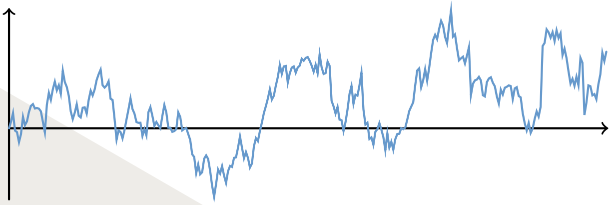
Markov additive friendships

Leif Döring   Lukas Trottner   Alex Watson

🎂 18 July 2022

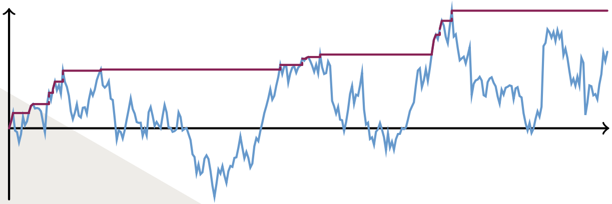
Lévy processes and the theory of friends

Wiener-Hopf factorisation (path picture)



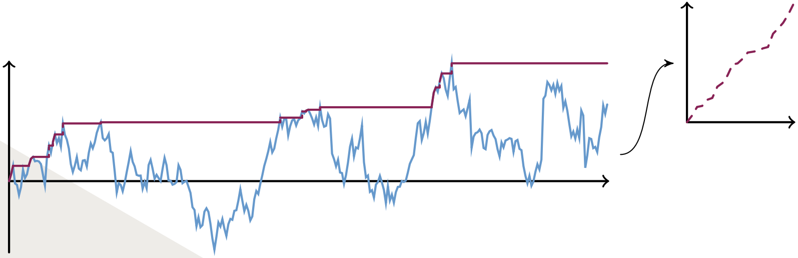
ξ, a Lévy process

Wiener-Hopf factorisation (path picture)



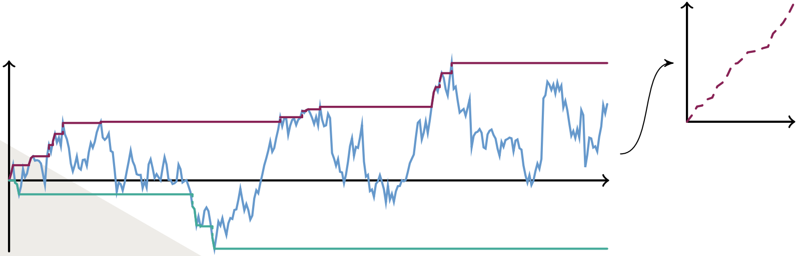
- ξ, a Lévy process
- $\bar{\xi}_t = \sup_{s \le t} \xi_s$, the running supremum

Wiener-Hopf factorisation (path picture)



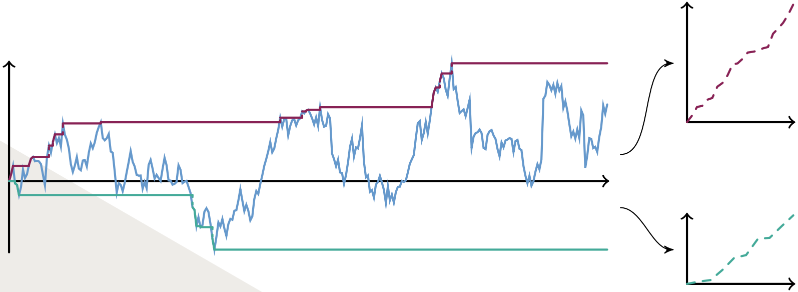
- ξ, a Lévy process
- $\bar{\xi}_t = \sup_{s \le t} \xi_s$, the running supremum
- H_t^+, the ascending ladder height process: suprema 'stitched together'

Wiener-Hopf factorisation (path picture)



