Modeling qualitative changes in bimanual movements

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In the cognitive sciences the study of complex rhythmic movements is a challenging problem which is a subject of extensive research. Experiments on bimanual movements are paradigms for studying the ability of humans on timing and coordination. Such experiments give insights into the control mechanisms of the central nervous system and also reflect the functional state and level of training of the person tested. In a recent study on bimanual polyrhythm production the existence of phase transitions in dependence on the speed of performance has been shown. In this paper we present an iterated map model to explain main findings of these experiments. The model consists of two iterated maps describing the dynamics of the finger movements. The essential properties of the model are a nonlinear correction function and a coupling mechanism between the two maps. Numerical simulations show that the model is in qualitative agreement with the experimentally observed phenomena.

I. INTRODUCTION

An important challenge of contemporary investigations in neuroscience is the understanding of the principles of movement timing and motor control. In particular, the coordination of movements is of high interest. The production of bimanual polyrhythms is ideally suited for studying how coordinated movements are controlled by the brain, because their performance leads to strong interactions between both hands. The analysis of experiments on bimanual rhythmic hand movements does not only give insight into the control mechanisms of the brain, but also reflects the functional state and level of training of the persons tested.

Qualitative aspects of coordination tasks (e.g. bifurcations and synchronization) of the underlying dynamical system are typically modelled by coupled oscillator systems [Haken *et al.*, 1985, Kelso, 1995, Haken, 1996]. Another approach is the investigation of covariances of the produced time sequences. These properties are usually discussed in the framework of timer-motor models [Wing & Kristofferson, 1973a, Wing & Kristofferson, 1973b, Vorberg & Wing, 1994]. A related approach for the study reported in this paper has been proposed by Krampe et al. [1996].

In this paper we present a top-down model for the explanation of recent experimental findings in an experiment on polyrhythmic hand movements [Engbert *et al.*, 1996a]. In this experiment pianists were required to produce a bimanual 3:4 polyrhythm. We have identified qualitatively different dynamical regimes in dependence on the speed of performance in these data. The transitions between these different regimes are interpreted as bifurcations [Ott, 1993]. The recorded data are discrete time intervals of key pressings. Our intention is to understand the dynamics of these discrete events. Therefore, we propose an iterated map model, which is discrete in time, as a possible explanation of the main experimental findings [Engbert *et al.*, 1996a]. The central objective of this paper is to find the principal mechanisms that underly the observed phenomena on a qualitative basis. At this stage it is not intended to achieve a quantitative correspondence between the data produced by the model and the experiments. The model describes the dynamical behavior of the length of the produced interval and its qualitative change under variation of the external control parameter, the cycle length. Simple fundamental mechanisms that can be motivated neurophysiologically are incorporated into the model. It describes the synchronization between the two fingers and, therefore it is related to a coupled oscillator approach. The model consists of the following main components: the tapping of each finger is modelled by a map with discrete time delay, which reflects a feedback loop control mechanism; errors in the movement of a single finger are corrected due to a nonlinear correction function; the synchronization of these movements is reflected by a special type of coupling, which acts at the end of every cycle only. Furthermore, we are studying the influence of noisy perturbations on the dynamical behavior of the model. This enables us to use the same technique of time series analysis previously applied to the experimental data. These methods are based on symbolic dynamics [Hao, 1989, Engbert *et al.*, 1996b].

In Sec. 2 the experiment and the result of the data analysis are briefly described. Section 3 explains the derivation of the model equations based on the findings of Sec. 2. In Sec. 4 we present results of numerical simulations of the model equations and compare them with the experimental data. Finally we give a conclusion and outlook in Sec. 5.

II. THE EXPERIMENT

In this section we briefly summarize the experiment and describe a method for analysing the data. This method is based on a symbolic transformation.

Our investigations are motivated by results of an experimental study in which pianists were required to produce a bimanual 3:4 polyrhythm at different tempos. Polyrhythms are ideally suited to study synchronization and coordination between two fingers because of the complicated phase relation between successive strokes during one cycle. The tempo of performance was used as an external control parameter [Krampe et al.]. A scheme of the rhythm for a cycle duration of 1200 ms is shown in Fig. 1. In the experiments the subjects initially could listen to the rhythm generated by a computer for as long as they wanted. After the subjects started to play on an electronic piano, the computer continued for four more cycles, playing in parallel. Then the computer stopped and the subjects had to continue the rhythm for 12 cycles. These data were recorded by the computer connected to the piano with a weighted keyboard mechanism. The time resolution of the recorded data is 1 ms. Fourteen different metronome tempos ranging from 800 ms to 8200 ms per cycle were presented in a randomized order (for details see Krampe et al. [1996]).
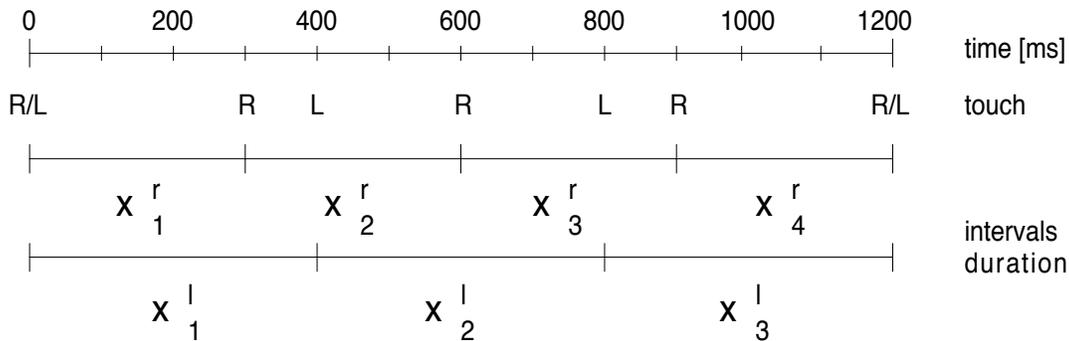
Figure 1: The scheme of the ideally performed polyrhythmic task. During each cycle the pianist should make 4 strokes using his right hand and 3 strokes using his left hand. The letters R and L denote the times of the strokes by right and left hands, $x_i^{l,r}$ are respective interstroke intervals. The scheme of the trial is shown for a cycle length of 1200 ms.

