

Correction to “The viscosity of a dilute
suspension of rough spheres”,
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1 Introduction

Two algebraic errors were committed in the calculations of [1], which are corrected here. Most of the results change only quantitatively, but we include a discussion of the physical meanings of those results which differ from those published previously.

2 Errors

The friction force in the case of slipping motion had a minus sign error in the tangential part; and the definitions of some of the constants β_i given in Appendix A were incorrect. To correct these errors, the following changes are necessary:

- The sign of terms proportional to ν should be changed in equations (2.15–2.19), (3.7–3.8), (3.11), (3.26–3.27), (4.14), (4.17), (4.37) and (4.38).
- The definitions (A1–A6) of the constants β_i in Appendix A should be amended as below:

$$\begin{aligned}\beta_1 &= 2a(y_{11}^a - y_{12}^a) - 2a^2 s_c (y_{11}^b - y_{12}^b) + \frac{1}{2} a^3 s_c^2 (y_{11}^c + y_{12}^c), \\ \beta_2 &= s_c [B^* - 1 + 2(y_{11}^h + y_{12}^h)], \\ \beta_3 &= 2a(x_{11}^a - x_{12}^a), \\ \beta_4 &= 2(y_{11}^h + y_{12}^h) + a^2 (\beta_2 / \beta_1) [(y_{11}^b - y_{12}^b) - \frac{1}{2} a s_c (y_{11}^c + y_{12}^c)], \\ \beta_5 &= s_c [B^* - 1 + 2(y_{11}^h + y_{12}^h)], \\ \beta_6 &= (s_c (1 - A^*) / \beta_3) [2a(y_{11}^a - y_{12}^a) / s_c + a^2 (y_{11}^b - y_{12}^b)].\end{aligned}$$

Note that $\beta_5 = \beta_2$. Changes have been made to β_1 , β_4 and β_5 .

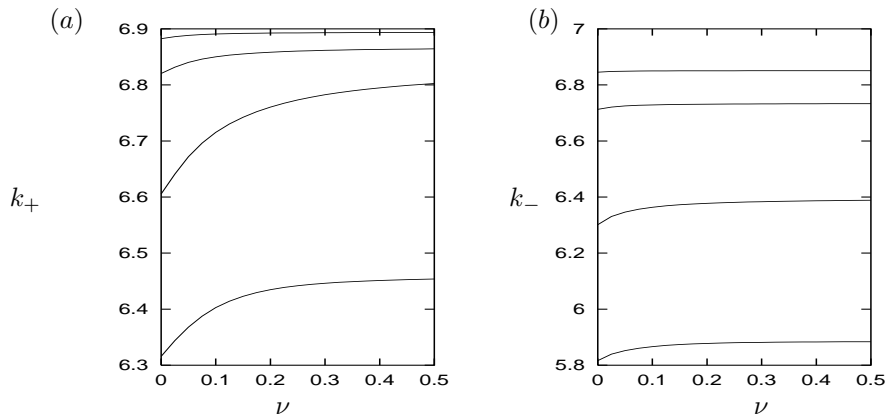


Figure 1: Plot of the c^2 viscosity coefficients (a) k_+ and (b) k_- against the friction coefficient ν for $\zeta = 10^{-7}, 10^{-5}, 10^{-3}$ and 10^{-2} (top to bottom).

3 Results: Straining Flow

Figures 1–3 show the viscosity results for axisymmetric straining flow. The qualitative difference between the behaviour here and that described in [1] is that the corrected viscosity increases rather than decreases with increasing friction coefficient, although the effect is weak. The contributions from the various parts of the flow are all of the same type as we had earlier claimed; however, the dominant terms are now mostly from the rolling region and not slipping as we had predicted. The reason for this change is the sign error on our slipping contact force, which erroneously promoted rather than retarded motion between the particle surfaces, thus causing unphysically large lubrication stresses.

4 Results: Shear Flow

Figures 4–6 show the shear normal stress results, as replotted from figures 6–8 of [1]. There is no qualitative change in the behaviour of the normal stresses with roughness height; however, as figure 5 demonstrates, the trend of normal stresses against friction coefficient has changed. The correct behaviour, shown here, is a very weak dependence of normal stresses on friction, with N_1 decreasing in magnitude with increasing ν and N_2 showing no clear dependence. In all cases both N_1 and N_2 remain negative where roughness heights are of physical size.

