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A growth-fragmentation found in the cone excursions of Brownian motion (and in the quantum disc)

- Take a planar Brownian motion, consider two types of ‘cone excursion’:

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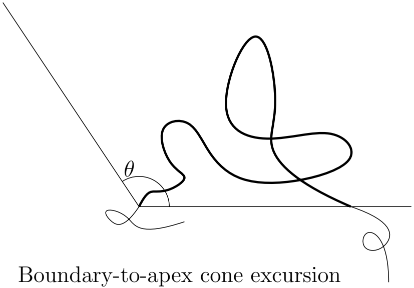
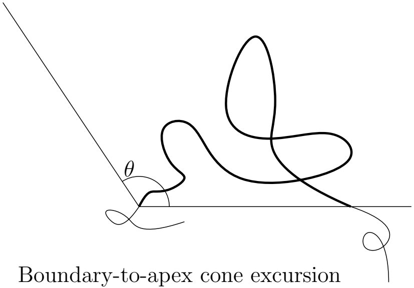
  - Boundary-to-apex cone excursions



Boundary-to-apex cone excursion

Cone excursions

- Take a planar Brownian motion, consider two types of 'cone excursion':
  - Boundary-to-apex cone excursions
  - Whole-path cone excursions



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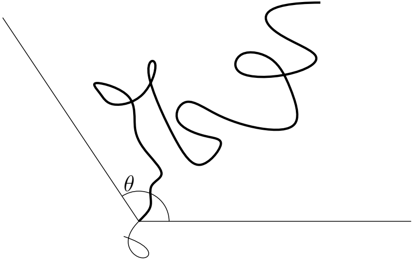
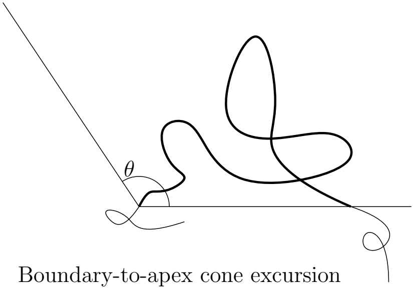


Whole-path cone excursion

Cone excursions

- Take a planar Brownian motion, consider two types of ‘cone excursion’:
  - Boundary-to-apex cone excursions
  - Whole-path cone excursions
- ‘Cone-free times’ (between boundary-to-apex excursions) form a regenerative set



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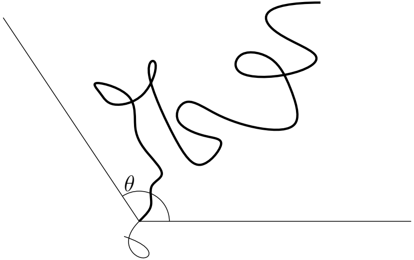

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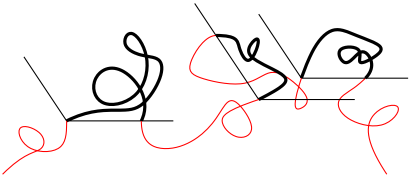
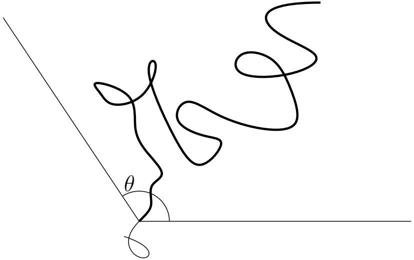


Boundary-to-apex cone-free times
and **cone excursions**



Whole-path cone excursion

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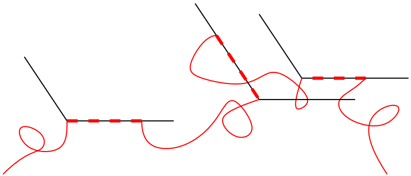
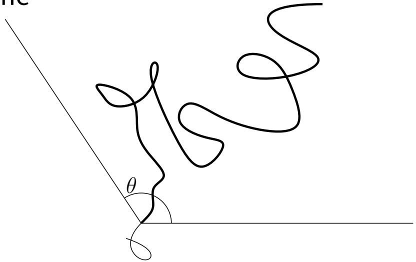
- The path is cut into cone excursions between said times
- Write $\tau^{>}$ for boundary-to-apex inverse local time



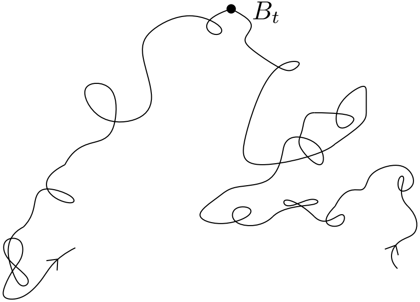
The path at boundary-to-apex cone-free times (with **jumps**)



Whole-path cone excursion

Targeting a time

- Take a point on a Brownian path B (at time t), and progressively...



