NAIVE DECISION MAKING

1. Book

p.67 Example 2.5.16. The point is to show that there can't be a Tchebychev-type inequality for these random variables, so the second displayed equation in the example should be $P(|\bar{X}_n - 1| \leqslant \delta) \leqslant \epsilon$ and the next one should be $P(|\bar{X}_n - 1| > \delta) \geqslant 1 - \epsilon$. I think the proof needs some corrections beyond those in the errata. Since $|\bar{X}_n - 1| \leqslant \delta$ iff $1 - \delta \leqslant \bar{X}_n \leqslant 1 + \delta$ (*), choose N so that $2^{k+1}/N < 1 - \delta$. (The book has $2^{k+1}/N < \delta$ which must be wrong: it's when δ is close to 1, not close to 0, that you will need an enormous N to make it very likely $|\bar{X}_n - 1| > \delta$). Now for $n > N$, with probability at least $1 - \epsilon$ all of $X_{k+1}, X_{k+2}, \ldots$ are zero, so with probability at least $1 - \epsilon$ the mean $\bar{X}_n$ satisfies

$$\begin{aligned} \bar{X}_n &= \sum^n X_i/n \\ &\leqslant \sum^k X_i/N \\ &\leqslant 2^{k+1}/N \\ &< 1 - \delta \end{aligned}$$

so (*) fails, that is, with probability at least $1 - \epsilon$ we have $|\bar{X}_n - 1| > \delta$.

p.69 Exercise 2.5.19 (ii) "jth grottos" should be "jth grotto."

p.75 with the correction of the Kelly factor to 1/12, the first displayed equation should be $\frac{11}{12}\left(\frac{11}{12} + \frac{11}{5}\frac{1}{12}\right) = 121/120$.

p.79 Exercise 2.7.1: there are two (ii)s. In the first (ii), the first term on the right hand side of the displayed equation should be $p_1 \log p_1$.

p.109 Exercise 3.5.10: there's no such c_p (the right hand side can be zero by taking $a = -b$, but the left hand side will not be unless $a = 0$): I think the inequality should instead read $|a + b|^p \leqslant c_p(|a|^p + |b|^p)$.

p.159 Exercise 5.3.9 (ii) "are turned over before the mth card is $(1-t)^m$" should be "$\ldots (1 - t)^k$." "...the probability we stop at the last card is t" should read "...is less than or equal to t", and "...before the mth card is $(1-t)^m$" should read "...is less than or equal to $(1 - t)^k$" (which is what is shown in the solutions).

p.161 last line: the term in the middle of the two inequality signs should be $\log(n!)$.

p.179 Exercise 6.1.7 last part: it's not true that if the process terminates, the pairing it produces must be stable. For example, with the preferences from p.177 the algorithm terminates with the unstable pairing (A, C), (B, D).

p.217 Exercise 7.6.2 (i) first line: 4 "$y > x_0$ and Fred knows the value of y" should be "$z > x_0$ and Fred knows the value of z." George gets to choose the value of z, not y.

p.281 Exercise 7.6.2 (ii) $y < x_0$ should be $z < x_0$. (iii) $y = x_0$ should be $z = x_0$.

p.228 Exercise 8.2.1: "which may be (x_0, y_0) itself" should read "which may be (x_1, y_1) itself" and "for all $(x, y) \in L$ should be "for all $(x, y) \in \tilde{K}$."

p.228 Exercise 8.2.2 (ii): the two conditions (a) and (b) aren't right, e.g. if K is the convex set $\{(x, y) : y \leqslant -x\}$ then K doesn't have any elements with both coordinates strictly positive, but contains elements with nonnegative x and nonzero y, like, $(1, -1)$, and elements with nonnegative y and nonzero x, like $(-1, 1)$. From the solutions I think the conditions are intended to be (a) . . . if $y = 0$ then $x \leqslant 0$ and (b) . . . if $x = 0$ then $y \leqslant 0$.

p.238 last displayed equation: α should be β, twice.

p.243, before Exercise 8.6.2: the Nash equilibria in this game are $(p, q) = (1, 1), (0, 0)$, and $(2/3, 1/3)$. $(1, 0)$ isn't a Nash equilibrium because if the row player is guaranteed to play left then the column player will switch from right to left as doing so improves their payoff from -1 to 0, similarly $(0, 1)$ isn't a Nash equilibrium as if the column player is guaranteed to play left then the row player will switch from right to left.

p.254: both the book and the errata are wrong here.

