Study of a Damped Oscillating Torsion Pendulum Driven into Resonance

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Abstract
An experiment was conducted to investigate the effects of resonance on an oscillating torsion pendulum and to determine the Q factor for the system, subject to a damping torque from (4.0±0.5) mm and (25.0±0.5) mm of silicone oil with a viscosity of 100cP. The aims of the experiment were to verify the resonant peak shape of the graph of amplitude of driven oscillations as a function of driving frequency and verify the general shape of the graph of phase angle of driven oscillations against driving frequency. It was found that the Q factor for the lightly damped system was 4.69±0.29, whereas for the heavily damped system, the Q factor was 1.32±0.03.

I. Introduction
There are many different systems, including mechanical and electrical ones, which oscillate with simple harmonic motion (SHM). SHM is periodic motion in which the resultant force on an object is proportional to the displacement from its equilibrium, and hence the acceleration is always directed towards the equilibrium. If a system is forced to oscillate by an external driving force of a frequency which is very similar to the natural frequency, $f_0$, of the object undergoing free oscillations, it will experience a phenomenon known as resonance, in which maximum amplitude oscillations occur.

Damping forces are introduced in order to reduce the effects of resonance, by decreasing the amplitude of oscillation. One of the most important examples of this is the case of suspension bridges being driven into resonance by wind or large crowds of people walking in phase, as they eventually become unstable and collapse.

IA. Theory
In the experiment conducted, a torsion pendulum, suspended from a steel wire and connected to a brass cylinder, was driven into resonance at (4.0±0.5) mm and (25.0±0.5) mm damping. The pendulum was connected to a chuck which was driven by a variable speed motor and which hence made small amplitude oscillations. The motion of the cylinder was damped by 100cP silicone oil, in which it was immersed, which provided the necessary viscous torque. The motion of the torsion pendulum is described by (1).

\[ I \frac{d^2\theta}{dt^2} + \lambda \frac{d\theta}{dt} + s\theta = T_o \cos(\omega t) \quad (1) \]

Here, $I$ represents the moment of inertia of the cylinder, $\lambda$ is the damping constant, $s$ is the torsion constant of the wire, $T_o$ is the maximum driving torque, $\omega$ is the angular frequency of the driving torque and $\theta$ is the angular deflection of the cylinder [1].

The frequency at which the driving mechanism oscillates is the same as for the oscillation of the cylinder, however the two oscillations are out of phase. The oscillation of the cylinder is expressed by (2).

\[ \theta = \theta_o \cos(\omega t + \phi) \quad (2) \]

Here, $\theta_o$ is the amplitude and $\phi$ is the phase angle [1]. A laser beam was fixed to the driving mechanism of the apparatus and reflected from two prism mirrors, which oscillated about mutually perpendicular axes. The two harmonic curves from the driving and driven frequency formed an ellipse, traced out by the laser onto a plotting table, after superposition from the perpendicular components produced from the motion of the prism mirrors. At resonance, the phase angle is close to $\pi/2$, producing an ellipse with vertical oscillations traced out by the laser.

Equations (3) and (4) describe the displacement of each axis with respect to time.

\[ x = A \cos(\omega t) \quad (3) \]
\[ y = B \cos(\omega t + \phi) \quad (4) \]
x and y represent the driven and driving displacement curves respectively that have an angular frequency \( \omega \) and where the maximum amplitudes are \( A \) and \( B \) [1]. From (3) and (4), (5) is derived which describes the relationship between the phase angle \( \phi \) and the dimensions of the ellipse. Refer to Fig. 1.

\[
\frac{ab}{cd} = \sin \phi
\]  (5)

\( ab \) represents the distance between where the ellipse intersects the x and y axis and \( cd \) represents the maximum width of the ellipse.

The sharpness of the resonant peak curve is measured by a factor ‘Q’, which is a dimensionless parameter and describes the rate at which energy is lost from damped oscillating systems. Resonators with high Q factors have low damping forces and hence oscillate for longer. Equation (6) describes how the Q factor is determined.

\[
Q = \frac{\omega_0}{\Delta \omega}
\]  (6)

Here \( \omega_0 \) is the natural frequency of the object and \( \Delta \omega \) is the angular half-power bandwidth, where the amplitude is reduced from its maximum by a factor of \( \sqrt{2} \).

The torsion pendulum apparatus and ellipses were used to verify the shapes of the amplitude and phase angle curves as functions of frequency, and were used to determine the Q factor for the system undergoing resonance at (4.0±0.5) mm and (25.0±0.5) mm of damping.

**II. Experimental method**

Initially, the time period was measured at resonance using a stopwatch, subject to a damping force of (4.0±0.5) mm immersion in silicone oil of viscosity 100cP. This occurred at 0.16 rev/sec motor speed, as at this speed the ellipse traced out by the laser produced vertical oscillations with maximum amplitude. The level of silicone oil was measured at eye level in order to eliminate parallax. Measurements of time period were repeated and averaged; hence the resonant frequency was calculated using (7).

\[
f = \frac{1}{T}
\]  (7)

Here \( f \) is the frequency and \( T \) is the time period of one oscillation.

The speed of the driver motor was then varied and hence the frequency at which the driving mechanism oscillated varied. Frequencies in the range (0.32-0.67) Hz in equal intervals were used and subsequent ellipses were drawn from the trace produced by the laser. Measurements of distances \( ab \) and \( cd \) were taken for both driving and driven displacements from each ellipse; refer to Fig. 1. Hence, the resulting phase angle \( \phi \) was calculated for each frequency using (5). To establish which quadrant \( \phi \) was in, the orientation of the ellipse on the plotting table was considered. The ellipses obtained were inclined to the right where \( |\phi| < \pi/2 \).