Targeting a time

- Take a point on a Brownian path B (at time t), and progressively...
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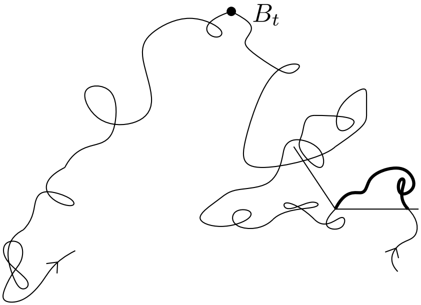

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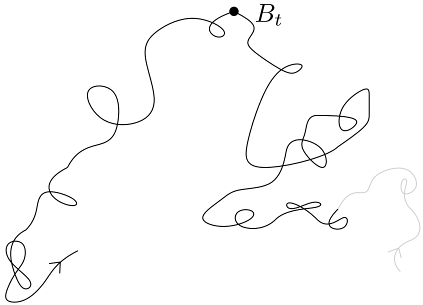

- Take a point on a Brownian path B (at time t), and progressively...
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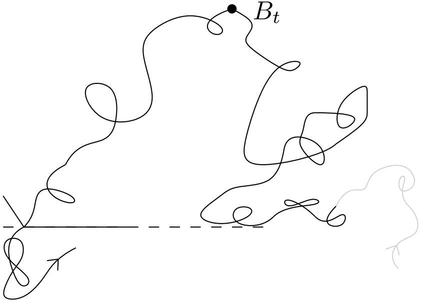

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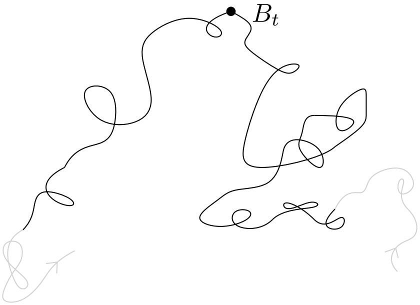

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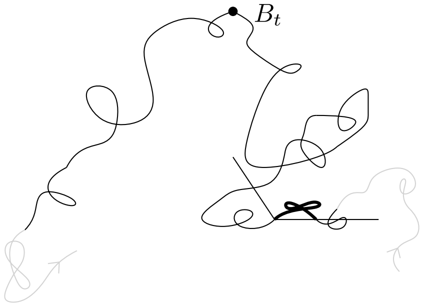

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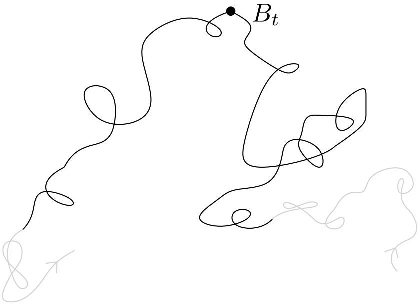

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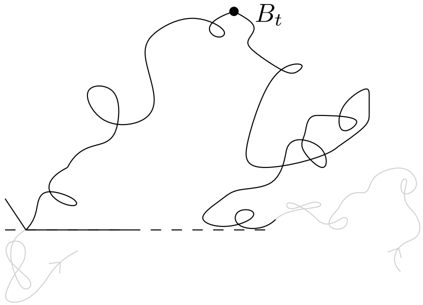

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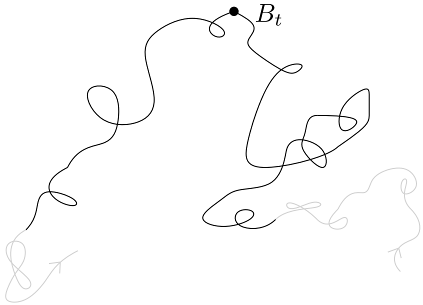

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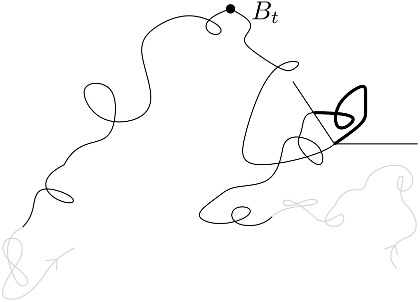

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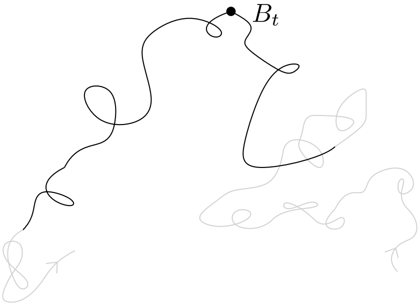