A powerful method of analysing the data is to apply a symbolic transformation [Wackerbauer et al., 1994]. This method includes a broad class of measures for characterizing complex systems.

Let us denote the data recorded during one cycle for the left and right hand respectively, by x_1^l, x_2^l, x_3^l and $x_1^r, x_2^r, x_3^r, x_4^r$ (Fig. 1). For every cycle we compute the mean interval length for the right hand x^r as $\frac{1}{4}(x_1^r + x_2^r + x_3^r + x_4^r)$ and for the left hand x^l as $\frac{1}{3}(x_1^l + x_2^l + x_3^l)$. We then apply a symbolic transformation $s_i^{r,l}$ to the interval lengths $x_i^{r,l}$ of each cycle according to the rule:

$$s_i^{r,l} = \begin{cases} 1 & \text{if } x_i^{r,l} > x^{r,l} , \\ 0 & \text{otherwise} . \end{cases} \tag{1}$$

Using this transformation each time series is converted into a symbol sequence. The results for all trials of two different subjects are visualized in Figs. 2 (a) and (c). The symbols are coded with black ("1") and white ("0") squares. The corresponding two-dimensional normalized distributions of the frequency of words are given in Figs. 2 (b) and (d). Here a word means a sequence of 3 (left hand) or 4 (right hand) symbols of 0 and 1 during one cycle. Further details of the analysis of symbol patterns can be found in Engbert et al. [1996a].

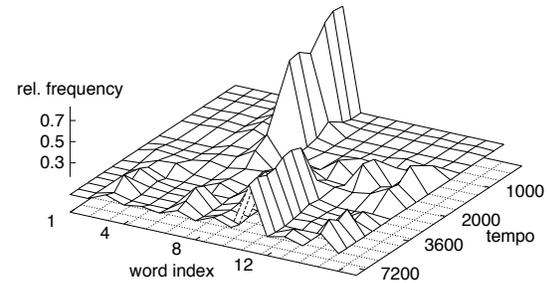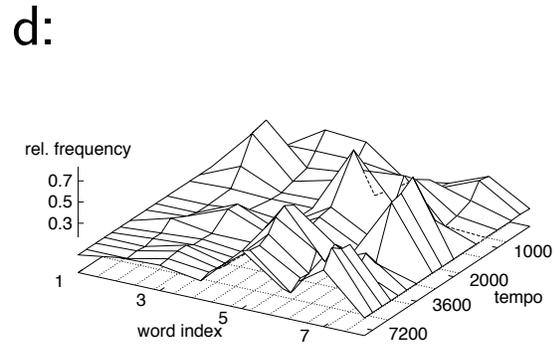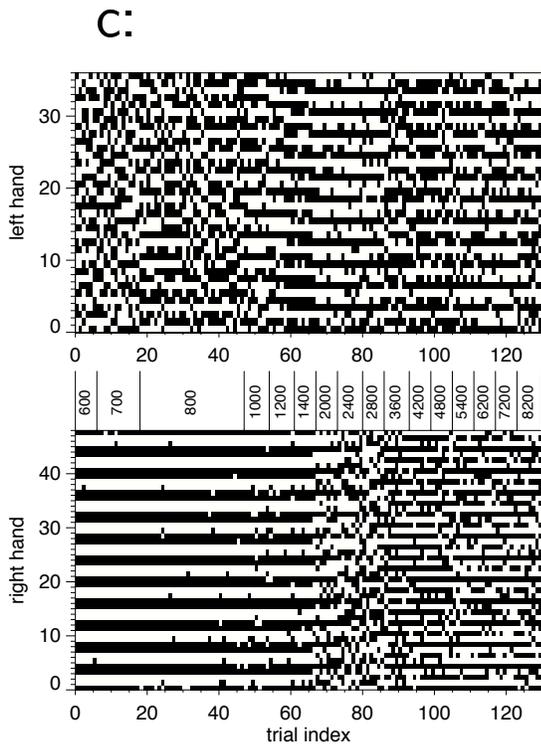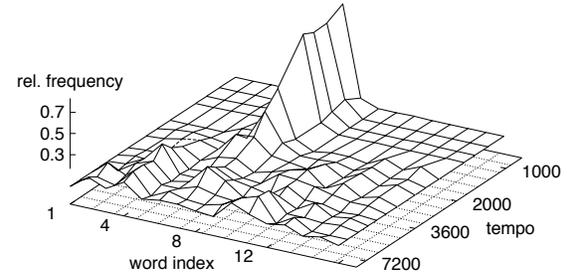Analysing the symbolic representation of the experimental data we summarize several typical properties:
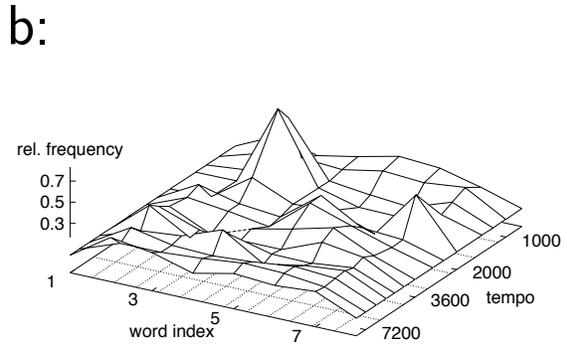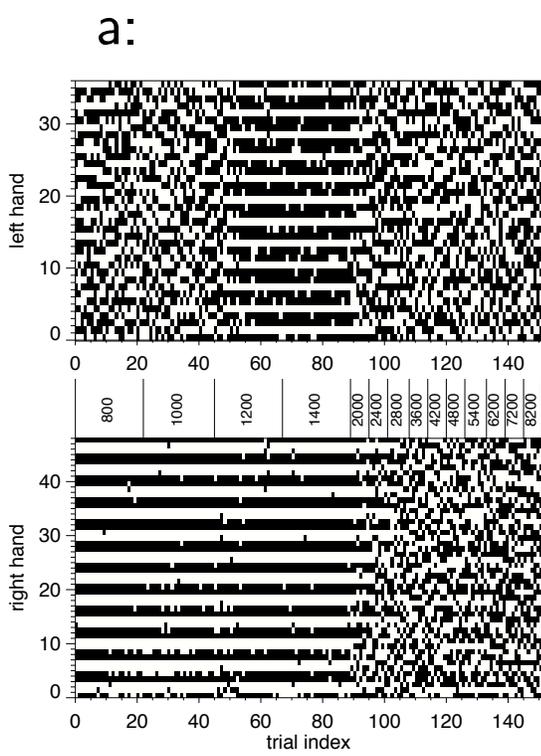
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Figure 2: Symbolic transformations (a and c) of the experimental data for two different subjects. On the abscissa the trial numbers are plotted, ordered with respect to performance tempo. Every trial consists of 12 cycles. On the ordinate time is increasing. 36 symbols are plotted for the left hand and 48 symbols for the right hand. The cycle lengths corresponding to the trial indices are plotted above the symbolic patterns of the right hands. The two-dimensional distribution of the frequency of appearance of certain combinations of black and white symbols within one cycle (distribution of words) is shown in (b) and (d). The presence of the sharp peak shows that one combination is dominating, i.e. the symbolic sequence is close to a periodic one.

