HTML ABSTRACT

Modeling Cognitive Control in Simple Movements

A.A. Zaikin and J. Kurths

Institute of Physics, University of Potsdam, Am Neuen Palais 10, 14469 Potsdam, Germany
zaikin@agnld.uni-potsdam.de

Abstract. Using a macroscopic approach we discuss appropriate minimal models to describe a performance of bimanual rhythmic movements. As a paradigm of such movements, a 3:4 polyrhythm is considered. An analysis of experimental data on the basis of symbolic transformation shows tempo-induced qualitative transitions. In attempting to get insights into the control mechanisms of the central nervous system behind these transitions, a model of the control system has been suggested. Two versions of the model, with discrete and continuous time are considered and both of them are in good correspondence with experimental data. Important ingredients of the model are time delay, nonlinear error correction and noise.

Keywords: cognitive control, brain, transitions, feedback, polyrhythmic movements.

INTRODUCTION

The way, how our brain performs control of movements is a challenging problem of contemporary science [1,2]. In the study of this problem one can distinguish traditional cognitive neuroscience which investigates cognitive processes through analysis of brain electrical and magnetic activity [3] and modeling brain control structures to compare model results and experimental data. Modeling in its turn can be based either on neural networks [4] or on a macroscopic or conceptual approach [5] (for review see also [6]). The key idea of the macroscopic approach is to model control structures by simple mathematical equations.

In the present paper we show the application of a macroscopic approach by consideration of two simple models for control structures. Here it is worth also to mention the well-known model introduced by H. Haken, J.A.S. Kelso and H.Bunz (see [7] and references therein) to describe the change in phasing of oscillatory motions of the two index fingers. The model consists of two differential equations with coupling. The model describes rather well a transition in phasing occured with the increase of the frequency.

Using the same approach we have suggested a minimal model to describe control structures, which are responsible for the production of polyrhythms [8]. Polyrhythms serve as a paradigm in the study of bimanual movements and are ideally suited to study synchronization and coordination between two fingers because of the complicated phase relation between successive strokes during one cycle.

This paper is organized as follows. In the Sec. 2 we review results of an analysis of the experimental data [8]. Using methods of symbolic dynamics it is possible to show that an increase of the tempo leads to clearly visible qualitative transitions. To understand the origin of these transition, a minimal theoretical model has been proposed [8,9]. This model and results of simulations are reviewed in the Sec. 3. In the Sec. 4 we extend this model by introducing a continuous time. This new model is a central point of the present paper. We suggest a compartmental model which consists of an excitable oscillator, which is "fired" by the control loop. Equations describing an oscillator originate from the famous Fitz-Hugh Nagumo model, which is usually used to describe nerve pulses. The main features of the model are nonlinearity, the influence of noise, simple control mechanism and feedback with a time delay. Results of numerical simulations are compared with experimental data. After a discussion about features of the new model we summarized results obtained.

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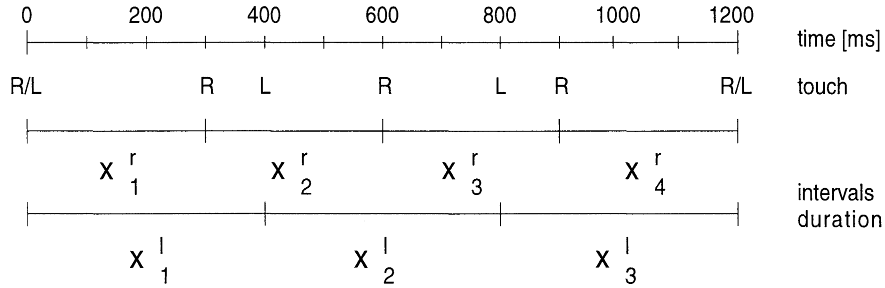

FIGURE 1. The scheme of the ideally performed polyrhythmic task for a cycle length of 1200 ms [8]. During every cycle subjects are required to produce 4 strokes with the right hand and 3 strokes with the left hand. Letters R and L denote the strokes by right and left hands, respectively.