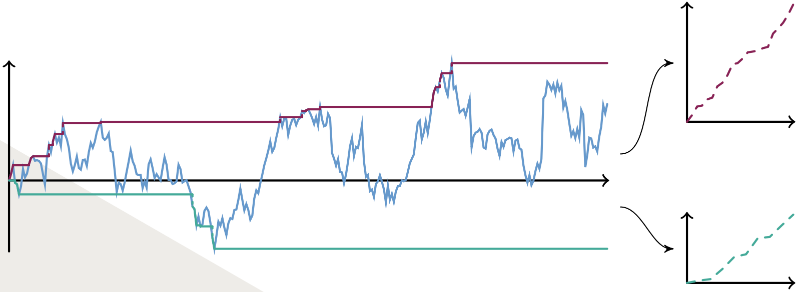
- ξ, a Lévy process
- $\bar{\xi}_t = \sup_{s \le t} \xi_s$, the running supremum
- H^+_t, the ascending ladder height process: suprema 'stitched together'

Wiener-Hopf factorisation (path picture)



- ξ, a Lévy process
- $\bar{\xi}_t = \sup_{s \le t} \xi_s$, the running supremum
- H_t^+, the ascending ladder height process: suprema ‘stitched together’
- H_t^-, the descending ladder height process

Wiener-Hopf factorisation (path picture)



- ξ, a Lévy process
- $\bar{\xi}_t = \sup_{s \le t} \xi_s$, the running supremum
- H_t^+, the ascending ladder height process: suprema ‘stitched together’
- H_t^-, the descending ladder height process

$H^\pm$ are subordinators (increasing Lévy processes).

Wiener-Hopf factorisation (analytic picture)

- Let Ψ be the characteristic exponent of ξ (i.e., $\mathbb{E}e^{\mathrm{i}\vartheta\xi_t} = e^{t\Psi(\vartheta)}$)

Wiener-Hopf factorisation (analytic picture)

- Let Ψ be the characteristic exponent of ξ (i.e., $\mathbb{E}e^{\mathrm{i}\vartheta\xi_t} = e^{t\Psi(\vartheta)}$)
- Let $\Psi^{\pm}$ be characteristic exponents of $H^{\pm}$

Wiener-Hopf factorisation (analytic picture)

- Let Ψ be the characteristic exponent of ξ (i.e., $\mathbb{E}e^{\mathrm{i}\vartheta\xi_t} = e^{t\Psi(\vartheta)}$)
- Let $\Psi^{\pm}$ be characteristic exponents of $H^{\pm}$
- Then

$$\Psi(\vartheta) = -\Psi^{-}(-\vartheta)\Psi^{+}(\vartheta).$$

The inverse problem

- Let $H^{\pm}$ be a pair of subordinators with CEs $\Psi^{\pm}$

The inverse problem

- Let $H^{\pm}$ be a pair of subordinators with CEs $\Psi^{\pm}$
- When is there a Lévy process ξ with CE Ψ such that $\Psi(\vartheta) = -\Psi^{-}(-\vartheta)\Psi^{+}(\vartheta)$?

The inverse problem

- Let $H^{\pm}$ be a pair of subordinators with CEs $\Psi^{\pm}$
- When is there a Lévy process ξ with CE Ψ such that $\Psi(\vartheta) = -\Psi^{-}(-\vartheta)\Psi^{+}(\vartheta)$?
- When such ξ exists, we call $H^{\pm}$ **friends** and ξ the **bonding process**

The theorem of friends

- $d^{\pm}$ = drift of $H^{\pm}$

THE THEOREM OF FRIENDS

- $d^{\pm}$ = drift of $H^{\pm}$
- $\Pi^{\pm}$ = Lévy measure of $H^{\pm}$

The theorem of friends

- $d^{\pm}$ = drift of $H^{\pm}$
- $\Pi^{\pm}$ = Lévy measure of $H^{\pm}$
- $H^{\pm}$ are **compatible** if $d^{\mp} > 0$ implies $\Pi^{\pm}$ is absolutely continuous and its density $\partial\Pi^{\pm}$ is the tail of a signed measure