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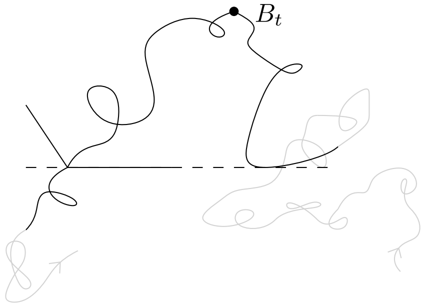

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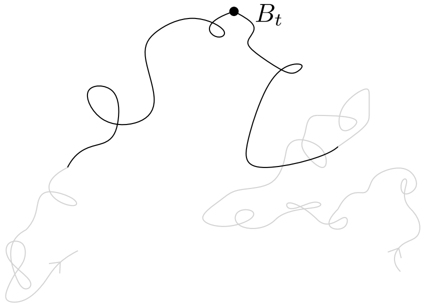
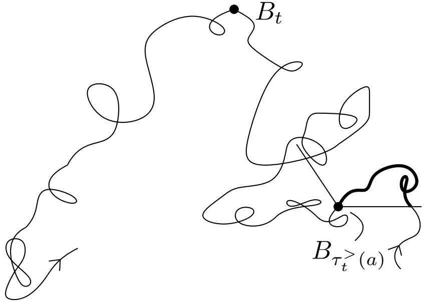


Targeting a time (more precisely)

- Fix $t > 0$, write $\tau_t^>$ for boundary-to-apex inverse local time of $(B_s \colon s \le t)$



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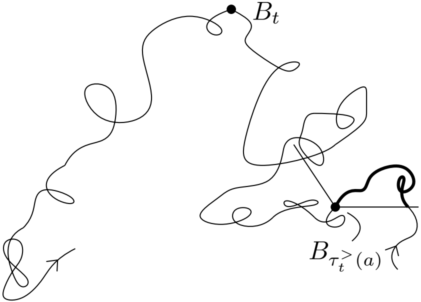
- Fix $t > 0$, write $\tau_t^>$ for boundary-to-apex inverse local time of $(B_s : s \le t)$
- Then fix $a \ge 0$, and...



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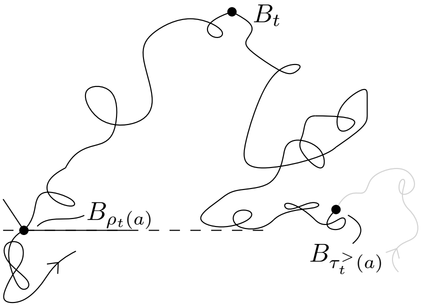

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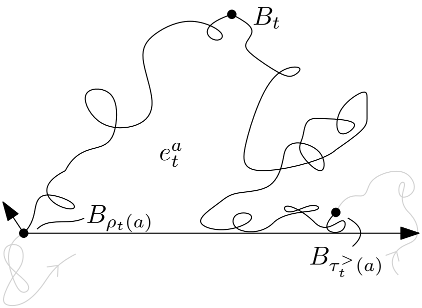

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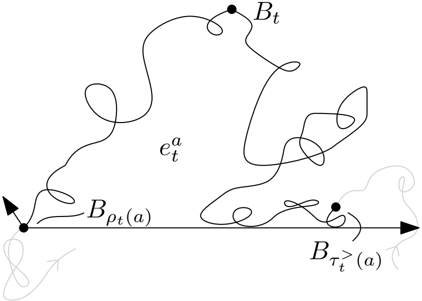

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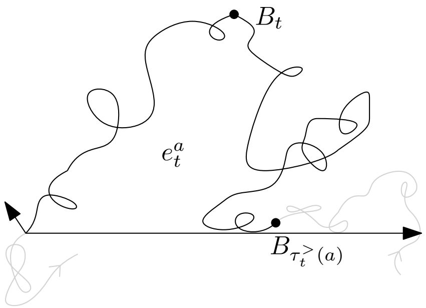

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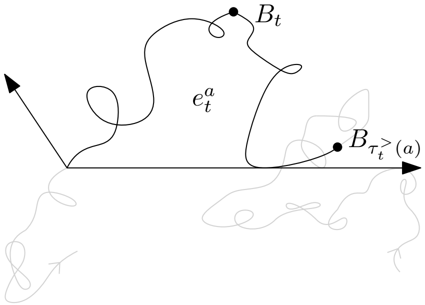
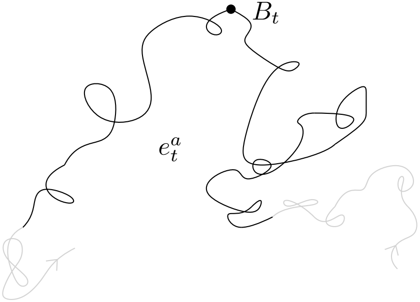


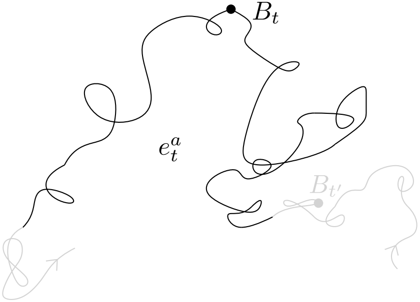
Targeting multiple times

- We have the excursion targeting t



Targeting multiple times

- We have the excursion targeting t
- What if we target some other t'?



