Reflection and Transmission of Plane Polarised Light

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Abstract: An experiment was conducted to see how the refracted intensity of light is affected by the incident angle and the plane of polarization of the incident light between two dielectric media (air and Perspex). From this the refractive index, \( n \), of Perspex was determined using Snell’s law and through the relationship with Brewster’s angle. The Fresnel relations were then verified by plotting a graph of the reflection coefficients against incident angle for light polarized parallel and perpendicular to the plane of interface. The value of the refractive index of Perspex was determined to be \( n_{\text{Perspex}} = (1.486 \pm 0.006) \) through the Snell’s law experiment and \( n_{\text{Perspex}} = (1.483 \pm 0.009) \) from measurement of Brewster’s angle, compared to the accepted value \( n_{\text{Perspex}} = 1.491 \) [1], indicating that the experiment conducted was accurate as both values of \( n_{\text{Perspex}} \) are in agreement with the accepted value.

I. INTRODUCTION

The intensity of light reflected from and transmitted through the surface of a dielectric medium depends not only on the angle of incidence of light, but also on the orientation of the plane of polarization of the light. The intensity of reflected and transmitted light with varying incidence angle was investigated for both orientations (parallel and perpendicular) of polarization at the interface between air and Perspex. Fresnel’s relations are derived from Maxwell’s equations, which show that when an electromagnetic wave passes through the boundary between two dielectric media, the angles of incidence and transmission is related by Snell’s law given by (1) [2].

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

(1)

Where \( n_1 \) and \( n_2 \) are the refractive indices of material 1 and 2 respectively, and \( \theta_1 \) and \( \theta_2 \) are the angles of incidence and refraction respectively.

By combining Snell’s law and relations derived from Maxwell’s equations it is shown that the reflection coefficient of the fraction of incident amplitude reflected for light polarized parallel to the plane of incidence is given by (2) [5].

\[
R_// = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}
\]

(2)

Similarly, the reflection coefficient of the fraction of incident amplitude reflected for light polarized perpendicular to the plane of incidence is given by (3) [5].

\[
R_\perp = \frac{-\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}
\]

(3)

Equations (2) and (3) are known as the Fresnel relations.

From equation (2) it can be seen that when:

\[
\theta_1 + \theta_2 = 90^\circ
\]

(4)

\( R_// \) tends to zero, suggesting there exists an angle at which the fraction of incident amplitude reflected for light polarized parallel to the plane of incidence equals zero. Hence, all reflected light at this particular incidence angle is completely polarized perpendicular to the plane of incidence. This is known as Brewster’s angle \( \theta_B \), given by (5) [4].

\[
\theta_B = \tan^{-1} \frac{n_2}{n_1}
\]

(5)

II. EXPERIMENTAL METHOD

An experiment was conducted to determine the refractive index of Perspex, \( n_{\text{Perspex}} \), by varying the angle of incidence light polarized parallel to the plane of incidence and measuring the refracted angle. Prior to taking measurements, it was ensured that the normal to the plane face of the Perspex block was correctly aligned with the 0° on the protractor by rotating the detector arms so that the reflected light fell centrally on the detector. This ensured no systematic error was introduced by the block being in an incorrect position. It was also ensured that the contact with the curved and plane face of the Perspex block was limited, to minimize the amount of residue left on these faces; hence this would reduce the proportion of random uncertainties introduced in measuring \( \theta_2 \) and the propagated refractive index.

Additionally, when rotating the device arms to set \( \theta_1 \) and measure \( \theta_2 \), it was ensured to look from directly above the apparatus in order to eliminate parallax and reduce the proportion of systematic errors.
In the investigation a solid state laser diode of wavelength $\lambda = (670 \pm 1)$ nm was used. The incidence angle was varied between $0^\circ$-$90^\circ$ in intervals of $5^\circ$ and the corresponding refracted angles were measured. A graph of the sine of incidence angle against the sine of refracted angle was plotted, shown in Figure 3 and $n_{\text{Perspex}}$ was determined from Snell’s law using (1) and through a weighted mean calculation. The apparatus used is shown in Figure 1.

