Risk processes with tax

Dalal Al Ghanim Ronnie Loeffen Alex Watson

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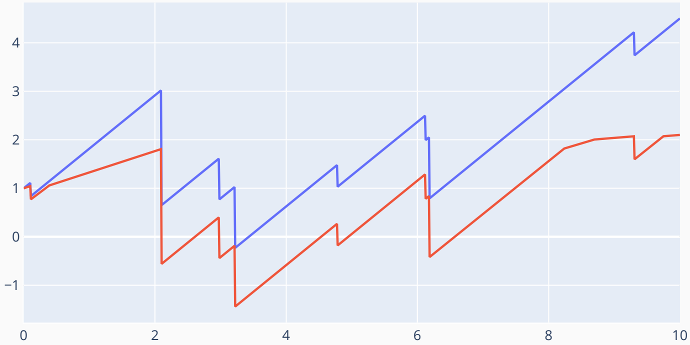
Taxation



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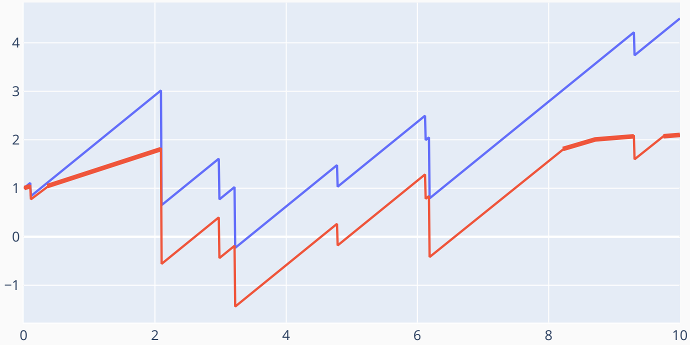

- Loss-carry-forward: tax paid at the maximum

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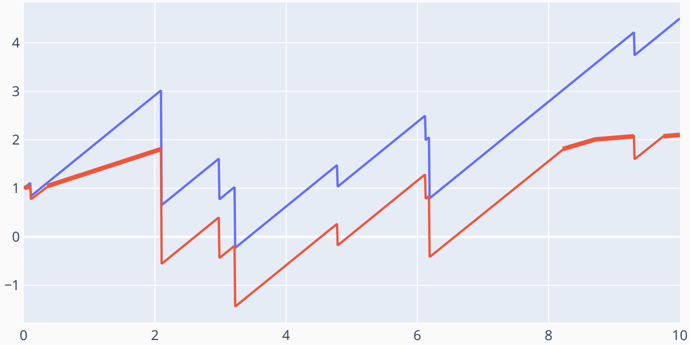

- Loss-carry-forward: tax paid at the maximum
- What is the best tax rate to maximise revenue?

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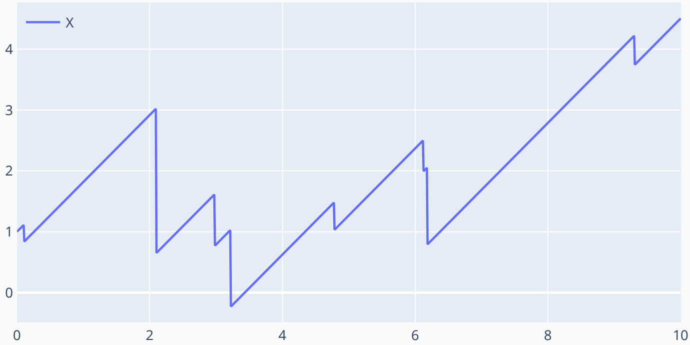