1. There exist two types of structures, periodic and irregular ones. Irregular structures correspond to ideal performance. For the parameter ranges, where one can observe periodic structures, the rhythm has been changed and the average values of the produced interstroke intervals are not equal to the required ones.

2. Transitions between periodic and irregular patterns may be observed with a monotonic variation of the tempo.

3. For a certain tempo, the structures of the symbolic representation for the left and right hand can be similar or different.

4. For periodic structures, the period of the symbol sequence corresponds to the order of the polyrhythm, that is 3 to 4 in our case. Periodic sequences with a higher period (beyond cycles) cannot be observed.

5. A perturbation of a periodic structure usually disappears immediately. This means that the periodic patterns are stable and therefore can be interpreted as dynamical "modes".

6. Slow tempos are typically characterised by irregular structures.

7. With increasing tempo, a transition within the periodic structure is observed in a few cases (e.g. Fig. 2 c, right hand).

In the next section we propose a model which explains the main features outlined above.

III. THE MODEL

The general idea of the proposed model is that periodic patterns appear as a result of excitation of oscillations in the control system, modulating the desired rhythm and producing the observed patterns. The model mainly aims at the role of the central nervous system. We assume that the influence of the properties of the muscle-skeleton system on the dynamics of the system is small and therefore instructions coming from some control mechanism are executed exactly and immediately.

In the following we consider the dynamics to be governed by means of a timer mechanism that provides the required duration of interstroke intervals for both hands; we denote these values by δ^l and δ^r, respectively. We assume that the time intervals δ^l and δ^r are generated by two different, but certainly coupled timers. In this case we can describe the control mechanism using two circuits, where each circuit controls the motion of a single hand.

Each of these circuits corrects the interstroke intervals based on the information on those intervals at some previous moment in time. In other words, we consider *time delay* as an essential property of the control system. For simplicity we consider one feedback loop for each hand, although the existence of proprioceptive, visual and audio feedback leads to different delay times.

The crucial point of our model is the coupling between these control circuits. We neglect the role of weak coupling within the cycle itself and suggest that it is "turned on" only at the end of the cycle. This coupling is motivated by the experimental fact that the simultaneous strokes at the end of each cycle are precisely performed by all subjects. As we are interested in the time intervals between strokes, we formulate a discrete model in the form of difference equations.

Let us consider a plausible control scheme based on the above assumptions (Fig. 3). The goal of the regulation is to keep the interval durations according to the predetermined rhythm and to correct errors while playing. As a first step we write the equation of motion for a single hand to illustrate the approach suggested above:

$$y_{i+1} = \delta - k \tanh(\alpha \cdot (y_{i-m} - \delta)), \tag{2}$$

where i represents time and $m \geq 0$ is the discrete delay time of the correction interval. In these equations y_i are time intervals produced by the control system. The correction function is chosen to be $\tanh(y_{i-m} - \delta)$. The correction acts with respect to the difference between the time interval y_{i-m} and the interval δ (the exact interval length of the system). The tempo dependent parameters $\alpha = \alpha(\delta)$ and $k = k(\delta)$ adjust the strength of the correction. The idea of the control is rather simple: if the fulfilled interval y_{i-m}, estimated with some time delay (where m is the delay time index) is performed with some error, then the control system will correct it.