EXPERIMENTAL SETUP AND DATA ANALYSIS

In the experiment pianists were required to produce a bimanual 3:4 polyrhythm. The key idea was to analyze a production of polyrhythm over a wide range of cycle lengths, i.e. tempos. A scheme of the ideally produced polyrhythm for a cycle duration of 1200 ms is shown in Fig. 1. The task was performed by well-trained amateur pianists on an electronic piano and intervals between strokes were recorded (for details see [9]).

The experiments were organized as follows. Subjects initially could listen to the rhythm generated by a computer as long as they wanted. After the subjects started to play on an electronic piano, the computer continued for four more cycles, playing in parallel. Then the computer stopped and the subjects had to continue the rhythm for 12 cycles. The intervals between strokes were recorded by the computer connected to the piano with a weighted keyboard mechanism. The time resolution of the recorded data is 1 ms. Fourteen different metronome tempos ranging from 800 ms to 8200 ms per cycle were presented in a randomized order (for details see [10]).

To analyze the data we applied a symbolic transformation [11]. For this we denoted the data recorded during one cycle for the left and right hand respectively, by x_1^l, x_2^l, x_3^l and $x_1^r, x_2^r, x_3^r, x_4^r$ (Fig. 1). For every cycle we computed the mean interval length for the right hand x^r as $\frac{1}{4}(x_1^r + x_2^r + x_3^r + x_4^r)$ and for the left hand x^l as $\frac{1}{3}(x_1^l + x_2^l + x_3^l)$. We then applied a symbolic transformation $s_i^{r,l}$ to the interval lengths $x_i^{r,l}$ of each cycle according to the rule:

$$s_i^{r,l} = \begin{cases} 1 & \text{if } x_i^{r,l} > x^{r,l} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Using this transformation each time series is converted into a symbol sequence. The results for all trials of two different subjects are visualized in Fig. 2. The symbols "0" and "1" are coded with black ("1") and white ("0") squares. From these plot we find that an increase of the tempo (decrease of the trial index) leads to clearly visible transitions in the structure of the pattern. This transition can be reentrant and also a situation is possible, when a transition happens only for one hand. Further details of the analysis of symbol patterns as well as a quantitative description of the transitions observed can be found in [8,9].

MODELING POLYRHYTHM WITH DISCRETE TIME

To understand the transitions observed, a minimal model has been suggested [8]. The key idea of this model is that periodic patterns appear as a result of excitation of oscillations in the control system, modulating the desired rhythm and producing the observed patterns.

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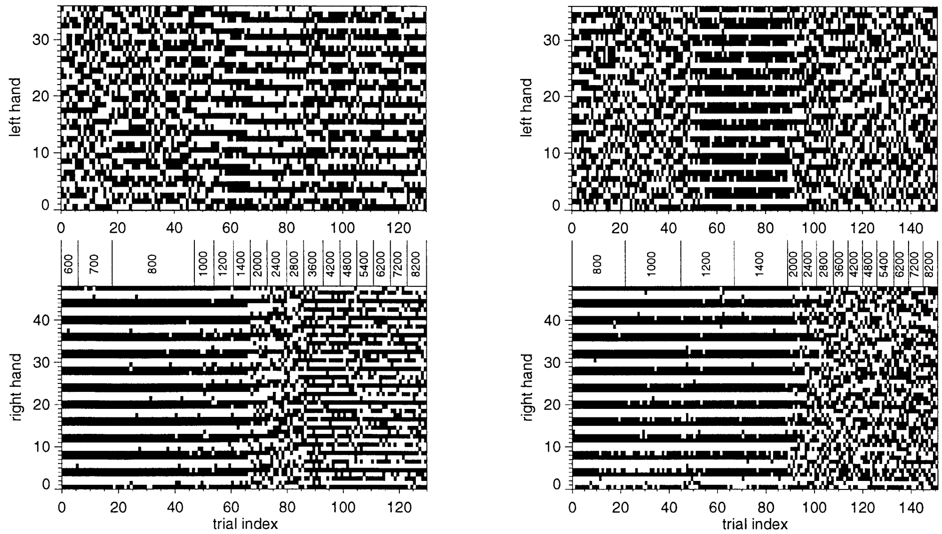