5 Conclusions

The correction of the errors has made little qualitative difference to the results except for the dependence on friction coefficient, ν . The strain viscosity now

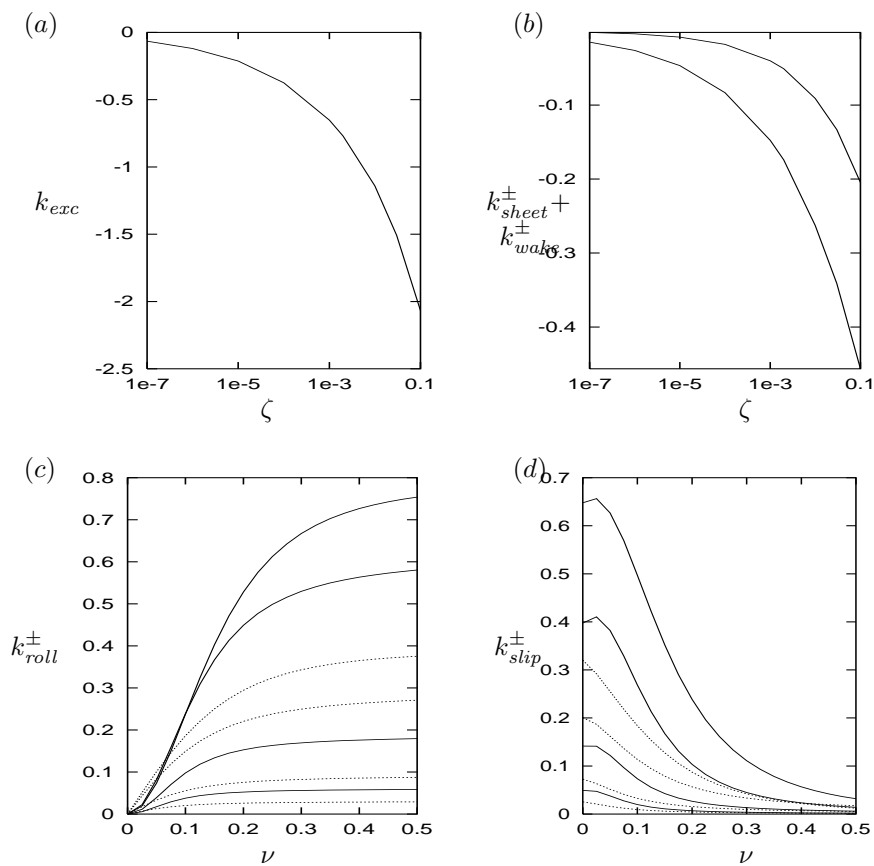


Figure 2: The contributions to the c^2 viscosity coefficient k_{\pm} from the (a) excluded volume, (b) sheet and wake, (c) rolling contact and (d) slipping contact regions of the straining flow. Since the contributions from the excluded region $s < s_c$, the sheet and the wake are independent of ν , parts (a) and (b) are plotted against ζ . In part (b) the upper curve is for $E > 0$ and the lower for $E < 0$. Parts (c) and (d) are plotted against ν for $\zeta = 10^{-2}, 10^{-3}, 10^{-5}$ and 10^{-7} from top to bottom, and the contributions for $E < 0$ are solid curves where those for $E > 0$ are dotted.