Prior to taking measurements the phase lock amplifier was calibrated by taking the difference in DVM readings with the laser switched on through the Perspex block and off to determine the normalization constant. A Polaroid filter was used to polarize the light from the laser parallel to the plane of incidence for measuring $R_{\perp}^2$. The incident angle was then varied between $0^\circ$-$85^\circ$ in $5^\circ$ intervals at a constant gain and the corresponding reflection intensity was measured. It was noted that Brewster’s angle appeared between $55^\circ$-$65^\circ$ where the intensity tended to zero. Hence, measurements of reflection intensity were repeated within this range in intervals of $1^\circ$ increasing the gain to amplify the weak signal, in order to determine where the intensity is a minimum.

The readings from the DVM were normalized and rooted to obtain reflection coefficients $0 \leq R \leq 1$. A graph of reflection intensity parallel to the plane of incidence, $R_{//}$, was plotted against the incident angle, shown in Figure 4 and a second value for $n_{\text{Perspex}}$ was determined using Brewster’s angle (5).

The same procedure was repeated for $R_{\perp}$ adjusting the Polaroid filter to polarize the light perpendicular to the plane of incidence. The Fresnel relations were then compared with the experimental data by plotting the predictions made by (2) and (3) alongside them (see Fig. 4).

### III. RESULTS & ANALYSIS

The value obtained for the refractive index of Perspex was $n_{\text{Perspex}} = 1.493 \pm 0.378$ using Snell’s law equation and the inverse gradient of Figure 3. The refractive index of air, $n_1$, was taken to be 1 as the precision of its uncertainty is much greater than the precision of this experiment. Hence, this would have had a negligible effect on the propagated uncertainty for $n_{\text{Perspex}}$. Manipulating (1) gives rise to equation (6).

\[
    n_2 = \frac{\sin \theta_1}{\sin \theta_2} \tag{6}
\]

However, by calculating the gradient using the method of weighted least squares fitting, $n_{\text{Perspex}} = 1.486 \pm 0.006$, hence the proportion of uncertainty was reduced and was much closer to the accepted value given by [1].

The main source of uncertainty in calculating the refractive index of Perspex was the measurement of $\theta_1$ and $\theta_2$, as readings were taken to be up to a precision of $0.5^\circ$, which was half the precision of the protractor used. This added to the uncertainty in $n_{\text{Perspex}}$ when propagating values. The graph plotted of $\sin(\theta_1)$ against $\sin(\theta_2)$ used to determine $n_{\text{Perspex}}$ is shown in Figure 3.
Fig. 3 Plot of sine of incidence angle against sine of refracted angle of light polarized parallel to the plane of interface through a Perspex Block. The plot was used to determine a value for $n_{\text{Perspex}}$ using both Snell’s law (1) and a weighted mean calculation [3].

From Figure 3, it can be seen that $\sin \theta_1$ is proportional to $\sin \theta_2$, which is in agreement with the theory. The errors in both $\sin \theta_1$ and $\sin \theta_2$ are negligible, hence the error bars are small and indicating that $n_{\text{Perspex}}$ determined from Snell’s law was both accurate and precise.

When measuring Brewster’s angle, it was not physically possible to determine where the intensity of reflected light reached zero. When the gain was increased to amplify readings from the weakest possible signal, this was not normalizable with the apparatus available as the output gain became fully saturated when obtaining a normalization constant. Table 1 shows the data acquired for measurements around Brewster’s angle.

<table>
<thead>
<tr>
<th>$\theta_1$ (°)</th>
<th>Intensity of reflected light (V)</th>
<th>$n_{\text{Perspex}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.0 ± 0.5</td>
<td>0.0149</td>
<td>1.428 ± 0.009</td>
</tr>
<tr>
<td>56.0 ± 0.5</td>
<td>0.0101</td>
<td>1.483 ± 0.009</td>
</tr>
<tr>
<td>57.0 ± 0.5</td>
<td>0.0098</td>
<td>1.540 ± 0.009</td>
</tr>
</tbody>
</table>

Table 1 Data obtained for light polarized parallel to the plane of interface for measurements around Brewster’s angle, varying the incidence angle in intervals of 1° and measuring the intensity of reflected light at a gain of 100 [6].