Now let us come to the general case of bimanual rhythm production. For the given order of a polyrhythm, three vs. four in our case, the values δ^l and δ^r are related as $3:4$. For an arbitrary polyrhythm of order $N^l : N^r$ these quantities are related as $N^l \delta^l = N^r \delta^r = \delta$. If the rhythm is played perfectly, then all within-hand intervals y_i^l and y_i^r, respectively, will be of equal length, i.e. $y_i^{l,r} = \delta/N^{l,r}$. A simple scheme to realize such a control mechanism can be described by the equations

$$y_{i+1}^{l,r} = \delta^{l,r} - \Delta_i^{l,r} G^{l,r}(x_{i-m}^{l,r} - \delta^{l,r}) - \Theta_i^{l,r} F^{l,r}(\tilde{\delta}^{l,r} - \tilde{\delta}^{r,l}), \qquad (3)$$

$$x_i^{l,r} = y_i^{l,r} + \xi_i^{l,r}, \qquad (4)$$

where $x^{l,r}$ are observed time intervals and the superscripts r, l refer to the left and right hand, respectively. These observed intervals $x_i^{l,r}$ differ from the intervals $y^{l,r}$ produced by the control system, because they are corrupted by some noise, which is discussed later. The second and third terms describe within-cycle control and end-of-cycle coupling, respectively.
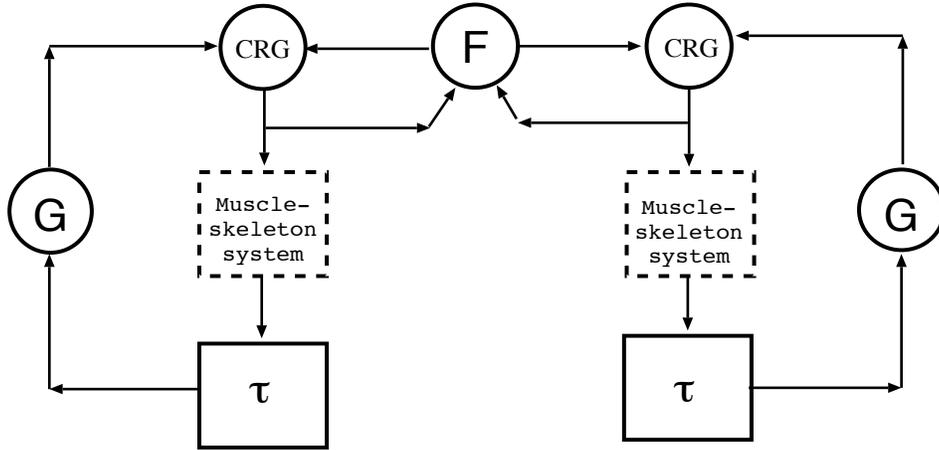

Figure 3: The scheme of the control system. CRG are central rhythm generators. The delay time in the control loop is denoted by τ. Blocks G and F reflect within-cycle regulation and end-of-cycle coupling respectively.

The within-cycle correction assumed above is implemented by the term

$$\Delta_i^{l,r} G^{l,r}(x_{i-m}^{l,r} - \delta^{l,r}) \quad ,$$

where

$$\Delta_i^{r,l} = \begin{cases} 1 & \text{if } (i+1) \bmod N^{l,r} \neq 0, \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

The index m is the delay time in the correction loop, and G is the correction function, which depends on the deviation of the produced interval from the ideal one. We assume that we start the iterations with index $i = 0$ and initial conditions $x_0^{l,r}, \ldots, x_{-m}^{l,r}$.

The synchronization that is needed at the end of each cycle is described by the coupling mechanism

$$\Theta_i^{l,r} F^{l,r}(\tilde{\delta}^{r,l} - \tilde{\delta}^{l,r}),$$

where

$$\Theta_i^{r,l} = \begin{cases} 1 & \text{if } (i+1) \bmod N^{l,r} = 0, \\ 0 & \text{otherwise,} \end{cases} \qquad (6)$$

and $\tilde{\delta}^{r,l}$ are the estimated lengths of the cycles. The introduction of this coupling enables our modell to carry out the strokes at the end of the cycle simultaneously. This is an essential ingredient of the model, since we observe in experiments that the subjects do not have problems to perform the simultaneous strokes. Note that we consider this end-of-cycle control to dominate over the within-cycle one. Therefore, the action of the latter one during the last strokes of each cycle is neglected.

To take into account the inaccuracy of timers, their weak interaction with other physiological subsystems, fluctuations in the properties of the muscle–skeleton system etc., we introduce additive noise $\xi_i^{l,r}$ into the model equations. It is uniformly distributed, uncorrelated and considered to be limited, i.e. its absolute value $|\xi_i^{l,r}|$ cannot exceed some critical value ξ_c. For simplicity we describe all the sources of fluctuations for a single hand by one noise term.

IV. SIMULATION RESULTS

In this section we present results of numerical simulations of the proposed model for some particular forms of correction and coupling functions G and F. In our model the transition between regular and irregular structures in the symbol patterns of Fig. 2 can be explained by excitation of oscillations in the control system, i.e. by the occurrence of a Hopf bifurcation. In the absence of oscillations, even a very small amount of noise leads to irregular patterns in the symbol sequences. This case corresponds to a correct production of the rhythm. If oscillations arise, one can observe periodic symbol patterns. Using the symbolic transformation, the onset of the Hopf bifurcation is mapped to a disorder − order phase transition in the symbol sequences.