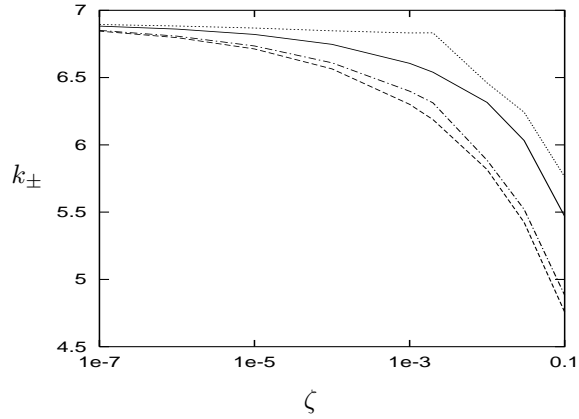


Figure 3: Plot of the c^2 viscosity coefficient k_{\pm} against the dimensionless roughness height ζ in the limits $\nu \rightarrow 0$ (no friction) and $\nu \rightarrow \infty$ (no slipping). The curves for $E > 0$ (biaxial expansion) are solid for $\nu = 0$ and short-dashed for $\nu \rightarrow \infty$; the curves for $E < 0$ (uniaxial expansion) are long dashed for $\nu = 0$ and dot-dashed for $\nu \rightarrow \infty$.

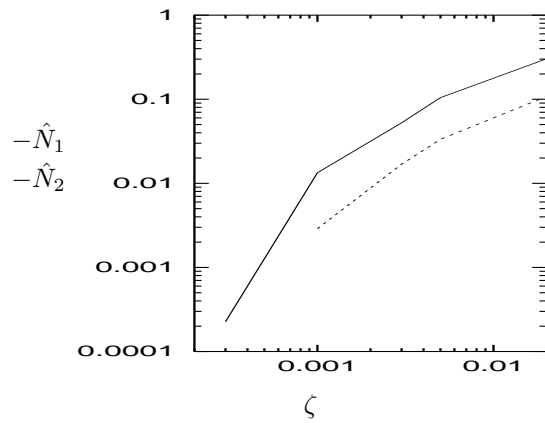


Figure 4: The negative scaled normal stresses plotted against the dimensionless roughness height for $\nu = 0$, with the solid and dotted lines representing $-\hat{N}_1$ and $-\hat{N}_2$ respectively.

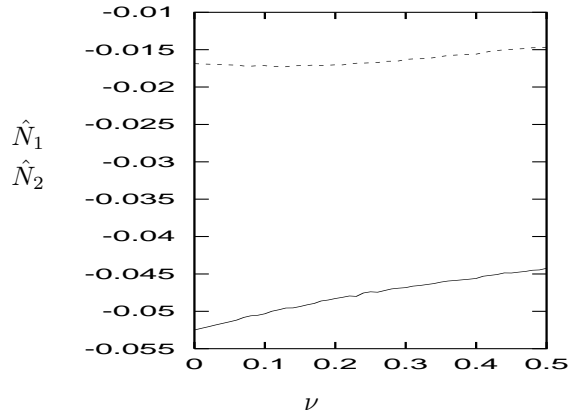


Figure 5: The dependence of the normal stress differences \hat{N}_1 (solid line) and \hat{N}_2 (dotted line) on the friction coefficient, ν , for a physically plausible value of the roughness height, $\zeta = 3 \times 10^{-3}$.

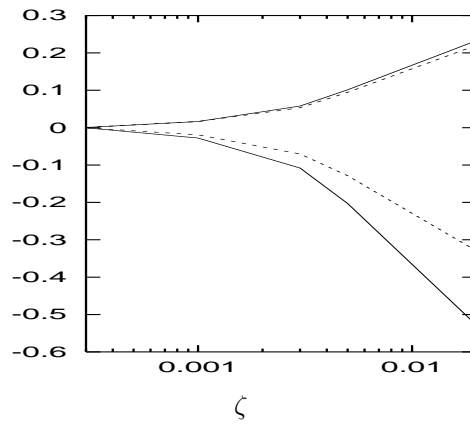


Figure 6: The contributions to the scaled normal stresses, plotted against the dimensionless roughness height, with fixed $\nu = 0.1$. The solid and dotted lines are for \hat{N}_1 and \hat{N}_2 respectively, with the upper curves being the contact contribution to \hat{N}_1 and the bulk and sheet regions contribution to \hat{N}_2 , and vice versa for the lower curves.

rises weakly with increasing ν and the shear normal stresses exhibit only the very weakest dependence on ν .

References

- [1] H J Wilson and R H Davis. The viscosity of a dilute suspension of rough spheres. *J. Fluid Mech.*, 421:339–367, 2000.