Brewster’s angle could only be determined by inspection as normalization at a gain of 100 was not possible. Therefore, Brewster’s angle was identified to be $\theta_B = (56.0 \pm 0.5)°$, as the value for $n_2$ obtained corresponding to $\theta_B$ is $n_{\text{Perspex}} = 1.483 \pm 0.009$ which is within one uncertainty range of the accepted value given by [1].

The main source of uncertainty when determining Brewster’s angle were the measurements of the intensity of reflected light. This was due to the fact that when the gain was increased to 100, there were many fluctuations in the reading on the DVM. This may be a result of possible aberrations on the plane and curved faces of the Perspex block, hence introducing random errors in the data. This was accounted for by taking a maximum and minimum voltage reading and calculating the average. Additionally, the uncertainty of the phase-lock amplifier was negligible and would not have had a significant effect.

Moreover, it was found that at incident angles $\geq 80°$ there were consistent distortions in the transmitted and reflected laser beam, as it appeared on the detector as a line rather than a point. This may be due to the fact that the solid state laser will encounter higher dispersion effects at larger incident angles, as the laser has a cross-sectional area rather than being a point charge, hence it will spread out when hitting the plane face of the Perspex block. Consequently, this was accounted for by taking the measurement of $\theta_2$ to be at the centre of the distortion appearing on the detector.

Figure 4 shows the graph plotted of reflection intensity of light against incidence angle for both polarization orientations, with predictions made from the Fresnel relations.

From Figure 4 it can be seen that the experimental data obtained for the reflection coefficients with varying incident angle for both polarization orientations are in quantitative agreement with the theoretical predictions made by the Fresnel equations. The values for $R_\perp$ are much more consistent with the theoretical predictions compared with $R_\parallel$, this becomes more evident when reaching higher incident angles. This may be a result of the laser beam undergoing dispersion at larger incident
angles; hence the laser beam spreads out when hitting the Perspex block and becomes distorted. The propagated errors in the reflection intensity coefficients were small; therefore they are not clearly visible on Figure 4.

IV. CONCLUSIONS

In this experiment the value of \( n_{\text{Perspex}} \) was determined using two methods; through manipulation of Snell’s law (1) and also through measurement of Brewster’s angle. The results obtained were in accordance with what was expected and were similar to colleagues’ results. The value obtained for the refractive index of Perspex from Snell’s law using a weighted mean calculation was \( n_{\text{Perspex}} = 1.486 \pm 0.006 \) and from determining Brewster’s angle \( n_{\text{Perspex}} = 1.483 \pm 0.009 \). Both of these values are in agreement with the accepted value as they are both within one uncertainty range of the value given by [1]. This indicates that the investigation carried out was accurate. Nevertheless, the precision may be improved by using a protractor with smaller degree intervals. Brewster’s angle was identified to be \( \theta_B = (56.0 \pm 0.5)^\circ \), which gave a value for \( n_{\text{Perspex}} \) in agreement with the accepted value [1].

With reference to Figure 4, the data acquired for \( R_\parallel \) and \( R_\perp \) are in accordance with the theoretical predictions made by equations (2) and (3), indicating that the investigation verified the Fresnel relations. However, at incident angles \( \geq 80^\circ \) experimental data for \( R_\perp \) begin to diverge due to distortion of the laser beam. This may be improved by using a laser which is focused more into a point, rather than having a cross-sectional area which is more susceptible to dispersion. Possible further work would be to investigate how the refractive index of materials is dependent on wavelength of light incident on it, by considering the effects of dispersion.

REFERENCES


[6] Nisha Lad, Experiment 2, ‘Lab Book I’, Teaching Laboratory II, Department of Physics and Astronomy, University College London, WC1E 6BT, pp. 38