Targeting multiple times

- We have the excursion targeting t
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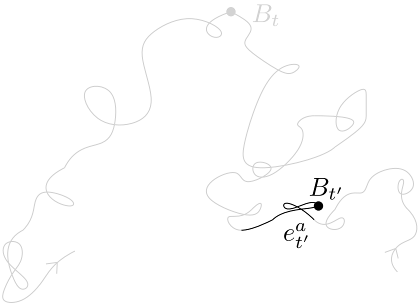
- Every piece cut out while targeting t is one which is included in the excursion targeting some other t'



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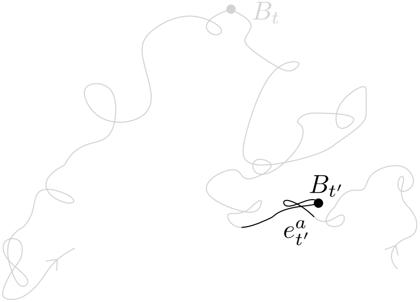

Targeting multiple times

- We have the excursion targeting t
- What if we target some other t'?
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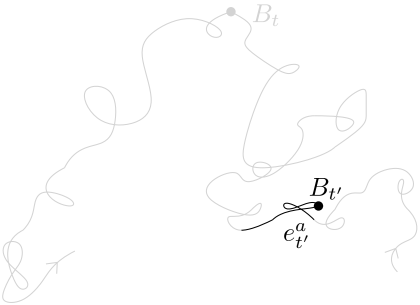
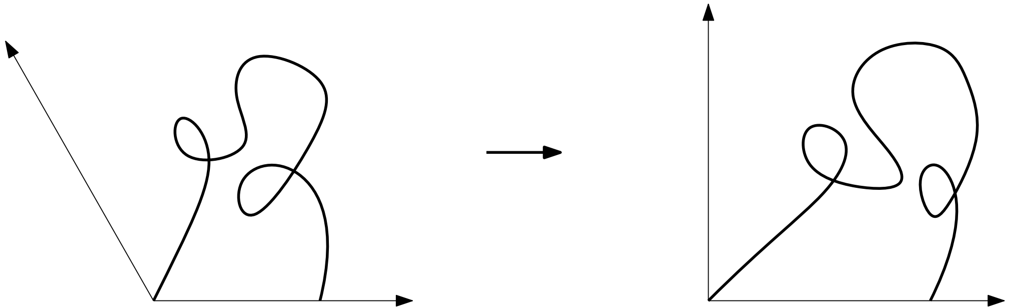
- Consider targeting every time simultaneously
- There is some kind of branching process for us to capture



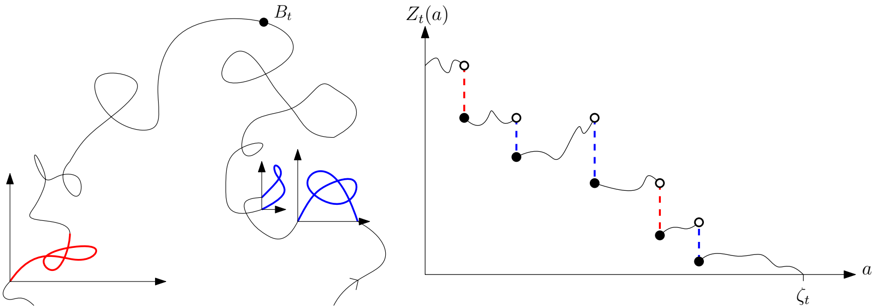
Summarising the path targeting t

- Map the cone with apex angle ϑ to the positive quadrant $\mathbb{R}^2_+$; standard Brownian motion becomes correlated