From the data collected, graphs of amplitude of driven oscillation and relative phase as functions of driving frequency were plotted. From the amplitude versus driving frequency graph, the angular half-power bandwidth was measured and hence, the Q factor was determined using (6). This procedure was repeated for the system subject to (25.0±0.5) mm of damping for frequencies in the range (0.22-0.76) Hz.

**III. Results and Analysis**

Initially, it was found that when taking measurements of \( ab \) and \( cd \) there were no reference axes. Hence, perpendicular tangents, used as x and y axis were drawn when the driver motor was set to 0.16 rev/sec,
as the oscillation produced on the plotting table by the laser was vertical. It was also found that the elevation of the plotting table and pendulum were not aligned correctly for the full oscillation of the laser to be visible. Therefore, the plotting table was correctly placed and fixed in position to avoid random movements.

Additionally, it was observed that the systematic error that was present from the kink in the pendulum wire was impossible to avoid and hence it was predicted that the results would become skewed from the true values of Q. Moreover, when measuring the time period of oscillation, ten oscillations were measured, and then the time for one oscillation was calculated. This reduced the proportion of random error produced from human reaction time.

The following time periods, and hence the resonant frequencies, were found for the (4.0±0.5) mm (25.0±0.5) mm damping respectively [2]:

\[
\begin{align*}
T_4 &= (1.90±0.30) \text{ s} \\
\omega_0 &= (0.53±0.01) \text{ Hz} \\
T_25 &= (2.04±0.30) \text{ s} \\
\omega_0 &= (0.49±0.01) \text{ Hz}
\end{align*}
\]

It was found that the general shape of the amplitude against driving frequency graph formed a peak (Fig. 2). The amplitude of oscillations were found to increase as the driving frequency reached the natural frequency of the pendulum and then decreased as the frequency increased further. Additionally, it was found that the greater the damping force the smaller the amplitude of oscillation was produced at the resonance frequency and hence the peak amplitude significantly decreased. The rate at which the amplitude of oscillation increased differed for the two damping levels, as the heavier damping level produced a shallower and lower resonant peak compared with the lighter damping level. Therefore, a greater amount of energy was dissipated from the system as the amount of damping increased. Refer to Table 1 for results at the resonant frequency for both damping levels. From the data collected the graph in Figure 2 was plotted.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Damping} & \omega_0 & \theta_o & \phi \\
(±0.5\text{mm}) & (±0.01) & (±0.5) & (\text{radians}) \\
\hline
4.0 & 0.53 & 140.8 & (-1.51±0.04) \\
25.0 & 0.49 & 54.0 & (-1.57±0.11) \\
\hline
\end{array}
\]

Table 1 Results. Amplitude $\theta_0$ and Phase Angle $\phi$ at the resonant frequency for both damping levels [2].

Measurements of $ab$ and $cd$ were used to calculate the phase angle for each corresponding ellipse by using (5). From this data the graph in Figure 3 was plotted. It was found that as the level of damping increased, the rate of change of phase angle decreased, hence there is a steeper curve for the system subject to light damping. Additionally, the ratio of $ab/cd$ at the resonant frequency for both levels of damping were very close to 1, hence both curves were found to have a phase angle that was very similar to $\pi/2$ radians at their resonant frequency.

Fig. 2. Graph of amplitude of driven oscillation as a function of driving frequency [2].

Fig. 3. Phase shift between driven and driving displacement as a function of driving frequency for different levels of damping [2].
The Q factor for each level of damping was determined by measuring the angular half-power bandwidth from the graph in Figure 2 and using (6). Results are shown in Table 2.

<table>
<thead>
<tr>
<th>Damping (±0.5mm)</th>
<th>ω₀ (±0.01Hz)</th>
<th>Δω (±0.007Hz)</th>
<th>Q Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>0.53</td>
<td>0.113</td>
<td>4.69±0.29</td>
</tr>
<tr>
<td>25.0</td>
<td>0.49</td>
<td>0.370</td>
<td>1.32±0.03</td>
</tr>
</tbody>
</table>

Table 2 Results. Determination of the Q factor for both damping levels [2].

It was found that at the lower level of damping the Q factor is greater and hence describes that the system will oscillate for longer, compared to a system with heavy damping. This shows that the rate at which energy is dissipated away from the system increases as the damping force increases.

IV. Conclusion

In conclusion, the results of the experiment verify the effects of damping on an oscillatory system undergoing resonance, in which a resonant peak graph is obtained and when the driving frequency is equal to the natural frequency, maximum amplitude oscillations occur. The results also verify that the larger the damping force, the smaller and shallower the amplitude of the resonant peak becomes. Therefore a greater amount of energy is transferred away from the system and hence the Q factor decreases as the damping force increases. Additionally, the results also confirm the shape of the phase angle versus driving frequency graph and that at both damping levels the phase angle produced at resonance is very close to π/2 radians.

Suitable improvements for accuracy include taking more measurements with smaller intervals above and below the resonant frequency, as it is a turning point on the graph. Also, to ensure a straight wire is used for the torsion pendulum.

It was found that the findings in this experiment are important as they are related to testing the resonant frequencies of bridges and other systems that oscillate with SHM. This is to ensure structures do not become unstable and collapse.

V. References

[1] Dr P. Bartlett, “Experiment NX3 Study of an Oscillating Mechanical System Driven into Resonance” (UCL Physics and Astronomy Department, Lab 1, 03/09/2013)

[2] Nisha Lad, Lab Book 1, Experiment 2, “Study of an Oscillating Mechanical System Driven into Resonance” (UCL Physics and Astronomy Department, Lab 1)