Let us discuss possible implementations of the correction function G. As the goal of the regulation is to decrease the deviation of the performed interval length from the ideal value, the correction function must be an odd function of its argument. To allow stable periodic oscillations, it must be nonlinear. A typical choice in biological modelling for such a function is a sigmoid shaped one, e.g.

$$G^{l,r}(x) = k_1^{l,r} \tanh(\alpha x)$$

or as the piece-wise linear function

$$G^{l,r}(x) = \begin{cases} k_1^{l,r} x & \text{if } |x| < \alpha, \\ k_1^{l,r} \cdot \text{sign}(x) & \text{otherwise.} \end{cases}$$

In a similar way we choose the coupling function F as

$$F^{l,r}(x) = k_2^{l,r} \tanh(\alpha x) \qquad \text{or}$$

$$F^{l,r}(x) = \begin{cases} k_2^{l,r} x & \text{if } |x| < \alpha, \\ k_2^{l,r} \cdot \text{sign}(x) & \text{otherwise.} \end{cases}$$

Here $\alpha = \alpha(\delta)$ and $k_{1,2}^{l,r} = k_{1,2}^{l,r}(\delta)$ are control parameters which depend on the speed of performance δ. Using this choice we can write the equations for the numerical simulations:

$$y_{i+1}^r = \delta^r - \Delta_i^r k_1^r \tanh(\alpha \cdot (x_{i-m}^r - \delta^r)) - \Theta_i^r k_2^r \tanh(\alpha \cdot (\tilde{\delta}^r - \tilde{\delta}^l)) \tag{7}$$

$$y_{i+1}^l = \delta^l - \Delta_i^l k_1^l \tanh(\alpha \cdot (x_{i-m}^l - \delta^l)) - \Theta_i^l k_2^l \tanh(\alpha \cdot (\tilde{\delta}^l - \tilde{\delta}^r)) \tag{8}$$

$$x_i^{r,l} = y_i^{r,l} + \xi_i^{r,l}. \tag{9}$$

The role of the coupling is to decrease the difference between the lengths of the cycles performed by different hands. As this correction is produced by the control system *before* the cycles are finished, the argument of this function is the *estimated* difference between cycles lengths $\tilde{\delta}^r - \tilde{\delta}^l$. We assume that the length of the cycle is estimated as a sum of the intervals produced by the regulating system (i.e. not corrupted by noise). As the last interval has to be predicted by the control system, it seems reasonable to suppose that the "ideal" value is used instead. Hence, we use

$$\tilde{\delta}^{l,r} = \sum_{j=0}^{N^{l,r}-2} y_{i-j}^{l,r} + \delta^{l,r}. \tag{10}$$

Using this coupling we observe a Hopf-like bifurcation under variation of either parameter α or $k = k_{1,2}^{l,r}$. Varying two parameters simultaneously we can obtain a forward and a backward bifurcation. This case is illustrated in the bifurcation diagram in Fig. 4. The diagrams were obtained in the absence of noise. The excitation of periodic oscillations with periods in the ratio 3:4 explains the regular patterns observed in experimental data.

Obviously, after the bifurcation the amplitude of oscillations increases with increasing k (α constant). If the parameter α is decreased and k is held constant, we can hold the amplitude of oscillation approximately constant, or even decrease it up to the reverse bifurcation. Our simulations show the appearance of periodic structures for fast tempos (Fig. 5 (a) and (b)). These pictures were obtained by monotonic variation of α and a constant value of k (Fig. 5 a) and by monotonic variation of k for a constant α (Fig. 5 b). The same symbolic transformation as for the experimental data was applied. The situation in the experiment shows no significant increase in the amplitude of oscillation after the bifurcation value, therefore the variation of the parameter α seems to be more realistic.
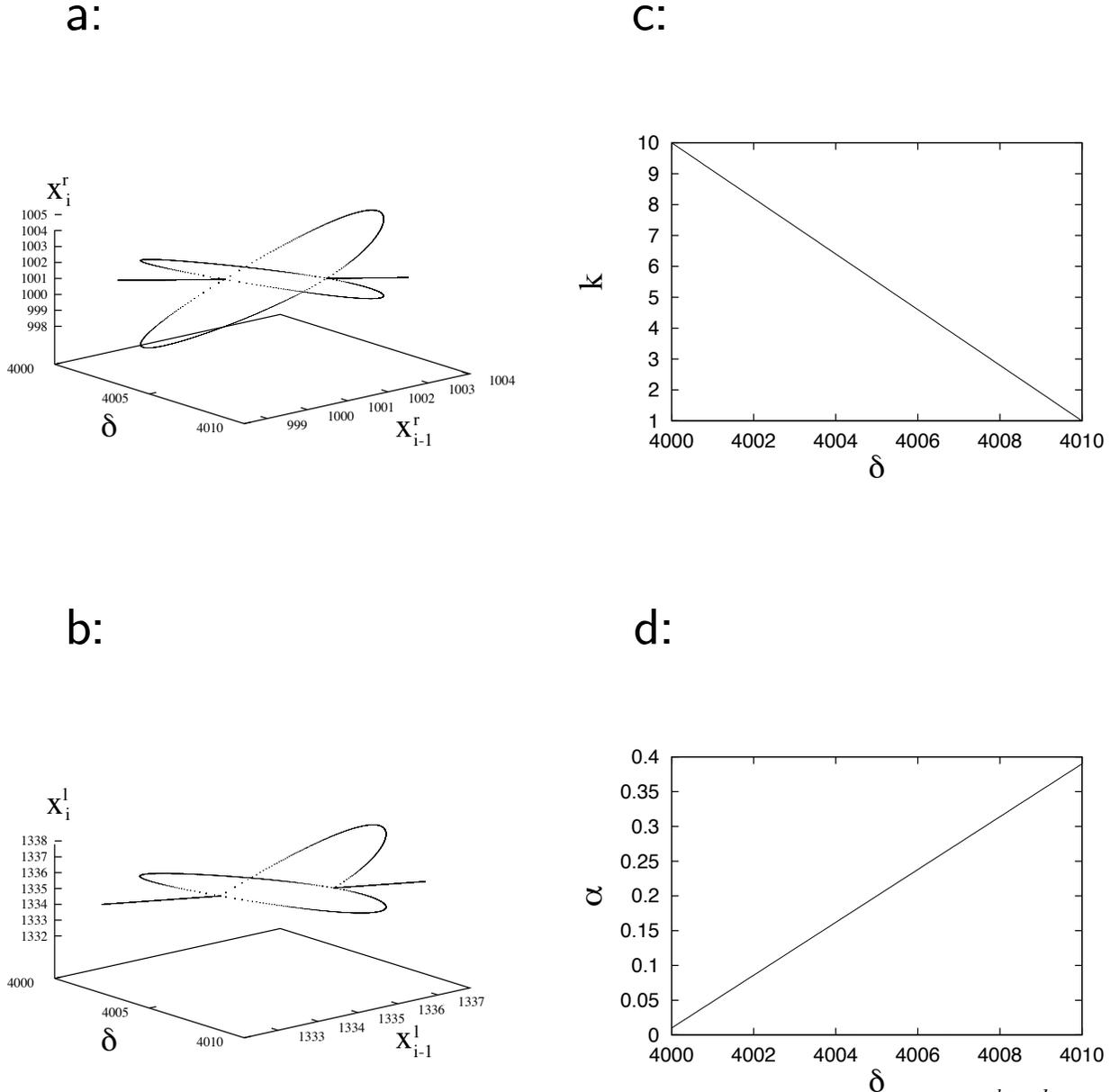
a:

c:

b:

d:

Figure 4: Bifurcation diagrams for left and right hands. In Figs. (a) and (b), the coordinates $x_i^{l,r}, x_{i-1}^{l,r}$ are shown in dependence on the control parameter δ. The cycle length δ is chosen to be the control parameter. The dependence of α and k on δ is shown in (c) and (d), respectively.

To take into account natural fluctuations in the mechanical and neural systems discussed above, noisy perturbations of the dynamics (3) were introduced. The noise was considered to be uncorrelated and uniformly distributed in the interval $[-\xi_c, \xi_c]$, with $\xi_c = 8$.

The pictures were obtained after discarding transients of 10 cycles. It can be seen from the patterns that the symbol sequences demonstrate stability with respect to small perturbations.

The model enables us to obtain different periodic structures with respect to different initial conditions. The initial conditions can be considered as the individual strategy in fulfilling the task, because different patterns are the result of the execution of the experiment by different subjects, but the same subject shows always the same specific patterns. We have chosen the following initial conditions in all simulations: $x_0^r = \delta^r + 10$, $x_{-1}^r = \delta^r + 10$, $x_0^l = \delta^l + 10$, $x_{-1}^l = \delta^l + 10$.

To demonstrate the role of time delay, we have used the parameter value that gives a periodic structure and have varied the delay time index m in (3). As a result we observe different periodic structures (Fig. 6). The effect of
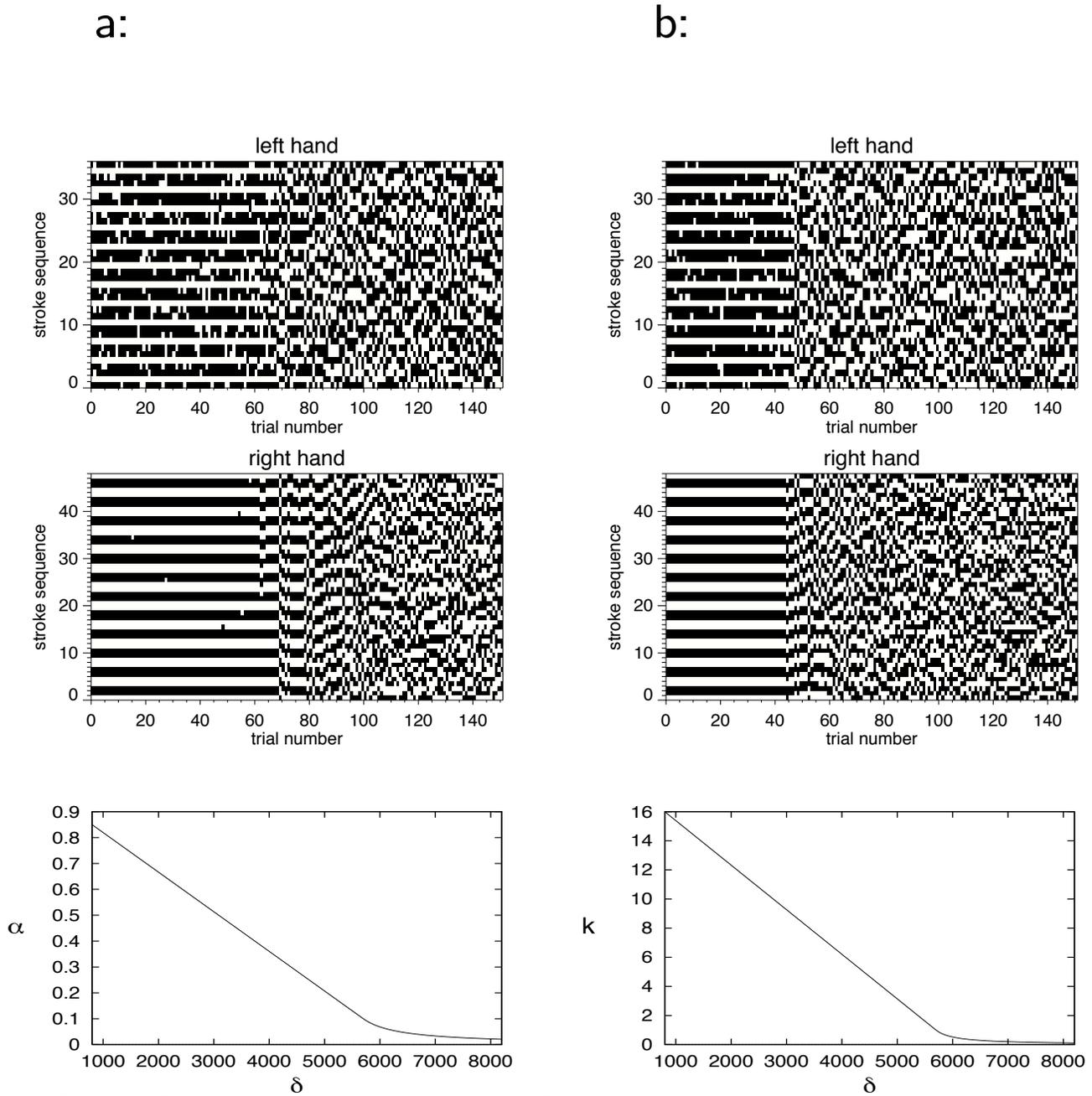
different delays could serve as an explanation of similar transitions seen in the experimental data (Fig. 2c).