The theorem of friends

- $d^{\pm}$ = drift of $H^{\pm}$
- $\Pi^{\pm}$ = Lévy measure of $H^{\pm}$
- $H^{\pm}$ are **compatible** if $d^{\mp} > 0$ implies $\Pi^{\pm}$ is absolutely continuous and its density $\partial\Pi^{\pm}$ is the tail of a signed measure

Theorem (Vigon)

Define

$$\Upsilon(x) = \begin{cases} \int_{(0,\infty)} \big(\Pi^-(y,\infty) - \Psi^-(0)\big)\, \Pi^+(x + \mathrm{d}y) + d^-\partial\Pi^+(x), & x > 0, \\ \int_{(0,\infty)} \big(\Pi^+(y,\infty) - \Psi^+(0)\big)\, \Pi^-(-x + \mathrm{d}y) + d^+\partial\Pi^-(-x), & x < 0. \end{cases}$$

*$H^{\pm}$ are friends **if and only if** they are compatible and Υ is decreasing on $(0,\infty)$ and increasing on $(-\infty,0)$.*

Then, Υ is a.e. the right/left tail of the Lévy measure of the bonding process.

Philanthropy

- Let $H_t^+ = d^+ t$. A subordinator H^- is called a **philanthropist** if it is a friend of H^+.

Philanthropy

- Let $H_t^+ = d^+ t$. A subordinator H^- is called a **philanthropist** if it is a friend of H^+.
- **Equivalently**, a subordinator is called a philanthropist if its Lévy measure admits a decreasing density.

Philanthropy

- Let $H_t^+ = d^+ t$. A subordinator H^- is called a **philanthropist** if it is a friend of H^+.
- **Equivalently**, a subordinator is called a philanthropist if its Lévy measure admits a decreasing density.

Theorem (Vigon)

Any two philanthropists can be friends.

Example (spectrally negative processes)

- Let H^{+} be a (killed) pure drift

Example (spectrally negative processes)

- Let H^{+} be a (killed) pure drift
- Let H^{-} be a philanthropist

Example (spectrally negative processes)

- Let H^+ be a (killed) pure drift
- Let H^- be a philanthropist
- Then $H^\pm$ are friends and the bonding process is a spectrally negative Lévy process

Example (spectrally negative processes)

- Let H^{+} be a (killed) pure drift
- Let H^{-} be a philanthropist
- Then $H^{\pm}$ are friends and the bonding process is a spectrally negative Lévy process
- All spectrally negative Lévy processes are of this form

Markov additive processes

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \ldots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \ldots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

- **Equivalently**, a Markov additive process is a regime-switching Lévy process:

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \dots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

- **Equivalently**, a Markov additive process is a regime-switching Lévy process:
  - $\xi^{(i)}$ is a Lévy process for each i

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \ldots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

- **Equivalently**, a Markov additive process is a regime-switching Lévy process:
  - $\xi^{(i)}$ is a Lévy process for each i
  - J is a Markov chain with transition matrix Q

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \dots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

- **Equivalently**, a Markov additive process is a regime-switching Lévy process:
  - $\xi^{(i)}$ is a Lévy process for each i
  - J is a Markov chain with transition matrix Q
  - when J is in state i, run $\xi^{(i)}$

Markov additive processes

- A process (ξ, J) with state space $(\mathbb{R} \cup \{\partial\}) \times \{1, \dots, N\}$ is a **Markov additive process (MAP)** if

$$\text{given } \{J_t = i\},\ (\xi_{t+s} - \xi_t, J_{t+s}) \text{ is independent of the past up to } t,$$
$$\text{and has the same distribution as } (\xi_s, J_s) \text{ under } \mathbb{P}^{0,i}$$

- **Equivalently**, a Markov additive process is a regime-switching Lévy process:
  - $\xi^{(i)}$ is a Lévy process for each i
  - J is a Markov chain with transition matrix Q
  - when J is in state i, run $\xi^{(i)}$
  - when J moves to j, make a jump from distribution F_{ij} and run $\xi^{(j)}$