FIGURE 2. Example of the symbolic transformation of the experimental data for two chosen subjects [8]. The transitions in the symbolic patterns are clearly visible. On the abscissa the trial numbers are plotted, on the ordinate the coding for the time intervals between the strokes. Every trial consists of 48 strokes for a right hand and 36 for a left one. Every stroke is coded either by black or white color. The black color is chosen if the interval between strokes is less than 1/4 or 1/3 of the produced cycle for the right and left hand, respectively.

A possible scheme of the control system is shown in the Fig. 3. Here by "CRG" we denote central rhythm generator. A produced interval is analyzed by a control system with a *time delay* and corrected. For simplicity we consider a feedback loop with one delay time for each hand, although the existence of proprioceptive, visual and auditive feedback leads to different time delays. There are two error correction loops. The first one is a correction with respect to the produced interval just for one hand. This correction is performed by the block "G". If the produced interval was too short, then the following interval will be corrected to be longer, and vice versa. It occurs with a time delay τ. The second correction loop is the coupling between these control circuits. We neglect the role of weak coupling within the cycle itself and suggest that it is "turned on" only at the end of the cycle. This coupling is motivated by the experimental fact that the simultaneous strokes at the end of each cycle are precisely performed by all subjects. This correction is denoted by the block "F" and fulfilled with respect to the difference between produced cycle durations for two hands.

Now let us write equations for a plausible control scheme based on the assumptions above. First we do it for one hand - the uncoupled case:

$$y_{i+1} = \delta - k\ \tanh(\alpha \cdot (y_{i-m} - \delta)), \tag{2}$$

where i represents time and $m \geq 0$ is the discrete time delay of the correction interval. In these equations y_i are time intervals produced by the control system. The correction function is chosen to be $\tanh(y_{i-m} - \delta)$. The correction acts with respect to the difference between the time interval y_{i-m} and the interval δ (the exact interval length of the system). The tempo dependent parameters $\alpha = \alpha(\delta)$ and $k = k(\delta)$ adjust the strength of the correction. The idea of the control is rather simple: if the fulfilled interval y_{i-m}, estimated with some time delay (where m is the delay time index) is performed with some error, then the control system will correct it.

A correction function is chosen to be nonlinear and there is a motivated reason for it. If the correction is linear, then no stable 3:4 deviation in the errors can be observed, but this is a case in the experimental data. An extended discussion about it can be found in ([9], Sec.IV,a).

On the base of a correction loop for one hand (eq.(2)) we construct a model for the description of bimanual performance. For this we add coupling to the model:

$$\begin{aligned} y^r_{i+1} &= \delta^r - \Delta^r_i\, k^r_1\ \tanh(\alpha \cdot (x^r_{i-m} - \delta^r)) - \Theta^r_i\, k^r_2\ \tanh(\alpha \cdot (\tilde{\delta}^r - \tilde{\delta}^l)) \\ y^l_{i+1} &= \delta^l - \Delta^l_i\, k^l_1\ \tanh(\alpha \cdot (x^l_{i-m} - \delta^l)) - \Theta^l_i\, k^l_2\ \tanh(\alpha \cdot (\tilde{\delta}^l - \tilde{\delta}^r)) \end{aligned} \tag{3}$$

