within $1/N$

7. Undulator characteristics

i) Strongly peaked spectrum, approaching monochromatic.
Bandwidth $\sim 1/N$. Flux $\propto N^2$ (only N for wiggler)

ii) Only odd harmonics on-axis.

iii) Spectrum softens distinctly off-axis

iv) As $K \to 0$ (ideal case) uninteresting, since $K \propto B_0$
Since $\omega_c \propto B$, more harmonics present at larger K.
Closed gap: max K, B_0, λ + many harmonics.
Open gap: min K, B_0, λ_1 only

[tunable gap (und A)
gap = 10mm K = 2.4
gap = 20 K = 1
40 = 0.14

v) Strongly focussed beam.
Width = $1/\gamma$ in horizontal & vertical
$1/\gamma = 0.07$ mrad $\Rightarrow$ 0.7mm @ 10m like a laser beam

vi) Flux inside central cone (odd harmonic)

$$F(\text{ph/sec/0.1\%}) = 1.43\times10^{14}\, I[\text{Amp}]\, Q_m(K)\, N$$

$$B(\text{ph/sec/mrad}^2\text{/0.1\%}) = 4.15\times10^{13}\, I[\text{Amp}]\, E^2[\text{GeV}]\, F_m(K)\, N^2$$

Combine with machine params to get Brilliance