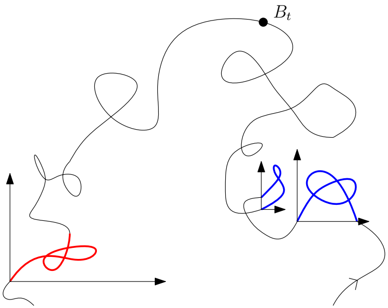
Summarising the path targeting t

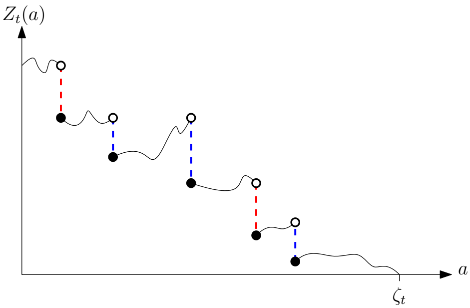
- Map the cone with apex angle ϑ to the positive quadrant $\mathbb{R}_+^2$; standard Brownian motion becomes correlated
- The initial displacement of the excursion targeting t at local time a: $e_t^a(0) \in \mathbb{R}_+^2$



Summarising the path targeting t

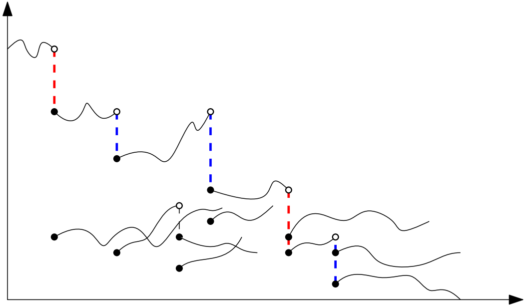
- Map the cone with apex angle ϑ to the positive quadrant $\mathbb{R}_+^2$; standard Brownian motion becomes correlated
- The initial displacement of the excursion targeting t at local time a: $e_t^a(0) \in \mathbb{R}_+^2$
- In the case $\vartheta = 2\pi/3$ look at its ℓ^1-norm: $Z_t(a) = \|e_t^a(0)\|_1$





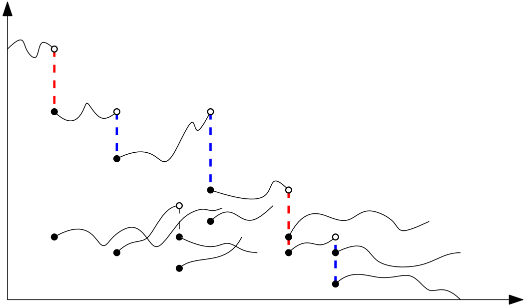
A growth-fragmentation is:

- a system of particles (excursions targeting each time t)...



A growth-fragmentation is:

- a system of particles (excursions targeting each time t)...
- ...each summarised by a trait (ℓ^1-norm of its initial displacement)...



Growth-fragmentations

A growth-fragmentation is:

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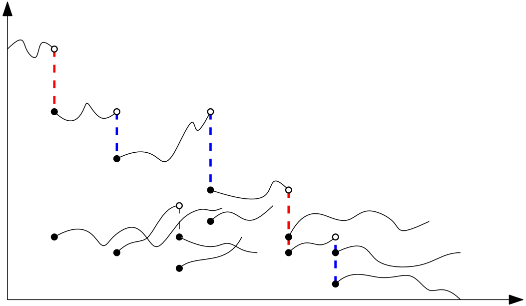

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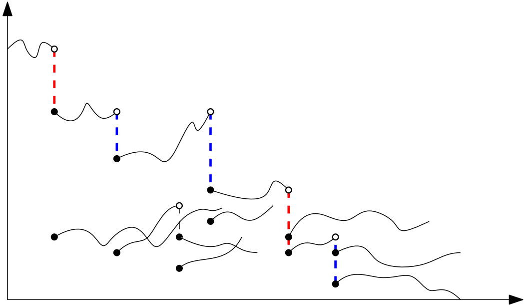

Growth-fragmentations

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- a system of particles (excursions targeting each time t)...
- ...each summarised by a trait (ℓ^1-norm of its initial displacement)...
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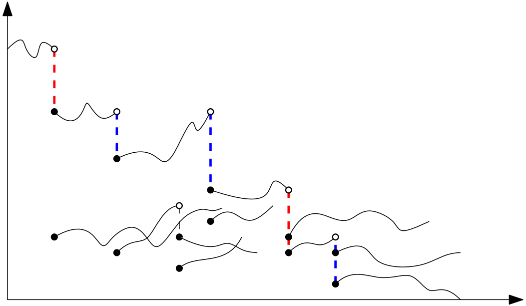
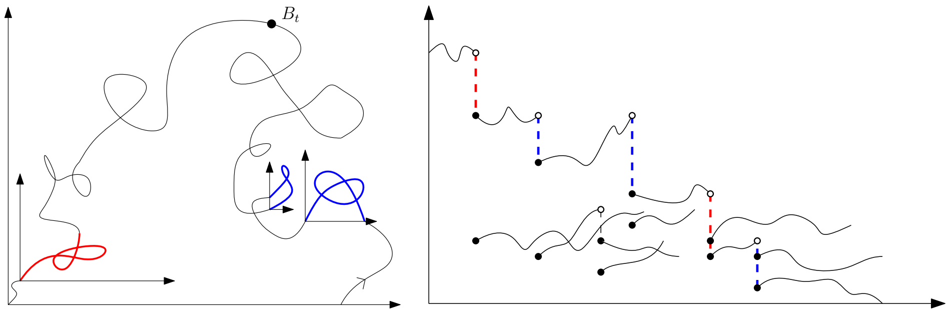
- ...whose path only jumps down...
- ...and each jump of which is accompanied by the birth of another particle, conditionally independent given initial trait value