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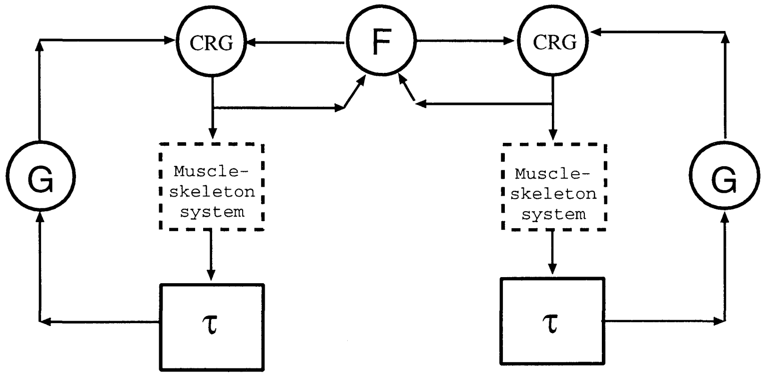

FIGURE 3. The scheme of the control system. CRG are central rhythm generators [8]. A correction is performed at the end of the cycle (denoted by F) and within a cycle (denoted by G) with a time delay τ. A scheme reflects that essential compartments of the control system is a correction loop with a delay and a coupling between hands.

$$x_i^{r,l} = y_i^{r,l} + \xi_i^{r,l}.$$

Here within-cycle correction is implemented by the term

$$\Delta_i^{l,r} \tanh(x_{i-m}^{l,r} - \delta^{l,r}) \quad ,$$

where

$$\Delta_i^{r,l} = \begin{cases} 1 & \text{if } (i+1) \bmod N^{l,r} \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The synchronization at the end of each cycle is maintained by the coupling term

$$\Theta_i^{l,r} \tanh(\tilde{\delta}^{r,l} - \tilde{\delta}^{l,r}),$$

where

$$\Theta_i^{r,l} = \begin{cases} 1 & \text{if } (i+1) \bmod N^{l,r} = 0, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

and $\tilde{\delta}^{r,l}$ are the estimated lengths of the cycles.

We assume this end-of-cycle control to dominate over the within-cycle one. Therefore, the action of the latter one during the last strokes of each cycle is neglected.

The index m is the delay time in the correction loop and set equal to 1. The role of the coupling is to decrease the difference between the lengths of the cycles performed by different hands. As this correction is produced by the control system *before* the cycles are finished, the argument of this function is the *estimated* difference between cycle lengths $\tilde{\delta}^r - \tilde{\delta}^l$. We assume that the length of the cycle is estimated as a sum of the intervals produced by the regulating system (i.e. not corrupted by noise). As the last interval has to be predicted by the control system, it seems reasonable to suppose that the "ideal" value is used instead. Hence, we use

$$\tilde{\delta}^{l,r} = \sum_{j=0}^{N^{l,r}-2} y_{i-j}^{l,r} + \delta^{l,r}. \tag{6}$$

To take into account the inaccuracy of timers, their weak interaction with other physiological subsystems, fluctuations in the properties of the muscle–skeleton system etc., we introduce additive noise $\xi_i^{l,r}$ into the model equations. For the sake of simplicity we describe all the sources of fluctuations for a single hand by one noise term.

An excitation of oscillations in the control system, i.e. the occurrence of a Hopf bifurcation, explains the transition between regular and irregular structures in the symbol patterns of Fig. 2. In the absence of

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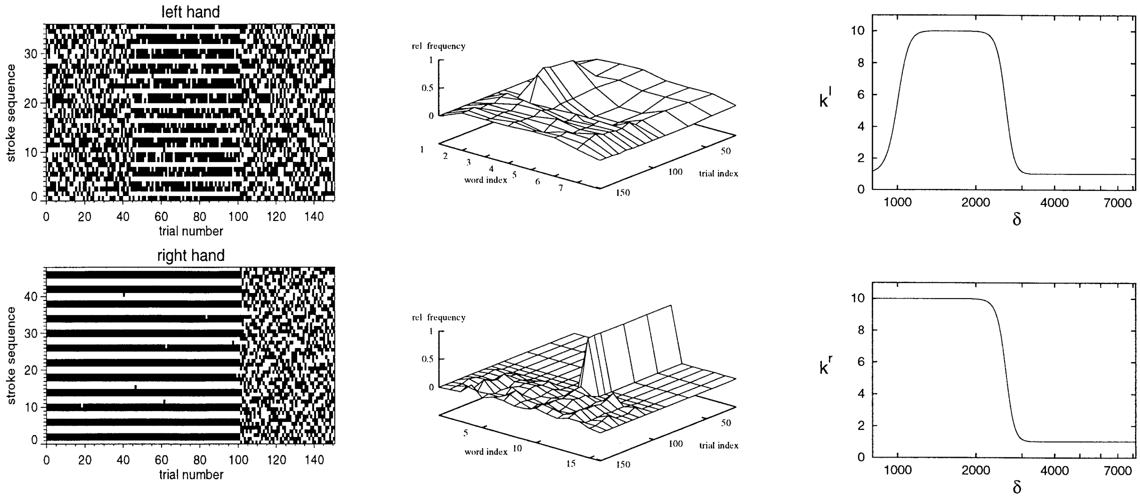