a: **b:**

Figure 5: Results of the numerical simulations. Transitions between irregular and regular patterns occur under variation of the control parameter $\alpha(\delta)$ (a) as well as under variation of the control parameter $k(\delta)$ (b). Shown are the symbol patterns for the left and the rigth hand and the variation of the control parameters (bottom figures).

Increasing the delay m in Eq. (3) while decreasing the cycle length can be motivated physiologically. The delay in the feedback loop is determined by the speed of propagation of signals within the central nerveous system and can be considered to be constant. The relative delay (i.e. the delay expressed in cycles) changes with variation of tempo.

The above diagram and graphical visualization of the transition are obtained for the case when the parameters of regulation and coupling for both hands are the same and changed simultaneously. That is why the amplitudes of oscillations are the same for the right and left hand. If these parameters differ, then the amplitudes for both hands

will be different and the bifurcation will occur for different values of the tempo. We used these properties of the model to reproduce the symbol patterns observed in the experiment (Fig. 7).
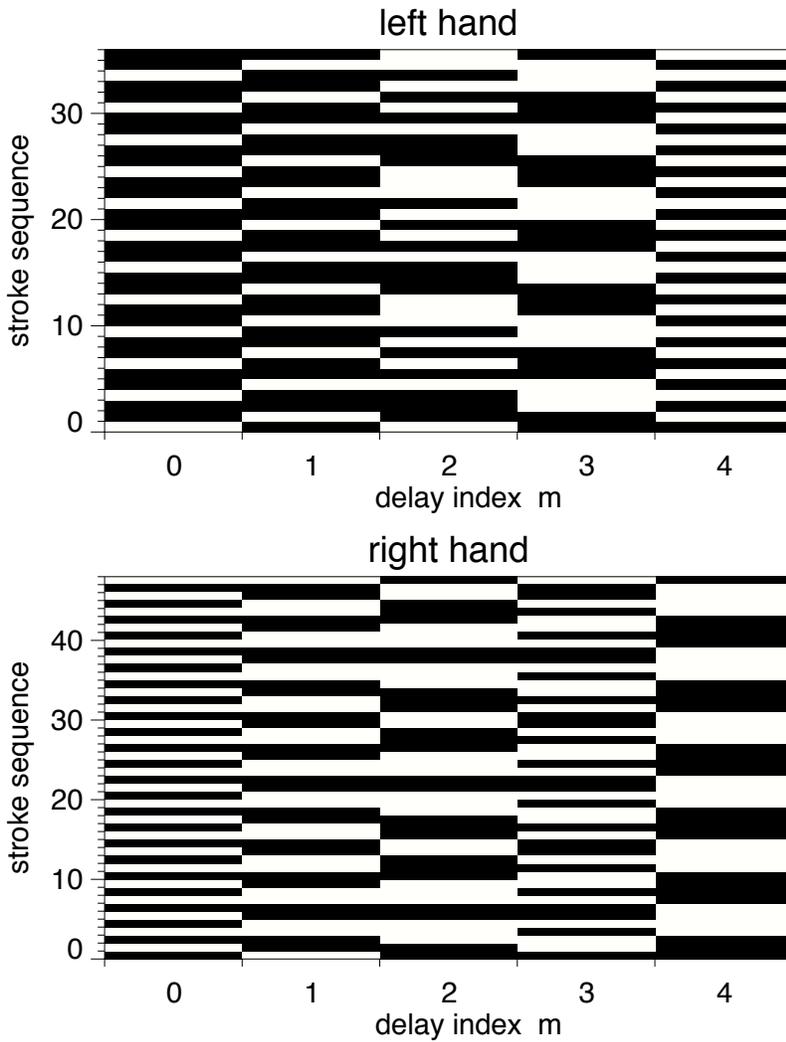
Figure 6: Results of the numerical simulations. Changes of the periodic patterns due to different values of the delay time index m can be seen clearly for both hands.

The functional dependence of the parameters $k^l = k_1^l = k_2^l$ and $k^r = k_1^r = k_2^r$ on δ seems to be somewhat arbitrary, but it is demonstrated, that with this model it is possible to obtain qualitatively the same symbol patterns as for the experimental data.