Markov additive processes (notation)

- Let Π_{ii} be the Lévy measure of $\xi^{(i)}$ and $\Pi_{ij} = q_{ij}F_{ij}$ when $i \neq j$

Markov additive processes (notation)

- Let Π_{ii} be the Lévy measure of $\xi^{(i)}$ and $\Pi_{ij} = q_{ij}F_{ij}$ when $i \neq j$
- Call $\mathbf{\Pi} = (\Pi_{ij})_{i,j=1,\ldots,N}$ the **matrix Lévy measure** of ξ

Markov additive processes (notation)

- Let Π_{ii} be the Lévy measure of $\xi^{(i)}$ and $\Pi_{ij} = q_{ij}F_{ij}$ when $i \neq j$
- Call $\mathbf{\Pi} = (\Pi_{ij})_{i,j=1,\ldots,N}$ the **matrix Lévy measure** of ξ
- **Matrix characteristic exponent** $\mathbf{\Psi}$: $\mathbb{E}^{0,i}[e^{\mathrm{i}\vartheta\xi_t}; J_t = j] = (e^{t\mathbf{\Psi}(\vartheta)})_{ij}$

Markov additive processes (notation)

- Let Π_{ii} be the Lévy measure of $\xi^{(i)}$ and $\Pi_{ij} = q_{ij}F_{ij}$ when $i \neq j$
- Call $\boldsymbol{\Pi} = (\Pi_{ij})_{i,j=1,\dots,N}$ the **matrix Lévy measure** of ξ
- **Matrix characteristic exponent** $\boldsymbol{\Psi}$: $\mathbb{E}^{0,i}[e^{\mathrm{i}\vartheta\xi_t}; J_t = j] = (e^{t\boldsymbol{\Psi}(\vartheta)})_{ij}$
- Structure:

$$\boldsymbol{\Psi}(\vartheta) = \begin{pmatrix} \Psi_1(\vartheta) + q_{11} & \widehat{\Pi}_{12}(\vartheta) & \cdots \\ \widehat{\Pi}_{21}(\vartheta) & \Psi_2(\vartheta) + q_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

where Ψ_i is CE of $\xi^{(i)}$ and $\widehat{\Pi}_{ij}$ is the characteristic function of Π_{ij}

Markov additive processes (notation)

- Let Π_{ii} be the Lévy measure of $\xi^{(i)}$ and $\Pi_{ij} = q_{ij} F_{ij}$ when $i \neq j$
- Call $\mathbf{\Pi} = (\Pi_{ij})_{i,j=1,\ldots,N}$ the **matrix Lévy measure** of ξ
- **Matrix characteristic exponent** $\mathbf{\Psi}$: $\mathbb{E}^{0,i}[e^{i\vartheta\xi_t}; J_t = j] = (e^{t\mathbf{\Psi}(\vartheta)})_{ij}$
- Structure:

$$\mathbf{\Psi}(\vartheta) = \begin{pmatrix} \Psi_1(\vartheta) + q_{11} & \widehat{\Pi}_{12}(\vartheta) & \cdots \\ \widehat{\Pi}_{21}(\vartheta) & \Psi_2(\vartheta) + q_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

  where Ψ_i is CE of $\xi^{(i)}$ and $\widehat{\Pi}_{ij}$ is the characteristic function of Π_{ij}
- Let π be the invariant measure of J

Wiener-Hopf factorisation

- (H^+, J^+) and (H^-, J^-): ladder height processes of (ξ, J)

Wiener-Hopf factorisation

- (H^{+}, J^{+}) and (H^{-}, J^{-}): ladder height processes of (ξ, J)
- They are MAP subordinators (increasing MAPs) with matrix exponents $\mathbf{\Psi}^{\pm}$

Wiener-Hopf factorisation

- (H^+, J^+) and (H^-, J^-): ladder height processes of (ξ, J)
- They are MAP subordinators (increasing MAPs) with matrix exponents $\mathbf{\Psi}^{\pm}$
- Path picture is the same