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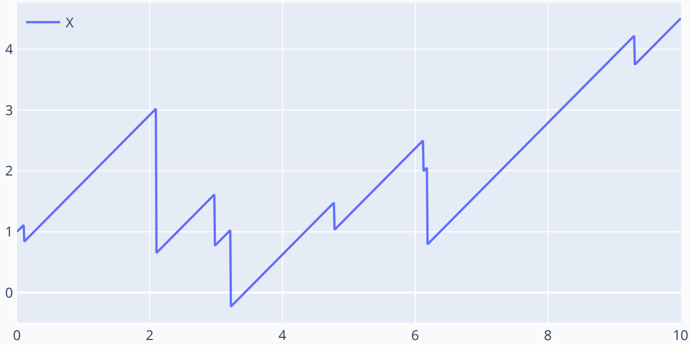

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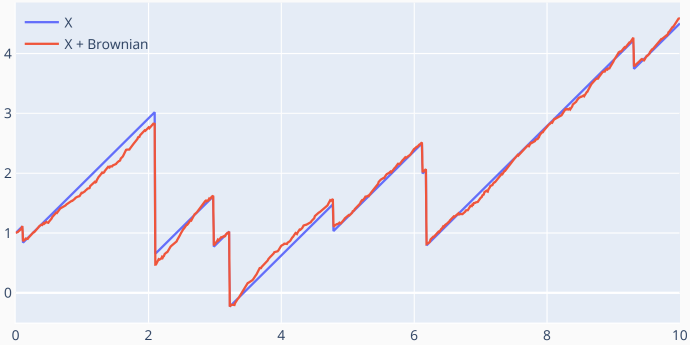

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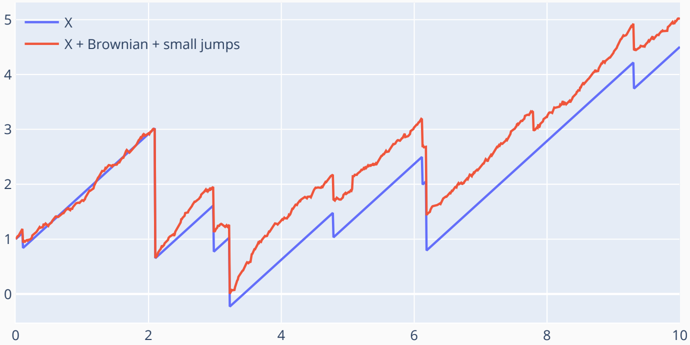

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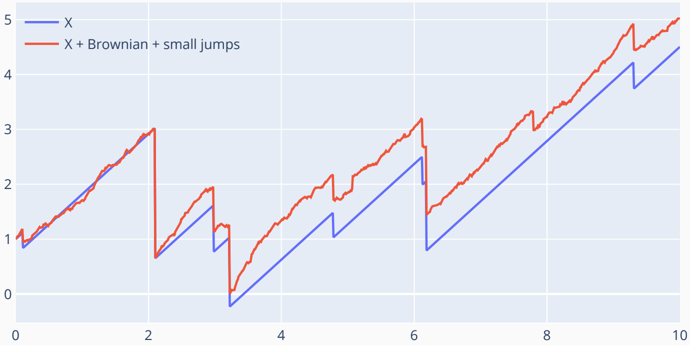

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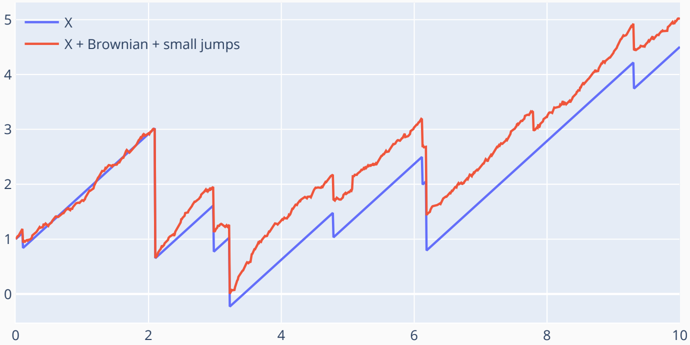

