Special relativity: the conjecture, the consequences and the curiosities

Author: James Cockburn
Tutor: Dan Browne

Contents	Page

Intended audience

This report contains mathematics and principles which would suit an audience already familiar with Newtonian mechanics and Galilean transformations.

Aim

The aim of this report is to discuss the importance of the theory of special relativity, and how it has helped solve some physical problems yet created new ones.

Introduction

In 1905, Albert Einstein published his special theory of relativity. The result of this had intriguing results, fascinating scientists and science fiction writers alike. Einstein claimed never to have a genius level of intellect, but rather an out-of-control imagination. Indeed, he famously remarked that 'imagination was more important than knowledge'.

Report

Galilean Transformations

In classical mechanics, it is a taken that its laws are consistent in any inertial reference frame, since there is no 'absolute' or 'preferred' frame. Take two observers, one moving with a truck with velocity **v**, the other standing at rest on the Earth. If the observer in the truck throws a ball straight upwards, it experiences gravitational acceleration in the negative y-axis direction and so falls straight back down again. Therefore, the ball travels along a vertical line.

For the stationary observer, however, since there is an x-component of velocity from the truck, the path the ball follows is parabolic. Although the two observers disagree on certain aspects of the situation, they both agree on the validity of Newton's laws. The resulting equations of taking the situation from one frame to another (for this purpose, say the stationary observer has unprimed coordinates and the moving has primed) have the form;

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

This implies that the x-component of velocity as seen by the stationary observer has magnitude equal to the x-component of velocity measured by the moving observer added to the velocity of his frame (taking derivatives). This agrees well with our everyday experience, but Einstein proved that this is not the case with speeds comparable to the speed of light.

Einstein's Insight

Einstein postulated that the speed of light should be the same in all reference frames. The Michelson-Morley experiment in 1881 showed no sign that the speed of light changed due to interactions with the 'ether', an imaginary medium whereby electromagnetic waves were supposed to propagate through. Quite rightly, the experiment predicted that as the Earth was moving through the ether, the speed of light would shift by a tiny (but measurable) amount to satisfy the Galilean equations. The experiment showed, however, no change in speed at all. To explain this, Einstein suggested the following thought experiment.

Imagine again the observer moving with a truck, except this time he shines a beam of light onto a mirror directly above him. Let the distance between the source of the light and the mirror be *d*. For the moving observer, the time taken for the light to hit the mirror and return to its source is just the distance it travels divided by the speed, that is, 2d/c.

For the stationary observer however, the extra velocity component makes it so the light travels slightly horizontally also, so that its path looks like an isosceles triangle. Because Einstein predicted the speed of light to be the same in all reference frames, the *time taken for the light to travel must change*. By solving the triangle (using t_p for the time taken as measured by the moving observer as 'the proper time');

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is a phenomenon known as *time dilation*. In our everyday lives, we do not experience it, since we travel at speeds much less than the speed of light and so the bottom part of the fraction tends to 1 and the equation reduces to the Galilean transformation equation for time.

Consequences of special relativity

This theory puts in place a 'universal speed limit', that is, if $v > c$, the time difference equation no longer makes any sense and you are left with an answer involving complex numbers, which is of little use in a real, physical quantity.

The theory also brought to an end a problem involving the decay of a muon. Slow-moving muons in laboratories have a lifetime in the proper time interval of around 2.2μs. Muons moving at the speed of light, then, would travel 6.6×10^2 m before decaying, so that one created at the top of our atmosphere would be unlikely to reach the surface of the earth. However, a large number of muons do, in fact, reach the surface. This is because of time dilation, with v being around 0.99c, increasing the distance travelled to 4.8×10^3 m, easily enough to be detected on the ground.

Curiosities of special relativity

Related to time dilation, there is a phenomenon known as *length contraction*. If the length of an object, L_p, as measured at rest relative to the object, then the contracted length can be written as;

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}}$$

So things moving close to the speed of light will appear (to the stationary observer) to be shorter than the 'proper length'. The contraction takes place only along the direction of motion.

The proper length and proper time are measured differently, but can be used independently to understand the same phenomena. The proper length is measured by an observer for whom the end points remain fixed in space and the proper time is measured by an observer for whom two events take place at the same position in space.

If something moves at exactly the speed of light, so that $v = c$, the time equation is such that you divide by zero, and so the time becomes infinite. Similarly, the measured length becomes zero. Both of these mathematical possibilities appear to suggest what may happen near to or in a black hole, but are far from being understood. The next step for physicists may well be to 'tweak' these equations to explain the physical cataclysm that is the black hole.

Summary

The theory of special relativity has helped solve:

- The problem of muon decay in the atmosphere
- The age of the universe
- The relativity of time
- The distances to far away planets/stars

References

1. Physics for Scientists and Engineers (7th Edition) – Jewett/Serway
2. http://www.youtube.com/watch?v=wteiuxyqtoM – a video explaining the simultaneity of events