We do all this starting with B given by a boundary-to-apex excursion with fixed initial value $B_0 = z \in \mathbb{R}^2_+$.

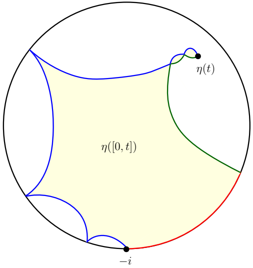
Theorem (Da Silva-Powell-W, vague version)

The particles t with traits $(Z_t(a) : 0 \le a \le \zeta_t)$ (the ℓ^1-norm summary of initial displacements of excursions targeting t) form a growth-fragmentation process whose law we can characterise.



Relationship with the quantum disc

- The quantum disc is a ball of radius 1 in the complex plane, loosely speaking endowed with a Riemannian metric $e^{\gamma h(z)}(dx^2 + dy^2)$ at $z = x + iy$, where h is a Gaussian free field



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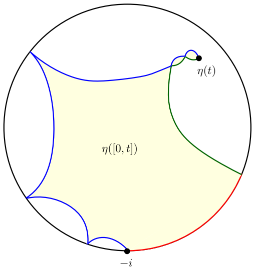
- Take $\gamma = \sqrt{8/3}$ and mark the point $-i$ on the boundary



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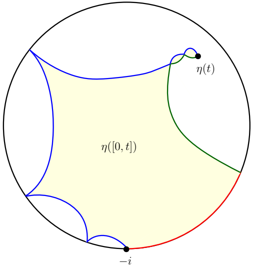
- Take $\gamma = \sqrt{8/3}$ and mark the point $-i$ on the boundary
- Draw counterclockwise space-filling SLE_6 curve started at $-i$, targeting the same point



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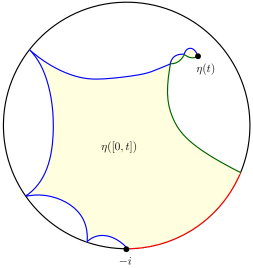

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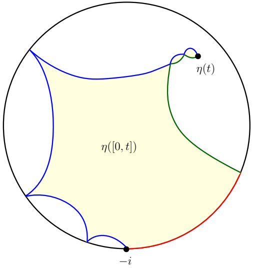
- Our growth-fragmentation describes the total (left and right) boundary length of the branches



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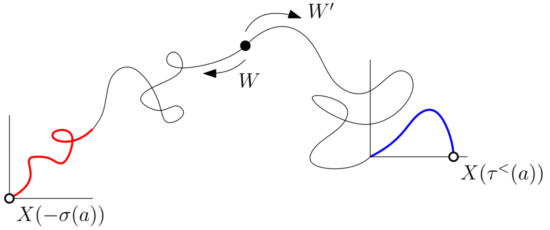
Theorem

S has the law of a 3/2-stable process conditioned to stay positive (and this characterises the growth-fragmentation)

A beautiful construction of a conditioned 3/2-stable process

- Everything here is in reverse time!



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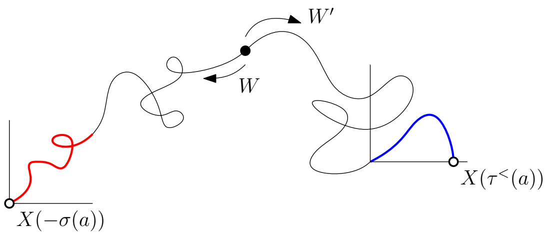

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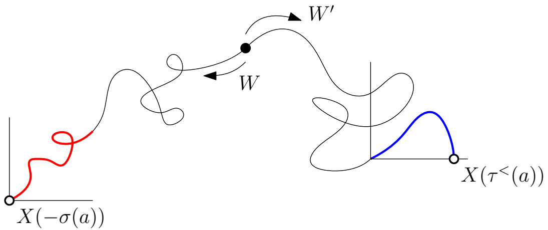