FIGURE 4. Numerical simulations [8]. Parameters are chosen to correspond to the structure observed in the experiment for one certain subject (cf. Fig. 2, *right*). To prove the existence of a transition distributions of the frequencies of words is shown. Corresponding dependencies of a control parameters k^r and k^l are shown on the left.

oscillations, even a small intensity of noise leads to irregular patterns in the symbol sequences. This case corresponds to a correct production of the rhythm.

If parameters of correction strength are different for the left and right hand, then the amplitudes will be also different and the bifurcation will occur for different values of the tempo. We used these properties of the model to reproduce the symbol patterns observed in the experiment (Fig. 4). The functional dependence of the parameters $k^l = k_1^l = k_2^l$ and $k^r = k_1^r = k_2^r$ on δ seems to be somewhat arbitrary, but it is demonstrated that with this model it is possible to obtain qualitatively the same symbol patterns as for the experimental data. The transient period for every trial was equal to 10 cycles. The same as in the experiment symbol sequences demonstrate stability with respect to small perturbations. Hence, numerical simulations performed for the model are in good correspondence with the transitions observed in the experiment.

To demonstrate the role of the time delay, we have changed a delay time index m in (3). As a result we observe different periodic structures (Fig. 5). The effect of different delays could serve as an explanation of similar transitions seen in the experimental data (Fig. 2). Increasing the delay m in Eq. (3) while decreasing the cycle length can be motivated physiologically. The delay in the feedback loop is determined by the speed of propagation of signals within the central nervous system and can be considered to be constant. The relative delay (i.e. the delay expressed in cycles) changes with variation of tempo.

The model has received its further development in [9] by introducing random time intervals δ_r, δ_l. In this case, to produce a pattern observed in the experiment (like in the Fig. 4) it is enough now to use only linear dependencies for the control parameters that seems more realistic from a physiological point of view.

MODELING POLYRHYTHM WITH CONTINUOUS TIME

In this section we present a model with continuous time to describe the control of bimanual movements in the performance of polyrhythms.

Similar to the previous section we start with a consideration of a central rhythm generator for one hand. In the presented model it is described by the following system of equations:

$$\epsilon \dot{x} = x - \frac{x^3}{3} - y,$$

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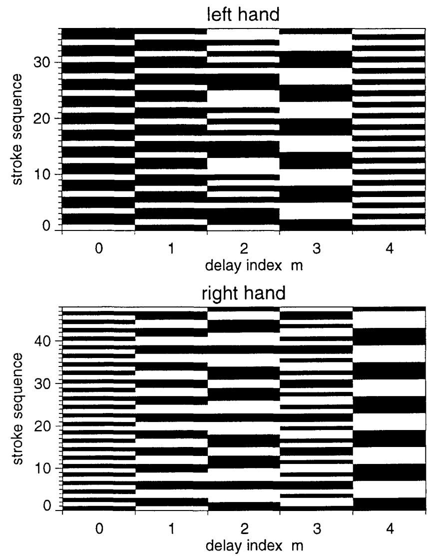