Wiener-Hopf factorisation

- (H^+, J^+) and (H^-, J^-): ladder height processes of (ξ, J)
- They are MAP subordinators (increasing MAPs) with matrix exponents $\mathbf{\Psi}^{\pm}$
- Path picture is the same

Theorem (Dereich, Döring and Kyprianou)

$$\mathbf{\Psi}(\vartheta) = -\Delta_\pi^{-1}\mathbf{\Psi}^-(-\vartheta)^T\Delta_\pi\mathbf{\Psi}^+(\vartheta),$$

where Δ_π is the diagonal matrix containing π.

Markov additive friendship

The inverse problem

- Two MAP subordinators $(H^{\pm}, J^{\pm})$ are **π-friends** if there is a MAP for which they satisfy the above matrix equation

The inverse problem

- Two MAP subordinators $(H^{\pm}, J^{\pm})$ are **π-friends** if there is a MAP for which they satisfy the above matrix equation
- Are there necessary and sufficient conditions for friendship?

The inverse problem

- Two MAP subordinators $(H^{\pm}, J^{\pm})$ are **π-friends** if there is a MAP for which they satisfy the above matrix equation
- Are there necessary and sufficient conditions for friendship?
- Is there a theory of philanthropy?

Compatibility

$(H^{\pm}, J^{\pm})$ are **π-compatible** if

- $d_i^{\mp} > 0$ implies $\Pi_i^{\pm}$ is absolutely continuous and its density is the tail of a signed measure

Compatibility

$(H^{\pm}, J^{\pm})$ are **π-compatible** if

- $d_i^{\mp} > 0$ implies $\Pi_i^{\pm}$ is absolutely continuous and its density is the tail of a signed measure
- Elements of the vector $-\Delta_\pi^{-1} \mathbf{\Psi}^-(0)^T \Delta_\pi \mathbf{\Psi}^+(0)\mathbf{1}$ are non-positive (row sums of Q-matrix of the putative bonding MAP)

Compatibility

$(H^{\pm}, J^{\pm})$ are **π-compatible** if

- $d_i^{\mp} > 0$ implies $\Pi_i^{\pm}$ is absolutely continuous and its density is the tail of a signed measure
- Elements of the vector $-\Delta_\pi^{-1}\boldsymbol{\Psi}^{-}(0)^T\Delta_\pi\boldsymbol{\Psi}^{+}(0)\mathbf{1}$ are non-positive (row sums of Q-matrix of the putative bonding MAP)
- ...and an additional positivity condition involving intensity of zero-jumps

Compatibility

$(H^{\pm}, J^{\pm})$ are **π-compatible** if

- $d_i^{\mp} > 0$ implies $\Pi_i^{\pm}$ is absolutely continuous and its density is the tail of a signed measure
- Elements of the vector $-\Delta_\pi^{-1}\mathbf{\Psi}^-(0)^T\Delta_\pi\mathbf{\Psi}^+(0)\mathbf{1}$ are non-positive (row sums of Q-matrix of the putative bonding MAP)
- ...and an additional positivity condition involving intensity of zero-jumps

Compatibility is a **necessary** condition for π-friendship!

The theorem of friends

Theorem

Define the matrix-valued function

$$\boldsymbol{\Upsilon}(x) = \begin{cases} \int_{0+}^{\infty} \Delta_\pi^{-1}\Big(\boldsymbol{\Pi}^-(y,\infty) - \boldsymbol{\Psi}^-(0)\Big)^\top \Delta_\pi\, \boldsymbol{\Pi}^+(x+\mathrm{d}y) + \Delta_{\mathbf{d}}^- \partial\boldsymbol{\Pi}^+(x), & x > 0, \\ \int_{0+}^{\infty} \Delta_\pi^{-1}\big(\boldsymbol{\Pi}^-(-x+\mathrm{d}y)\big)^\top \Delta_\pi \left(\boldsymbol{\Pi}^+(y,\infty) - \boldsymbol{\Psi}^+(0)\right) + \Delta_\pi^{-1}\big(\Delta_{\mathbf{d}}^+ \partial\boldsymbol{\Pi}^-(-x)\big)^\top \Delta_\pi, & x < 0, \end{cases}$$