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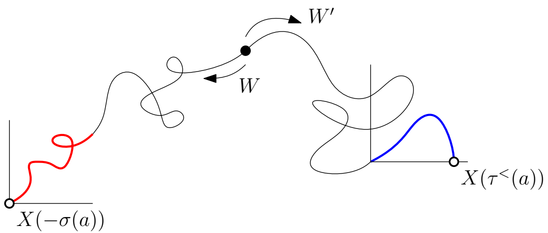

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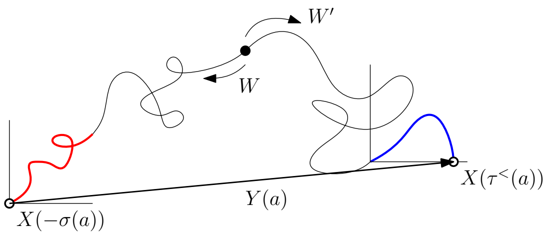

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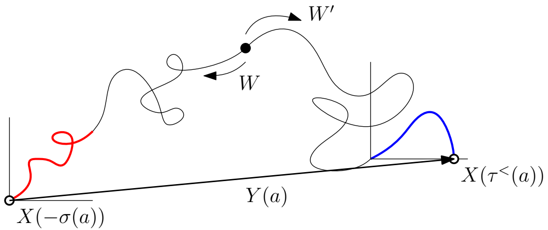
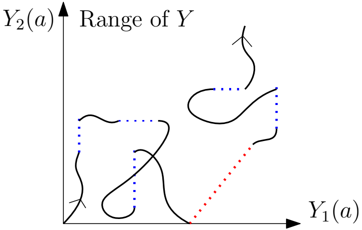
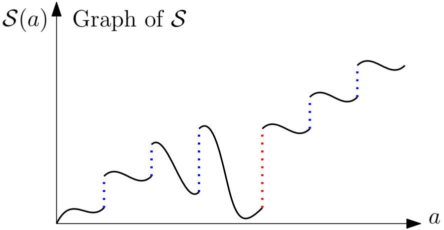
$\mathcal{S}$ is a 3/2-stable process, with only positive jumps, conditioned to stay positive.



Sketch of the proof





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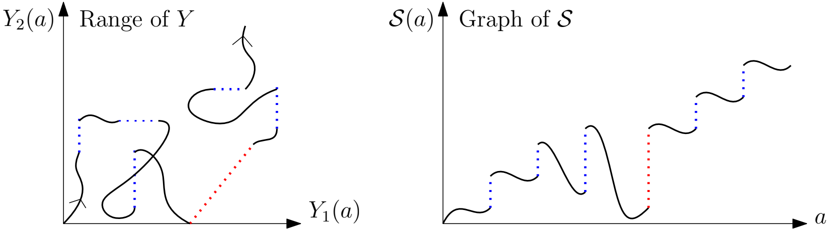

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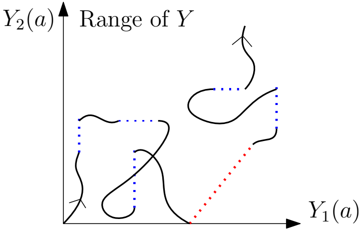

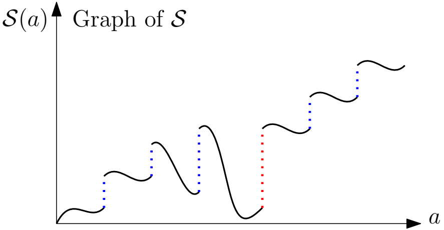

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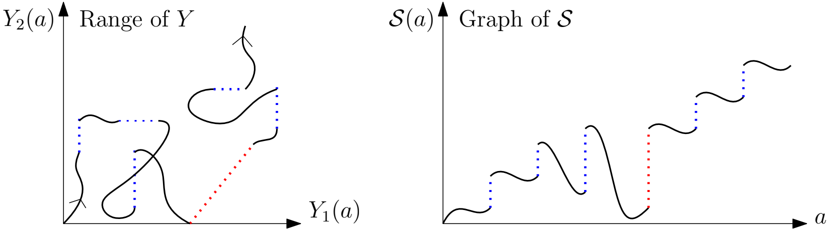

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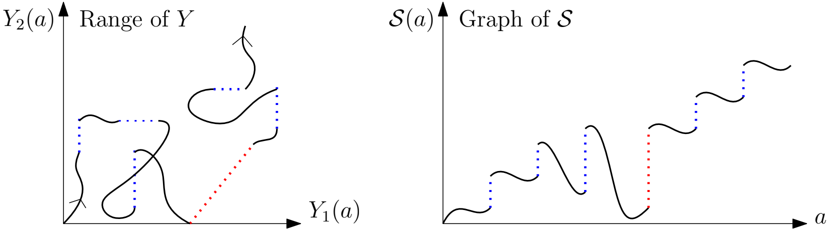