Our simulations have shown that the results do not depend qualitatively on the exact forms of the functions F and G; the piece-wise linear functions yield similar results.
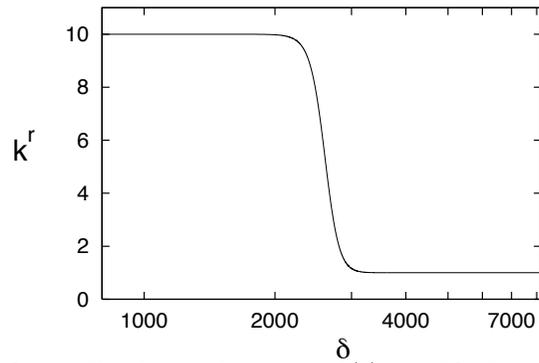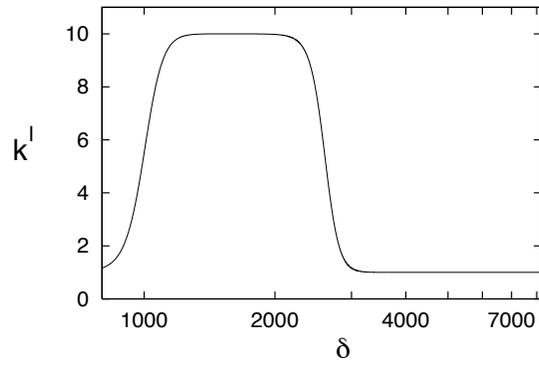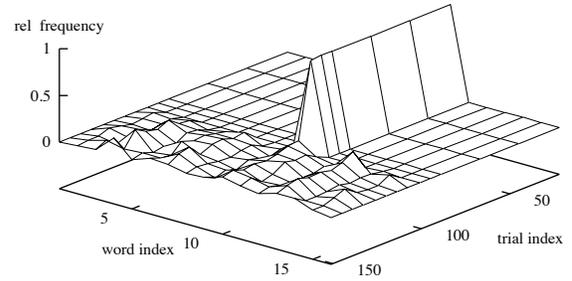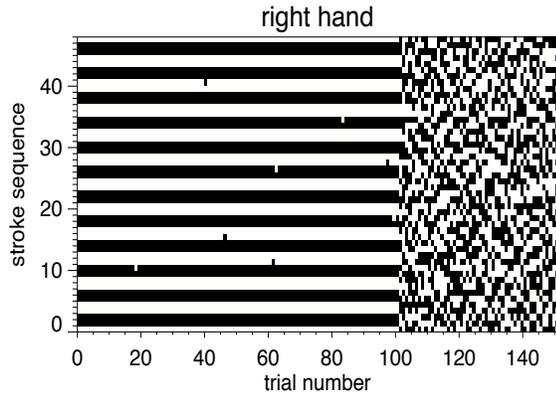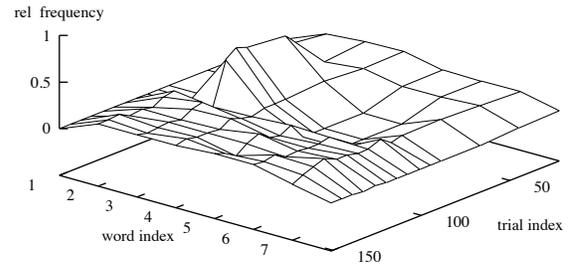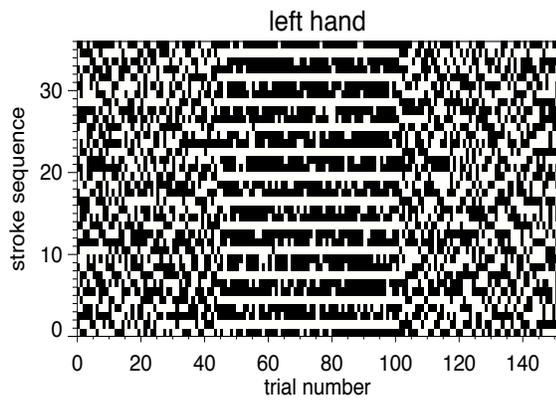
Figure 7: Simulation of the experimentally observed patterns: (a) graphical representation of the symbolic transformations (cf. Fig. 2 (a)), (b) distributions of the frequencies of words (cf. Fig. 2 (b)), (c) dependence of the coefficients k^l and k^r on δ which has been chosen to reproduce the experimental data of Fig. 2 (a).

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V. CONCLUSION

We have suggested a top-down model for explaining the main phenomena detected by time series analysis of a bimanual hand movement experiment. The key observation in the experimental data is a qualitative change of produced interval lengths under variation of the external control parameter, the speed. It is not possible to explain this phenomenon by linear correction mechanisms. With our proposed model we are able to reproduce main experimental findings. Under variation of the model parameter which corresponds to the tempo prescribed in the experiment, the model shows transitions between states with different dynamics. In addition the stability of the behaviour under the influence of noise is reproduced.

The idea of the model is based on a feedback control circuit. The main constituents are nonlinear feedback with time delay and a coupling mechanism between both hands. The nonlinear feedback yields qualitative changes in the model, while the coupling is necessary to obtain the $3:4$ periodic structures. This particular feature can only be obtained by including a delay. Introducing noise in the simplest way results in a qualitative correspondence of the symbol patterns obtained from experimental and model data.

We emphasize that this model is minimal in the sense, that it qualitatively describes the experimental results. It gives insights into the functionality of the central nerveous system. The validity of the model has to be tested further by performing new experiments. For instance, an artificial variation of the acoustic delay should result in different periodic structures in the symbolic representation. These are open problems for future work.

Although it is not completely clear how the fundamental mechanisms of our model are exactly realized neurophysiologically, they describe qualitatively the main aspects of a physiological control system, governing coordination of two hands. This model should be viewed as being complementary to existing ones, describing rhythmic movements.

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