FIGURE 5. Numerically obtained structures for different delay index [8]. The most typical pattern corresponds to the delay index equal to unity. But in the experiment (see Fig. 2, *left*) it is possible also to observe a pattern which is modelled by a delay index 0

$$\dot{y} = x + a + \sum_{j=1}^{\infty} A\delta(t - \sum_{i=1}^{j} l_i) + \xi(t), \tag{7}$$

$$l_{i+1} = \delta - k\tanh(l_{i-1} - \delta). \tag{8}$$

This model consists of two compartments. The first compartment (eq.(8)) is a rhythm generator described in the previous section by eq.(2). The same as above δ is a required produced interval, k is a strength of a correction and l_i is an interval which should be produced after a correction with strength k and delay equal to one interval. An interval l_i will determine a "firing" of the relaxation system, described by the Eqs. (7). A disturbance of this excitable system is performed via a term $\sum_{j=1}^{\infty} A\delta(t - \sum_{i=1}^{j} l_i)$ at times, which are set by the control system (8). An impulse in the variable $y(t)$ means that a stroke happens.

To understand the behaviour of this system we note that eqs.(7) with $A = 0$ describes an excitable oscillator – the famous Fitz Hugh-Nagumo system [12] with noise. Originally this model was suggested for the description of nerve pulses [13], that seems to be appropriate for the description of polyrhythm performance. The parameter $\epsilon << 1$ is a small parameter which allow us to separate a solution of eqs.(7) in the fast (only x changes) and slow ($y \approx x - x^3/3$) motions. The parameter a determines the existence of attractors: for $|a| > 1$ it is a stable fixed point, for $|a| < 1$ a limit cycle. We consider a case with $a = 1.05$. In this model one can observe noise-induced pulses and even a noise-induced temporal order [12] , but first we consider a case that the noise intensity σ^2 is too small to induce an impulse.

Now let us suppose that instead of a term with a sum of $\delta-$functions in eqs. (7), we have a term $A\delta(t-T)$, where $T = 50$, for example. In this case an impulse will be induced at the time $t = T$ (see Fig. 6). The parameter A determines a strength of "firing". To obtain a sequence of impulses we have a sum of $\delta-$functions in the equations. Hence a term $\sum_{j=1}^{\infty} A\delta(t - \sum_{i=1}^{j} l_i)$ implies that "firing" will occur with intervals l_i between impulses. Taking into account that eq.(8) describes a control system with correction, a performance of rhythm (impulses) is executed in the same way as in the previous section. An example of rhythm with a correction is shown in the Fig. 7 (*left*). This figure shows that a correction is fulfilled with a delay of one interval (shown by pointers).

The advantage of this rhythm generator is that not only a performance of intervals but also a movement of a hand is built-in. So, a coupling between two hands can be implemented, but in this paper we leave this as an open question.

To describe a bimanual rhythm, we use two control systems, described by eqs.(7,8) and add a coupling in a

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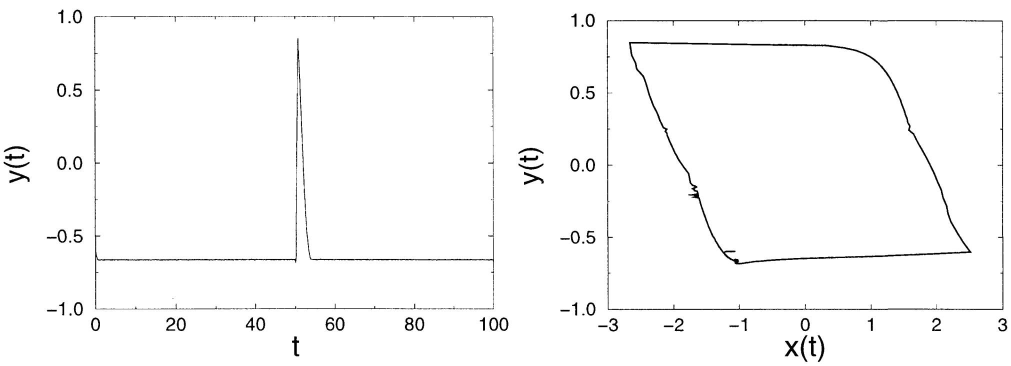

FIGURE 6. A model with a continuous time. The reaction to the $\delta(t-T)$ in the rhs. An impulse is induced at the time $t=T$ (*left*) which corresponds to a cycle in the phase space (*right*).

