Topology and Groups

Week 6, Thursday

1 Preparation

- 5.03 (Fundamental group of a mapping torus),
- 6.03 (Wirtinger presentation).

2 Discussion

1. (PCQ) Let $F\colon T^2 \to T^2$ be the map $F(x,y) = (y,x)$. What is the fundamental group of the mapping torus $MT(F)$?
2. (PCQ) The video on the Wirtinger presentation claimed that any braid gives a knot by taking the braid closure. Why is this false? What should I have said instead?

3 Classwork

1. Find a presentation for the fundamental group of
   - the Hopf link (2-strand braid closure of σ_1^2),
   - the trefoil knot (2-strand braid closure of σ_1^3),
   - the figure 8 knot (3-strand braid closure of $\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1$).
2. Prove that any knot in $\mathbf{R}^3$ can be unknotted by allowing ourselves to move parts of the knot through the fourth dimension.

3. Let K denote the trefoil knot in S^3 and let $C = S^3 \setminus K$. Hopefully you found that

$$\pi_1(C) = \langle a, b | aba = bab \rangle.$$

Let N be a solid torus $S^1 \times D^2$. Let γ be the loop $\{p\} \times \partial D^2$ in ∂N. Let $\phi \colon \partial N \to \partial C$ be a homeomorphism such that $\phi_* \gamma = bab^{-1}aba^{-2}$. Find a presentation for the fundamental group of $S^3_{K,\phi}$.