MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 6

Date: 25/11/2011

Due Date: 2/12/2011

Answer all questions marked with an asterisk (*).

1. Evaluate the following definite integrals

   (i) *
   $$\int_0^{2\pi} \cos^2(x) e^x \, dx$$

   (ii)
   $$\int_0^{\sqrt{2\pi}} x \sin(x^2 + 1) \, dx$$

   (iii)
   $$\int_2^3 \frac{x^3 + 4x^2 - 7x - 10}{x^3 + 2x^2 - x - 2} \, dx$$

   (iv) *
   $$\int_0^{1/10} \frac{1}{\sqrt{1 - 3x^2}} \, dx$$

   (v) *
   $$\int_1^2 \ln(x) \, dx$$

2. * A particle's moves in a straight line, whose velocity can be modelled by the following function
   $$v(t) = t \sin(t).$$
   Find the distance s, covered by the particle from time $t = 0$ to $t = 2\pi$. Sketch the graph for $v(t)$ over the domain $[0, 2\pi]$, and display what the value of s represents on your sketch. Hint: recall $\dot{s}(t) = v(t)$, also think about the sign of the function $v(t)$ for $t > \pi$.

3. * Using the trapezium method with 4 equal lengths, find the approximate value of
   $$\int_0^2 \frac{1}{1 + x^2} \, dx.$$
   By means of a sketch, illustrate the areas that correspond to the above integral and trapezium method and show the approximation gives an under-estimate int his case. Hint: Is the curve convex or concave?

4. The work done W by a force F in moving a body through a distance from $x = a$ to $x = b$ metres is given by

$$W = \int_a^b F(x)\,dx \text{ (Newtons)}.$$

Suppose we have the following values for F:

x (metres)	0	1	2	3	4	5	6	7	8
$F(x)$ (Newtons)	2	3	7	4	6	3	1	4	8

Use the trapezium method with 8 equal lengths to give an estimate for the work done in moving the body from $x = 0$ to $x = 8$.

5. Consider the following integral:

$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

Why can't we use the trapezium method to directly approximate this integral? What is the exact value of this integral?