$$\begin{aligned} P(\text{C wins}|\text{misses first shot}) &= P(\text{C wins}|\text{B hits A})b + P(\text{C wins}|\text{B misses A})(1-b) \\ &= \frac{cb}{b+c-bc} + c(1-b) \\ &= c\frac{b^2c - b^2 - 2bc + c + 2b}{b+c-bc} \end{aligned}$$

which doesn't factor any more. The condition for C to not deliberately miss their first shot is

$$c\frac{b^2c - b^2 - 2bc + c + 2b}{b+c-bc} > c\frac{1-b}{b+c-bc}$$

since the latter is the probability C wins given they miss the first shot at A, which is equivalent to $b(3-b) + (b-1)^2c > 1$. That's certainly true if $3b - b^2 > 1$ which is true for $b < \frac{3-\sqrt{5}}{2} \approx 0.38$. So for $b \geqslant 0.4$, say, C should deliberately miss the first shot.

p.261 first line: it's only a proof of part (ii) of that lemma.

p.267, Figure 9.1. There are missing lines from THH to HHT and HTT to TTH.

p.268 last line: should be p_{TH} not P_{TH} (first thing on the line).

p.274 Exercise 9.5.5: $B(A, S)$ should be $1/(1-p)$, not 0. An A-S game is press-press in every round played, so the expected winnings are $1 + p + p^2 + \cdots$.

p.274 Exercise 9.5.6, last line on the page: the second fraction should be $p/(1-p)$. The last fraction should be $p(1-2p)/(1-p^2)$.

p.283 after Exercise 10.1.3: the two roots should be $t = 1$ and $t = (1-p)/p$, not $p/(1-p)$. The next expression for q_n is right but the one in part (i) of Exercise 10.1.4 should be $q_n = A + B\left(\frac{1-p}{p}\right)^n$.

p.291 after the displayed equation beginning with 10000: it's not really the probability of bankruptcy since you'll still have 10^4 left over. You just won't be able to continue playing. On line 3 of that paragraph $1/p^8$ should be 2^8k^{-8} (rearranging the equation on the previous line). The next displayed equation should have k^{-8} at the end (when $k = 0$, winnings should be infinite). That makes the figure 1 698 356 on the next line wrong; it should be $\approx$ 3 858 790.

p.297 Exercise 10.3.11 has two parts (ii). The first part (ii) should ask for the general solution of $(E - I)u_n = \binom{n}{r-1}$.

p.302 Exercise 10.4.3 after the displayed equation for u_n there is $(1-p)u_{n+2} - u_{n+1} + ue_n = 1$. The last term on the left hand side should be pu_n and the right hand side should be 0.

p.303 Exercise 10.4.4 (ii) this expression for f_n isn't right (it doesn't obey the condition on f_N from (i)). The first fraction in the expression for f_n should be $\frac{1-p}{1-2p}$. The expression for f_N at the top of p.304 should also have $1-p$, not 1, in the numerator of the first fraction.

p.304 Exercise 10.4.7: the assumption $r \in \mathbb{N}$ is missing (the X_i are all whole numbers, so certainly $Y_m(r)$ will never be zero if r isn't a nonnegative whole number).

p.307 Exercise 10.4.10 (i). The sum in the denominator for e_m should start at $j = 0$ since $j = 0$ is possible. This expression is supposed to increase for a bit and then decrease, so the inequality in the paragraph after the displayed equation should be $e_m \geq e_{m+1}$ rather than $\leq$.

p.307 Exercise 10.4.11 V_0 is p/q, not q/p (when p is small, places are hardly ever occupied, so V_0 should be small).

p.313 Exercise 10.5.10 (i) $|x - \log(1 - x)| \leq x^2$ is false for $1/2 > |x|$, it should be $\geq$. But the inequality which is needed in the latter parts of the question is $|x - \log(1 + x)| \leq x^2$ for $|x| < 1/2$ (which is true).

p.313 Exercise 10.5.10 (iv) a factor of n is missing: the fraction outside the square brackets in the first displayed equation should be $\frac{2n}{N(n)^2}$, and the exponent in the second displayed equation should be $2n/N(n)^2$. The problem is that in the calculations in the solutions, a factor of n goes missing between the equation after "So" and the equation after "Thus."

p.323 part (iv) of Exercise 11.1.1 should ask for r *consecutive* heads or tails, at least, that's the problem which the solutions solve. The one stated is rather harder: the result is that the expected time to r heads or r tails, not necessarily consecutive, is $2r - 2^{-2r+1}r\binom{2r}{r}$ (OEIS A033504 although they write it in a less pleasing way for some reason).

