

Math 220: Discrete Mathematics
HW for Section 3.4

1. Prove the following:
 - (a) If $a \equiv b \pmod{n}$, and $m|n$, then $a \equiv b \pmod{m}$.
 - (b) If $a \equiv b \pmod{n}$, and $c > 0$, then $ca \equiv cb \pmod{cn}$.
2.
 - (a) Show that if a, b are relatively prime natural numbers, then n is divisible by the product ab if and only if $a|n$ and $b|n$.
 - (b) Show that if a, b are not relatively prime natural numbers, then there exists a natural number n which is divisible by both a and b but not ab .
3. State and prove the divisibility tests for the following numbers:
 - (a) 12
 - (b) 25
4. The number 25,730 is divisible by 10 and by 2. Is it divisible by 20? Explain why or why not.
5. Suppose a is a natural number such that the sum of the digits of a equals the sum of the digits of $5a$. Show that $9|a$.
6. Let n be a natural number, and let m be formed from n by switching two digits d_i and d_j . (For example, 86231 is formed from 83261 by switching d_3 and d_1 . Mixing up two digits like this is sometimes called a *transposition error*.) Show that $9|(m - n)$.