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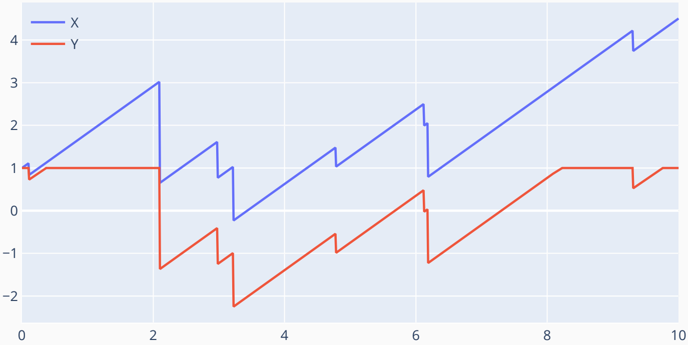
- Wealth of insurance company: c is premium rate, ξ_i are claims
- Intrinsic motivation: continuous equivalent of random walk

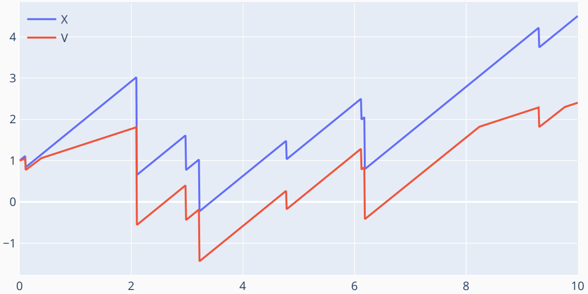
Reflected risk process



$X_0 = \bar{X}_0 = 1$

$$Y_t = X_t - (\bar{X}_t - \bar{X}_0),$$
$$\bar{X}_t = \sup_{s \le t} X_s \vee \bar{X}_0$$

Tax process – partially reflected



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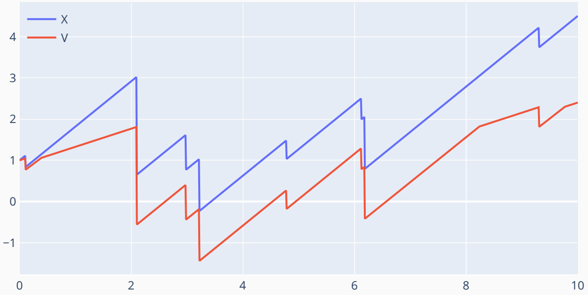

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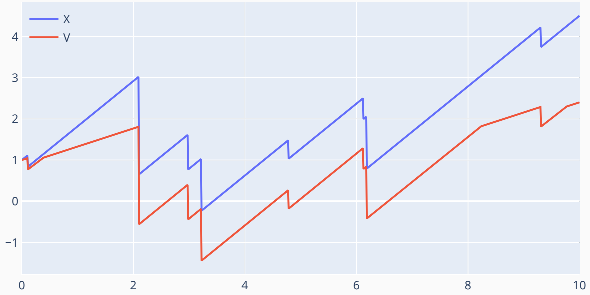

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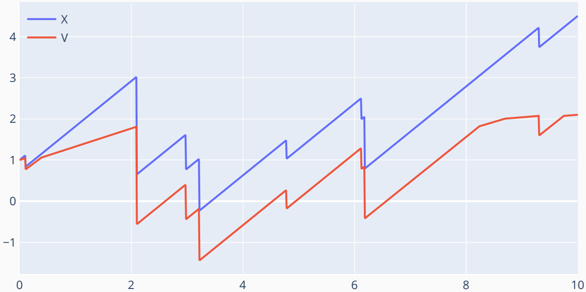

$$X_0 = \bar{X}_0 = 1,$$

$$\delta(x) = \begin{cases} 0.6, & x < 2, \\ 0.9, & x \geq 2 \end{cases}$$

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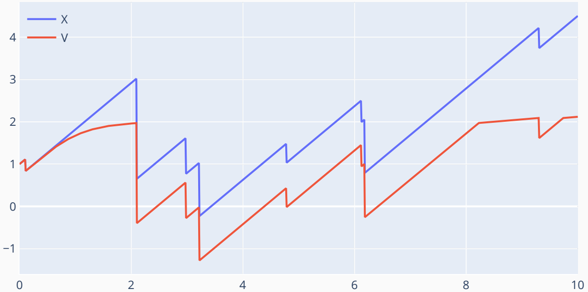