p.329 part (v) of proof of Theorem 11.3.1: "and apply (i)" should be "and apply (iv)."

p.330 Exercise 11.3.0 third displayed equation: the first inequality should be $\geq$ not $\leq$.

p.331 Exercise 11.3.4 the right hand side of the inequality doesn't seem correct to me. If np and nq differ by at least $2n\epsilon$ then it's certainly true that $P(|Y_n - nq| \leqslant n\epsilon) \leqslant P(|Y_n - np| \geqslant n\epsilon)$ and by (vii) of the previous exercise with $K\epsilon\sqrt{n}$ this is at most $2\exp(-\epsilon^2 n/(2p^2))$ (which is increasing with p) if $p \geqslant 1/2$ or $2\exp(-\epsilon^2 n/(2(1-p)^2))$ (which is decreasing) if $p \leqslant 1/2$. To get upper bounds for these we need an *upper* bound for p in the first case and a *lower* bound in the second so the best you can do is put $p = 1$ or $p = 0$ to get $\leqslant 2\exp(-\epsilon^2 n/2)$.

p.332 Exercise 11.4.1 "an a_n with the both following properties" should be "an a_n with both the following properties."

p.333 Exercise 11.4.2 u_r should be the probability we accept, not reject, the new drug (proof: if $p = 1$ we will accept with probability one, and if we put $p = 1$ in the expression $1/(1 + (q/p)^a)$ given for the *acceptance* probability we get 1.)

p.334 Exercise 11.4.4 (iv) $R(a, p)$ should be $R(p, a)$.

p.356 Exercise B.6: should refer to B.3, not B.4.

p.361 Exercise C.4: something is missing in "Jacob Bernoulli of pointed out…".

2. Solutions

Exercise 1.6.6: (ii) can't be right as stated: $p_1 + p_2$ should be 1, but here it's $\frac{t_1+t_2}{T+Z} = T/(T + Z)$.

Exercise 2.4.24 (i): the player has the option of asking for a third card in some circumstances, but the strategy described doesn't say when they should do that. I don't think the calculations are correct.

The player, P, and the banker, B, draw cards uniformly at random with replacement from the set $\{2, 3, 4\}$. Each gets one card face up. P may continue to draw more cards, up to a total of 3. When they stop, B must draw if they have 2 or 3 and must not draw if they have a 4. The scoring is then

- If P has a total of at least 7 they get -1, regardless of B's score.
- If P has < 7 and B has at least 7, P gets 1.
- If both P and B have less then 7 then if the scores are equal P gets 0, otherwise P gets 1 if they have the higher score and -1 if they have the lower score.

We then have the following payoffs and expected values:

	B's 1st card							P's EV if B's 1st card is		
	2			3			4			
	B's next card									
P stops at	2	3	4	2	3	4		2	3	4
3	-1	-1	-1	-1	-1	1	-1	-1	$-1/3$	-1
4	0	-1	-1	-1	-1	1	0	$-2/3$	$-1/3$	0
5	1	0	-1	0	-1	1	1	0	0	1
6	1	1	0	1	0	1	1	2/3	2/3	1
$\geqslant 7$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

P's first card could be 2, 3, or 4.

- P's first card is 2. They should draw again, getting 4, 5, or 6. Suppose they have 4.
  - If B has 4 then stopping gets 0 and drawing gets an expected value of $(1/3) \cdot 1 + (2/3) \cdot (-1) = -1/3$ so P should stop.
  - If B has 3, stopping has expected value $-1/3$ (from the table) and drawing has expected value $(1/3)\cdot$(expected score if B has 3 and P stops at 6)$+ (2/3) \cdot (-1) = -4/9$ so P shold hold.
  - If B has 2, stopping has an expected value of $-2/3$ from the table and drawing has expected value $(1/3)$(expected score if B has 2 and P stops at 6)$+ (2/3) \cdot (-1) = -4/9$ so P should draw.

  Now suppose P has 5 after the second draw. It can't make sense to draw again, so their expectation is 1/3 (conditioning on the three possibilities for B's card) Finally if P has 6 after the 2nd draw, they should not draw again. The expected score is $7/9 = (1/3)(2/3) + (1/3)(2/3) + (1/3)(3/3)$ from the table.