Two MAP subordinators $(H^\pm, J^\pm)$ are π-friends ***if and only if*** *they are π-compatible and Υ_{ij} is decreasing on $(0,\infty)$ and increasing on $(-\infty, 0)$.*

Then, $\boldsymbol{\Upsilon}$ is a.e. the right/left tail of the matrix Lévy measure of the bonding process.

Examples

Examples are hard to come by

- Only known MAP factorisation is from the **deep factorisation of the stable process**

Examples are hard to come by

- Only known MAP factorisation is from the **deep factorisation of the stable process**
- The MAP in question is the Lamperti-Kiu transform of the stable process

Examples are hard to come by

- Only known MAP factorisation is from the **deep factorisation of the stable process**
- The MAP in question is the Lamperti-Kiu transform of the stable process
- The factorisation is obtained using detailed knowledge of the stable process

Examples are hard to come by

- Only known MAP factorisation is from the **deep factorisation of the stable process**
- The MAP in question is the Lamperti-Kiu transform of the stable process
- The factorisation is obtained using detailed knowledge of the stable process
- No simpler proof is known; verifying the conditions of friendship appears difficult

Spectrally negative MAPs

- Let (H^+, J^+) be a pure drift, i.e., $H_t^+ = \int_0^t d^+_{J_s^+}\,\mathrm{d}s$

Theorem

A MAP subordinator (H^-, J^-) is π-friends with a pure drift if and only if they are π-compatible and

$$-\Delta_\pi^{-1}\boldsymbol{\Psi}^+(0)^T\Delta_\pi\boldsymbol{\Pi}^-(x,\infty) + \Delta_{\mathbf{d}^+}\partial\boldsymbol{\Pi}^-(x), \qquad x > 0,$$

is decreasing.

Spectrally negative MAPs

- Let (H^+, J^+) be a pure drift, i.e., $H_t^+ = \int_0^t d_{J_s^+}^+ \,\mathrm{d}s$

Theorem

A MAP subordinator (H^-, J^-) is π-friends with a pure drift if and only if they are π-compatible and

$$-\Delta_\pi^{-1}\boldsymbol{\Psi}^+(0)^T\Delta_\pi\boldsymbol{\Pi}^-(x,\infty) + \Delta_{\mathbf{d}^+}\partial\boldsymbol{\Pi}^-(x), \quad x > 0,$$

is decreasing.

- Allows us to construct spectrally negative MAPs

Spectrally negative MAPs

- Let (H^+, J^+) be a pure drift, i.e., $H_t^+ = \int_0^t d^+_{J_s^+}\, \mathrm{d}s$

Theorem

A MAP subordinator (H^-, J^-) is π-friends with a pure drift if and only if they are π-compatible and

$$-\Delta_\pi^{-1} \boldsymbol{\Psi}^+(0)^T \Delta_\pi \boldsymbol{\Pi}^-(x, \infty) + \Delta_{\mathbf{d}^+} \partial \boldsymbol{\Pi}^-(x), \quad x > 0,$$

is decreasing.

- Allows us to construct spectrally negative MAPs
- When the Π_i^- have completely monotone density, can make conditions more explicit

Spectrally negative MAPs

- Let (H^+, J^+) be a pure drift, i.e., $H_t^+ = \int_0^t d_{J_s^+}^+ \, \mathrm{d}s$

Theorem

A MAP subordinator (H^-, J^-) is π-friends with a pure drift if and only if they are π-compatible and

$$-\Delta_\pi^{-1}\boldsymbol{\Psi}^+(0)^T\Delta_\pi\boldsymbol{\Pi}^-(x,\infty) + \Delta_{\mathbf{d}^+}\partial\boldsymbol{\Pi}^-(x), \quad x > 0,$$

is decreasing.