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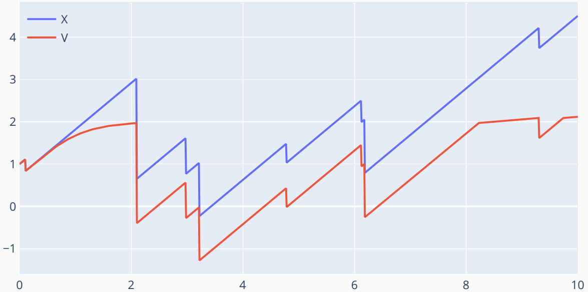

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- Introduces spatial dependence in a tractable way
- Questions: existence? computation of functionals? optimal δ?

Types of tax rate

Two ideas in literature:

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- $V_t = X_t - \int_0^t \delta(V_s)\, \mathrm{d}\bar{X}_s$ – not obvious it exists
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Existence

Theorem – Al Ghanim, Loeffen, W. (2020)

If the differential equation

$$y'(t) = 1 - \delta(y(t)), \quad y(0) = \bar{x},$$

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- Converse direction: uniqueness
- Core idea: look at V and X while they are drifting at the maximum

Functionals

Present value of tax collected until exiting $[0, a]$:

$$w(x, \bar{x}) = \mathbb{E}\left[\int_0^{\tau_0^a} e^{-qs} \delta(V_s)\, \mathrm{d}\bar{X}_s \,\middle|\, X_0 = x, \bar{X}_0 = \bar{x}\right],$$

$$\tau_0^a = \inf\{t \geq 0 : V_t \notin [0, a]\},$$

Functionals: computation

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$\delta \cdot \partial_x g + (\delta - 1) \cdot \partial_{\bar{x}} g = 0$ for $x = \bar{x}$ is the condition for g to be in the domain of the generator of $(V, \bar{V})$

Functionals: discussion

- Apply to get

$$w(x,\bar{x}) = \frac{W^{(q)}(x)}{W^{(q)}(\bar{x})} \int_{\bar{x}}^{a} \exp\left(- \int_{\bar{x}}^{y} \frac{W^{(q)\prime}(r)}{W^{(q)}(r)(1-\delta(r))} \, \mathrm{d}r \right) \frac{\delta(y)}{1-\delta(y)} \, \mathrm{d}y$$

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- Not the only way, but:
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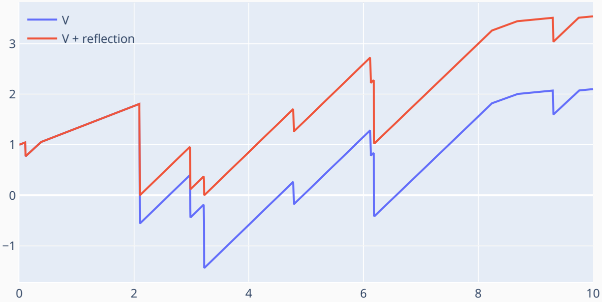
  - Reduces need for excursion theory or approximation by bounded variation processes

Taxation with bail-outs



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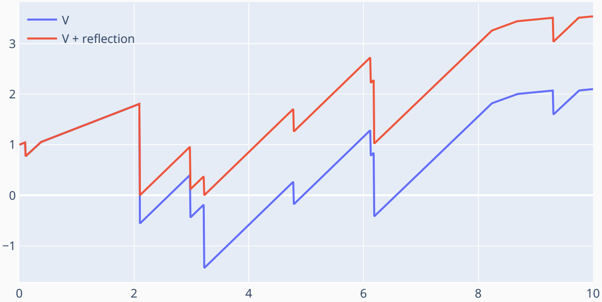