  Following this strategy, with first card 2 P has an EV of $(1/3)((1/3) \cdot 0 + (1/3)(-4/9) + (1/3)(-1)) + (1/3)(1/3) + (1/3)(7/0) = 23/81$.
- P's first card is 3. If B has 2 or 4 then clearly P must draw. If B has 3 then stopping has expectation $(1/3)(-1) + (1/3)(-1) + (1/3) \cdot 1 = -1/3$ and drawing expects $(1/3) \cdot 0 + (1/3)(2/3) + (1/3)(-1) = -1/9$ from the table, so P should draw in this case too. Their expectation is $(1/3)$(expectation if they stop at 5)$+(1/3)$(expectation if they stop at 6)$+ (1/3)$(expectation if they stop at 7) which from the table is $(1/3)(1/3) + (1/3)(7/9) + (1/3)(-1) = 1/27$.
- P's first card is 4. Stopping has an expected value of $-2/3$ if B has 2, $-1/3$ if B has 3, and 0 if B has 4. If P draws, the expectations are as in the first bullet point when P had 4 after their second draw. Thus P should stop if B has 3 or 4, expecting $-1/3$ and 0), and draw if B has 2, expecting $-4/9$. Their expected score is $(1/3)(-4/9) + (1/3)(-1/3) + (1/3) \cdot 0 = -7/27$.

Overall the expected score for P is $(1/3)(23/81) + (1/3)(1/27) + (1/3)(-7/27) = 5/243$. This is larger than the EV in the solutions. I tried simulation and it seems to be correct.

Exercise 2.5.15: there's a typo when $1 - n\sigma^2/(cn\mu)^2$ turns into $1 - c^2\sigma^2/(n\mu^2)$. It should be $1 - \sigma^2/(c^2n\mu^2)$, and this is $\geqslant 1 - b$ iff $n \geqslant \sigma^2/(c^2b\mu^2)$, so N should be 1 plus the integer part of $\sigma^2/(c^2b\mu^2)$.

Exercise 2.5.20 (ii): an extra factor of 2 has appeared in the displayed equation for EY: the previous displayed equation shows $EY_1 = k(2n-k)/(n(2n-1))$ so $nEY_1 = k(2n-k)/(2n-1)$, not two times this as written.

Exercise 7.6.6: the solution has a simpler form

$$y = \frac{x}{1+\frac{u}{v}\frac{v+w}{u+w}}.$$

Exercise 8.3.2: the p_i in the solutions seem to differ from those in the book, because the Jules-Jim symmetry means $p_1 = p_2$ and $p_3 = p_4$ at the Nash equilibrium in the book's notation. The Nash solution should be $p_1 = p_2 = 0$ and $p_3 = p_4 = 1/2$.

Exercise 8.6.2: some of the Nash equilibria here aren't correct. The notation is that (p, q) denotes the pair of strategies where the row player plays left with probability p and the column player plays left with probability q. For $a > -1$, $(0, 1)$ is not a Nash equilibrium as if the column player knows the row player will choose right, they will switch from left to right improving their expected score from -1 to a. Equally $(1, 0)$ isn't a Nash equilibrium. For $a > -1$ the equlibria are $(1, 1)$ with value $(a, 0)$, $(0, 0)$ with value $(0, a)$, and $((a+1)/(a+2), 1/(a+2))$ with value $-1/(a+2)$ to both players.

When $a = -1$ the Nash equilibria should be $(0, 0), (1, 1)$, and $(0, 1)$. The solutions are correct for $a < -1$.

Exercise 9.2.7: the table isn't displaying what it says it is, and the probability of winning seems wrong too. In the second part of the game, if your initial roll had probability q then since $P(7) = 6/36$, the probability p_w that you win satisfiew $p_w = q + (1-q-(1/6))q_w$ so $p_w = q/(q+1/6)$. If $q = r/36$ this is $r/(r+6)$, so if you roll $k \neq 2, 3, 7, 11, 12$ initially your chances of winning the second part are

k	r	win probability
4	3	1/3
5	4	2/5
6	5	5/11
8	5	5/11
9	4	2/5
10	3	1/3

so your win probability for the full game is

$$\begin{aligned}
P(\text{win}) &= P(\text{win}|\text{initial roll 7, 11})(8/36) \\
&\quad + P(\text{win}|\text{initial roll 2, 3, 12})(4/36) \\
&\quad + P(\text{win}|\text{initial roll not 2, 3, 7, 11, 12})(24/36) \\
&= 8/36 + 0 + (24/36)(2\cdot(3/24)\cdot(1/3) + 2\cdot(4/24)\cdot(2/5) + 2\cdot(5/24)\cdot(5/11)) \\
&= 244/495 \approx 0.493
\end{aligned}$$

where $2(3/24)(1/3)$ is for 4 or 10, $2(4/24)(2/5)$ is 5 or 9, and $2(5/24)(5/11)$ is 6 or 8. Wikipedia agrees with me, so I must be right.