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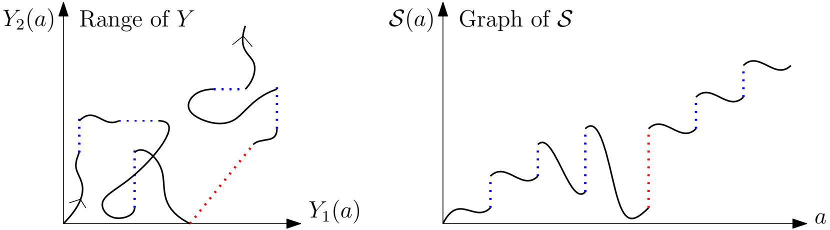

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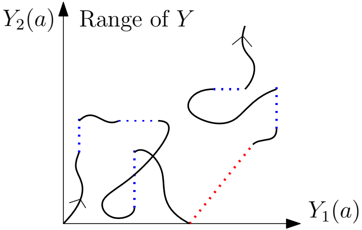

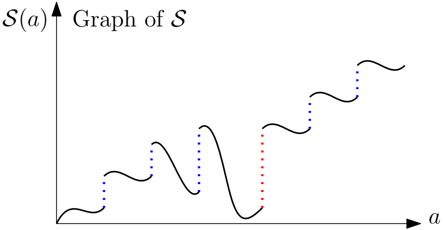

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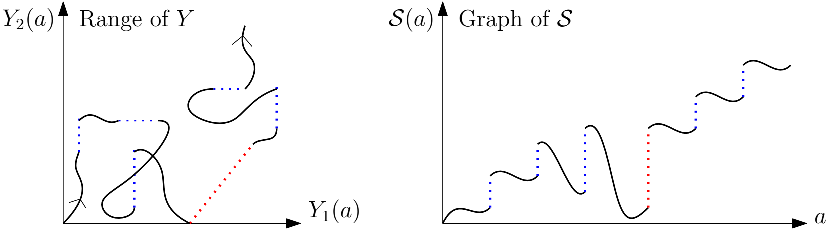

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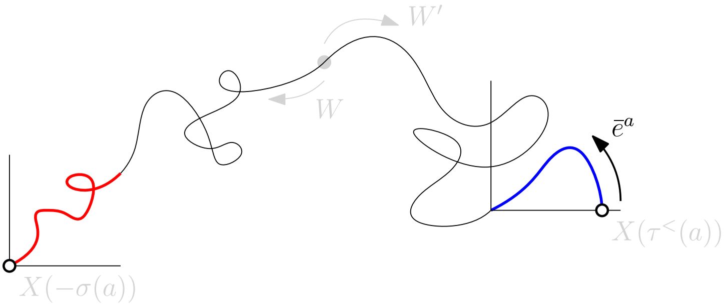

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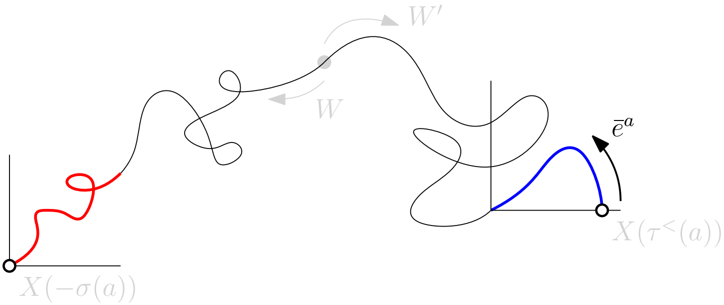

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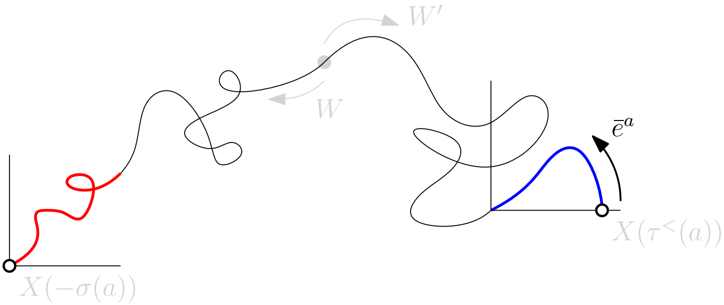
- Then $(\bar{e}^A, A)$ has the same distribution as a generic whole-path excursion together with a time uniformly chosen within its lifetime



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- ...and find a special martingale whose limit law is that of the lifetime of a typical excursion (recovering a result about the volume of Boltzmann triangulations)

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- It may be possible to derive our results with quantum gravity arguments, but we use nothing but an analysis of Brownian motion

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- Can we start things at zero?

Further reading

W. Da Silva, E. Powell and A. R. Watson
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Thank you!