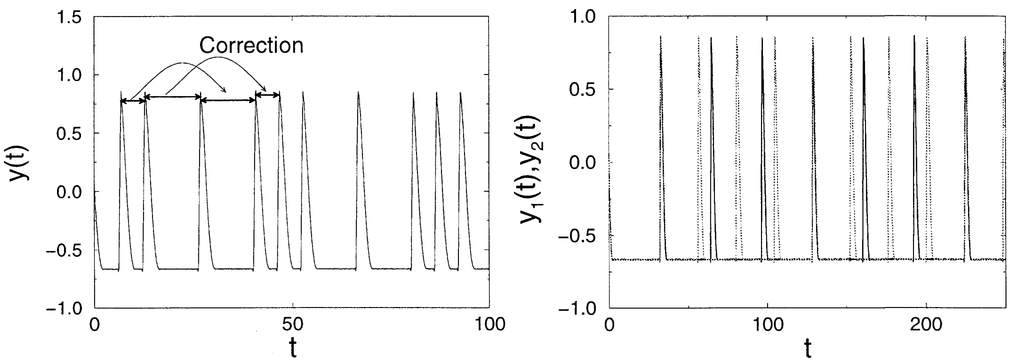

FIGURE 7. *Left*: Illustration of the correction mechanism. Intervals which are too short or too long are corrected with a delay of one interval. *Right*: Numerical simulations which correspond to the performance of an ideal 3:4 polyrhythm. Impulses for left and right hand are plotted by solid and dotted line, respectively. Every 4th stroke perfomred by right hand (dotted line) coincides with 3th stroke for the left hand (solid line).

similar way as in the model with discrete time.

Then the eqs. of the model are:

$$\epsilon \dot{x}_1 = x_1 - \frac{x_1^3}{3} - y_1,$$

$$\dot{y}_1 = x_1 + a + \sum_{j=1}^{\infty} \delta(t - \sum_{i=1}^{j} l_i) + \xi(t), \tag{9}$$

$$\epsilon \dot{x}_2 = x_2 - \frac{x_2^3}{3} - y_2,$$

$$\dot{y}_2 = x_2 + a + \sum_{j=1}^{\infty} \delta(t - \sum_{i=1}^{j} r_i) + \xi(t), \tag{10}$$

$$l_{i+1} = \delta^l - \Delta_i^l k \tanh(l_{i-1} - \delta_l) - \Theta_i^l k \tanh(\tilde{\delta}^r - \tilde{\delta}^l),$$

$$r_{i+1} = \delta^r - \Delta_i^r k \tanh(r_{i-1} - \delta_r) - \Theta_i^r k \tanh(\tilde{\delta}^l - \tilde{\delta}^r). \tag{11}$$

In comparison with a central rhythm generator for one hand, a coupling in the control system (eqs.(11)) is added. For eqs.(11) we use the same notations as in the previous sec. and an impulses described by $y_1(t)$ and $y_2(t)$ are strokes performed by left and right hands.