- Allows us to construct spectrally negative MAPs
- When the Π_i^- have completely monotone density, can make conditions more explicit
- Being friends with a drift does not make you friends with anything else: ‘philanthropy’, if it exists, is more complicated

Double exponential MAPs

- Let $(H^{\pm}, J^{\pm})$ be MAP subordinators with exponential jumps in every state

Double exponential MAPs

- Let $(H^{\pm}, J^{\pm})$ be MAP subordinators with exponential jumps in every state
- Given some balances between coefficients, such processes can be friends

Double exponential MAPs

- Let $(H^{\pm}, J^{\pm})$ be MAP subordinators with exponential jumps in every state
- Given some balances between coefficients, such processes can be friends
- The bonding MAP has double exponential jumps within and between every state

Double exponential MAPs

- Let $(H^{\pm}, J^{\pm})$ be MAP subordinators with exponential jumps in every state
- Given some balances between coefficients, such processes can be friends
- The bonding MAP has double exponential jumps within and between every state
- May be second example of two-sided MAP with known ladder processes?

Work in progress | open problems

Uniqueness of Wiener-Hopf factorisation

- We have studied the matrix equation $\mathbf{\Psi}(\vartheta) = -\Delta_\pi^{-1}\mathbf{\Psi}^-(-\vartheta)^T\Delta_\pi\mathbf{\Psi}^+(\vartheta)$

Uniqueness of Wiener-Hopf factorisation

- We have studied the matrix equation $\mathbf{\Psi}(\vartheta) = -\Delta_\pi^{-1}\mathbf{\Psi}^-(-\vartheta)^T\Delta_\pi\mathbf{\Psi}^+(\vartheta)$
- To be sure that $(H^\pm, J^\pm)$ really are the ladder processes, we need uniqueness

Uniqueness of Wiener-Hopf factorisation

- We have studied the matrix equation $\mathbf{\Psi}(\vartheta) = -\Delta_\pi^{-1}\mathbf{\Psi}^-(-\vartheta)^T\Delta_\pi\mathbf{\Psi}^+(\vartheta)$
- To be sure that $(H^\pm, J^\pm)$ really are the ladder processes, we need uniqueness
- We have partial results, for instance under absolute continuity conditions

Uniqueness of Wiener-Hopf factorisation

- We have studied the matrix equation $\mathbf{\Psi}(\vartheta) = -\Delta_\pi^{-1}\mathbf{\Psi}^-(-\vartheta)^T \Delta_\pi \mathbf{\Psi}^+(\vartheta)$
- To be sure that $(H^\pm, J^\pm)$ really are the ladder processes, we need uniqueness
- We have partial results, for instance under absolute continuity conditions
- Surprisingly, this does not seem to be known in full generality even for Lévy processes

Philanthropy

- Is there a notion of ‘philanthropist’ that implies friendship with other philanthropists?
- Are there conditions that do not depend on π?

Complex analytic structure

- In the ‘meromorphic’ Lévy processes (Lamperti-stable, simple/double hypergeometric), the factorisation can be deduced from the poles and zeroes of the CE

Complex analytic structure

- In the ‘meromorphic’ Lévy processes (Lamperti-stable, simple/double hypergeometric), the factorisation can be deduced from the poles and zeroes of the CE
- Is there such an approach for MAPs?

Complex analytic structure

- In the ‘meromorphic’ Lévy processes (Lamperti-stable, simple/double hypergeometric), the factorisation can be deduced from the poles and zeroes of the CE
- Is there such an approach for MAPs?
- This would give an alternative avenue of attack for ‘deep factorization’ type processes

Further reading

L. Döring, L. Trottner and A. R. Watson
Markov additive friendships
In preparation (working title)

Further reading

L. Döring, L. Trottner and A. R. Watson
Markov additive friendships
In preparation (working title)

Thank you!