There are two sets of solutions for 9.3.8, the first is really for 9.3.7.

Exercise 9.3.18: something goes wrong at the end of the upper bound for the probability they get fewer than N of poet j. This should be $P(S_M < N)$ (strict inequality) which is $\leqslant V(S_M)/(\epsilon^2 N^2)$. Using the estimate $V(S_M) \leqslant (1+\epsilon)N$ gives $P(S_M < N) \leqslant (1+\epsilon)N/(\epsilon^2 N^2) = (1+\epsilon)/(\epsilon^2 N)$. To make this $\leqslant \epsilon/n$ it's enough for $N \geqslant 2n\epsilon^{-3}$.

Exercise 10.1.5: the solutions refer to chapter 4 section 5 of *How to gamble* but in my Dover copy at least it's chapter 5 section 4.

Exercise 10.1.6: the first = in the first displayed equation should be a −. Your expected winnings are $q(l-k) + (1-q)(-k) = -k + ql$ (minus what appears on the right of the second displayed equation).

Exercise 10.4.9 (ii) (the solutions call this (iii) but there is no (iii)): in the displayed equation after "if and only if" a term has gone missing: $(h+2)(r-1)(g-1)+(h-1)(r+2)(g-1)+(h-1)(r-1)(g+2)-3hrg$ is $-3(h+r+g)+6$ so letting $N=h+r+g$, A should really be $4/(6-3N)$.

Exercise 10.4.10: in the statement, surely the sum in the denominator of e_m should start at $j=0$. We're then supposed to show that the e_m go up for a while and then go down again by examining $e_{m+1}-e_m$. The solutions introduce A_m and B_m and claim that mB_m is increasing but that seems wrong: B_m itself is decreasing with m, and

$$\begin{aligned} mB_m-(m+1)B_m &= -1-qm\sum_{j=0}^{m-1}p_j+qm\sum_{j=0}^{m}p_j+p\sum_{j=0}^{m}p_j \\ &= -1+qmp_m+q\sum_{j=0}^{m}p_j \\ &= qmp_m+q\sum_{j=0}^{m}p_j-1 \end{aligned}$$

If mB_m is to be increasing this should be $\leqslant 0$, that is $qmp_m+q\sum_{j=0}^{m}p_j\leqslant 1$, but that would fail if $q=1$ and $mp_m>1$ for example. Concretely, take $n=3$, $q=1$, and the p_r to be $1/12,1/12,3/4,1/12$. Then the mB_m are $0,11/12,1+8/12,3/12$ so aren't increasing or decreasing.

As an alternative argument, for there to be a unique maximum we just need that once the e_m start going down they keep going down, that is, $e_{m+1}\leqslant e_m$ implies $e_{m+2}\leqslant e_{m+1}$. The former is equivalent to

$$\begin{aligned} p_m\left(m-q\left(m\sum_{j=0}^{m}p_j+\sum_{j=m+1}^{n}jp_j\right)\right) &\geqslant 0 \\ m-q\left(m\sum_{j=0}^{m}p_j+\sum_{j=m+1}^{n}jp_j\right) &\geqslant 0 \end{aligned}$$

and $e_{m+1}\geqslant e_{m+2}$ is

$$m+1-q\left(\sum_{j=0}^{m}p_j+m\sum_{j=0}^{m}p_j+\sum_{j=m+1}^{n}jp_j\right)\geqslant 0$$

which follows because the bit on the left has gone up by 1 while the bit being subtracted has only increased by $q\sum_{j=0}^{m}p_j\leqslant 1$.

Exercise 10.5.10 (iv) a factor of n goes missing between the "So" and "Thus" equations. The result stated needed an extra factor of n as above to be correct. (v) the upper limit on the product should be $m-1$ not m (e.g. for $m=365$ we shouldn't get the answer 1 since they could still all have distinct birthdays). That's why there's an extra factor of $1/365$ in the second displayed equation.

Exercise 10.5.12: the question asked for 1000 chalets but the solutions deal with the case of 500 chalets, which is easier since we can use the approximation $500p\approx 1$. Probably the exercise should have had 500 instead of 1000.

Exercise 11.3.4: doesn't give any clues about where the final term comes from and I think it's wrong — see the note about p.331 above.

3. Errata

Errata to p.75: there's a typo in the fraction which should be $\frac{11}{12}\left(\frac{11}{12} + \frac{11}{5}\frac{1}{12}\right) = \frac{121}{120}$.