- Process reflected below at zero: ‘capital injections’ or ‘bail-outs’
- Conditions for ‘$f = w$’ are:
  - $f(x, \bar{x}) = f(0, \bar{x})$ if $x < 0$, and $f(x, \bar{x}) = 0$ if $x \geq a$,
  - $(\mathcal{L} - q) f(x, \bar{x}) = 0$ if $0 \leq x \leq a$,
  - $\delta(\bar{x}) \partial_x f(\bar{x}, \bar{x}) + (\delta(\bar{x}) - 1) \partial_{\bar{x}} f(\bar{x}, \bar{x}) = \delta(\bar{x})$
- Only change is in boundary conditions

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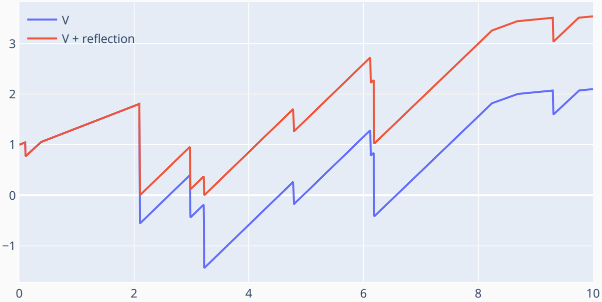

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$$H^* = \arg\max_H \mathbb{E}^H \left[\int_0^{\tau_0^\infty} e^{-qs} H_s \,\mathrm{d}\bar{X}_s \right] ?$$

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- If tax rate constrained to $H_t \in [\alpha, \beta]$: switch from α to β when V crosses some level b, i.e. $H_t^* = \delta(V_t)$,

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- Extension to maximising tax revenue minus bail-out cost, etc. – many possibilities

Optimal control

$$H^* = \arg\max_H \mathbb{E}^H \left[\int_0^{\tau_0^\infty} e^{-qs} H_s \, \mathrm{d}\bar{X}_s \right]$$

- If totally free to choose: reflection above (at some level), i.e. $H^* = \bar{X} \vee b$
- If tax rate constrained to $H_t \in [\alpha, \beta]$: switch from α to β when V crosses some level b, i.e. $H_t^* = \delta(V_t)$,

$$\delta(x) = \begin{cases} \alpha, & x \leq b, \\ \beta, & x > b \end{cases}$$

- Extension to maximising tax revenue minus bail-out cost, etc. – many possibilities
- Solutions are explicit (in terms of scale functions)

Optimal control: explicit result

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- $b = 0$ iff $R(0) \geq 0$, otherwise b is the unique root of $R(b) = 0$

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- We optimise over all tax rates (predictable H_s)

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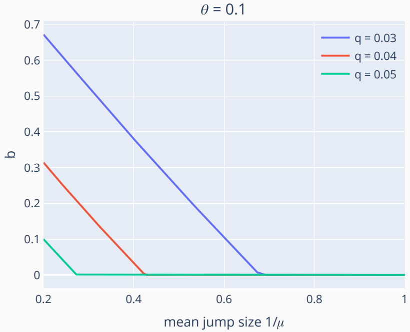

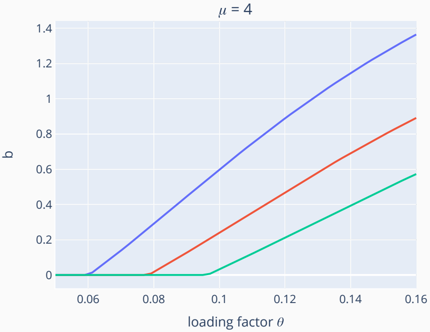

Further reading

D. Al Ghanim, R. Loeffen and A. R. Watson
The equivalence of two tax processes
Insurance Math. Econom., 2020. doi:10.1016/j.insmatheco.2019.10.002.

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Thank you!