## MATH6103 Differential & Integral Calculus MATH6500 Elementary Mathematics for Engineers

Problem Sheet 7

Date: 2/12/2011

Due Date: 9/12/2011

Answer all questions marked with an asterisk (\*).

1. Using the trapezium method with 7 equal intervals estimate the integral

$$J = \int_1^8 \frac{1}{x} \, dx$$

Hence, find an approximation to  $\ln(2)$ . Sketch the graph for f(x) = 1/x and illustrate the area that corresponds to your estimate for J. Decide whether your approximation to  $\ln(2)$  is greater or less than the actual value of  $\ln(2)$ .

- 2. Solve the following first order differential equations. State whether each one is separable and also if they are linear.
  - $y' + 7y = \cos(x)$
  - (ii) \*

(i)

- (iii) \*
  - $y' + 2y = 2e^{2x}$
- (iv)  $\cos^4(x)(y')^2 = 1 - y^2$
- (v) \*

$$y' = \frac{\ln(x)}{y}$$

 $y' = \frac{y}{1 - x^2}$ 

- 3. By finding the general solution to the homogeneous part of the equation, and finding a particular solution to the full non-homogeneous equation, solve the following first order differential equations.
  - (i)

$$y' = 2e^x + y$$

(ii) 
$$y' + xy = x$$

(iii) 
$$y' + 3y = 5x^2 - 2$$

4. Solve the following initial value problems (IVP).

(i) \*  
(i) \*  
(ii) 
$$y' + 2xy = -x, \quad y(0) = 1$$
  
(ii)  $y' + y = \cos(x), \quad y(\pi) = 1/2$   
(iii)  $y' + y = e^{2x}, \quad y(\ln(\sqrt{2})) = 1$   
(iv) \*

$$y' = y^3 x^3, \quad y(0) = 1$$