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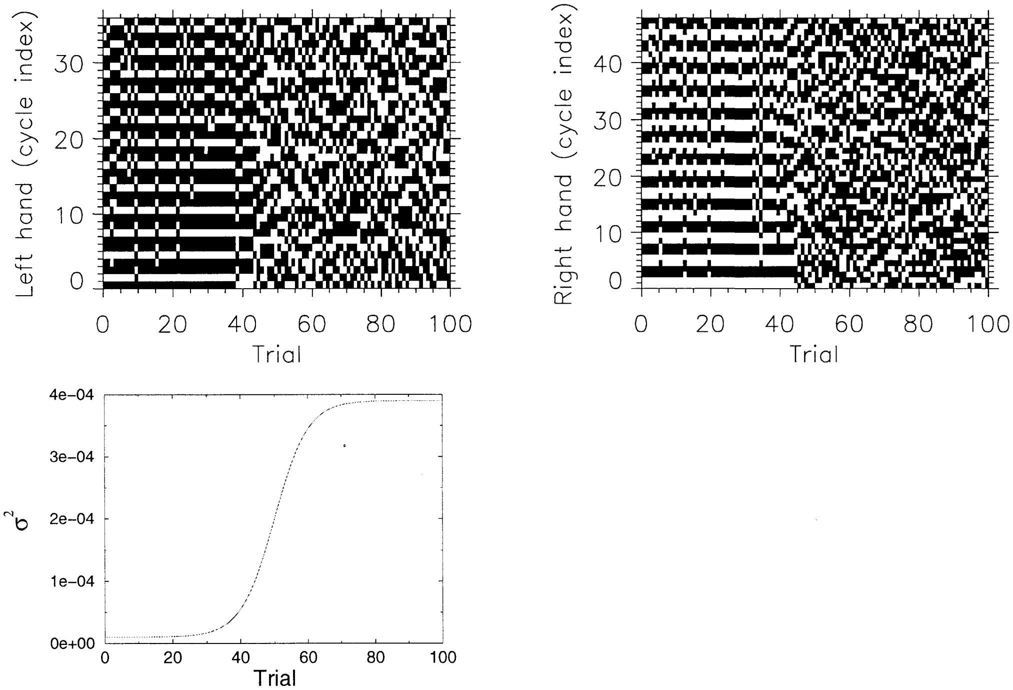

FIGURE 8. Numerical simulation of a model with continuous time. A transition between 3:4 pattern and disordered homogeneous one caused by the increase of noise intensity. A patterns for left and right hands are plotted together with a dependence of additive noise intensity on the trial index.

In the ideal case, when there is no need for correction (terms) an exact polyrhythm is executed. This situation is shown in the Fig. 7, *right.* If the produced interval differs from the required, it will be corrected with a delay of one interval. In this case using a symbolic transformation according to the same rules as in Sec. 1, we again observe a 3:4 structure that is shown in the Fig. 8 for the trials $0 - 40$. The corresponding dependence of additive noise intensity σ^2 on the number of trial is shown in the same figure. If the noise intensity is rather large in addition to the impulses "fired" by the control system, noise-induced impulses will occur and the periodic structure will be destroyed. In Fig. 8 one can observe this transition between order and disorder.

We have demonstrated only the general framework for modeling of a transition caused by the increase of noise intensity, but it is obvious that one can also achieve a transition by the variation of correction strength (as in Fig. 4) or by the introducing stochastic variables as in [9]; this is beyond a scope of the present paper.

Let us conclude this section by a statement that the presented model seems to be adequate for modeling bimanual rhythmic movements. Without any doubt, the model has a rich dynamics and thorough investigation of it is an open question now. Here it is worth to mention that one very important advantage of the model is the opportunity to introduce coupling between differential equations which describe movements of two hands. The coupling may lead to synchronization and therefore may enable us to explain also synchronization effects observed in the experiment.

CONCLUSIONS AND OPEN QUESTIONS

In conclusion, consideration of these two models has shown that surprisingly simple models can describe the algorithm of the control program responsible for different movements. Taking into account structuring of movements performed by these models one can speak about *anticipative cognitive control.* Despite the fact that noise plays an important role in the models, we note that, in contrast to a purely stochastic approach, both

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models are oriented to a dynamical explanation of the observed qualitative transitions. As it already happened with a study of nonlinear systems in physics, a next step could be a derivation of models demonstrating noise-induced or noise-sustained effects. That can be expected from the fact that noise-induced order has been observed in eqs., similar to eqs.(7) [12].

The open questions are how one control program replaces another one and how to find the connection between the presented macroscopic phenomenological approach for modeling and a fast developing powerful theory of the neural systems.

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