MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 4

Date: 1/11/2011

Due Date: 15/11/2011

Answer all questions marked with an asterisk (*).

1. * Bubonic plague has broken out in 13^{th} century London. On Monday morning the number of people infected is 10. On wednesday morning 360 people are infected. Assuming a simple exponential growth model, find the number of infected people on

   (i) Tuesday morning

   (ii) Thursday morning

   (iii) The previous saturday morning

   Is the model likely to be valid for the previous Saturday? How long (in hours) does it take for the infected population to double?

2. * A radioactive isotope has a half life of 500 years.

   (i) Find the decay rate for the isotope and write down a formula for the proportion of a sample that remains after t years.

   (ii) Using the formula, find what portion of the sample remains after 1500 years and 6000 years respectively.

   (iii) Use the formula to find out how long it takes for 95% of the sample to decay.

3. * You buy a supposedly genuine 2000 year old eggshell of the extinct giant elephant bird Aepyornis, however you are not sure of its authenticity. You use your Liquid Scintillation Counter to work out that the proportion of carbon-14, C^{14}, in a sample of the eggshell is 0.883% of the proportion that would naturally occur at the time it was laid. Using methods from the notes, make a decision whether the eggshell authentic or not. The decay constant of C^{14} is $\lambda = 0.000125$ per year.

4. [Past exam question] Suppose 1Kg of a known radioactive substance is buried for 10 years, and after this time is found to have mass 0.9Kg. If a simple exponential decay model is assumed, determine the mass of the sample at the following times after burial:

(i) 5 years,

(ii) 20 years,

(iii) 30 years.

How long after burial would the sample have a mass exactly half of its initial value?

for the purpose of this question, you are given the values of the logarithm: $\ln 2 = 0.693$, $\ln 3 = 1.099$, $\ln 5 = 1.609$. (The exam is a *non-calculator* paper.)