MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 3

Date: 25/10/2011

Due Date: 1/11/2011

Answer all questions marked with an asterisk (*).

1. * Recall that
$$\frac{d}{dx}(\tan x) = \sec^2 x = 1 + \tan^2 x.$$
By defining suitable $f(x)$ and $g(x)$ such that $f(g(x)) = x$, and using the chain rule, find the derivative of $\tan^{-1} x$ (this is the same as $\arctan x$) with respect to x.

2. Use the formula
$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}},$$
to find the derivative of $y(x) = \cos^{-1} x$. Hint: make sure you take the correct sign of any square root by considering carefully the domain of the inverse function.

3. * Differentiate the following functions:
   (i) $\cos(1/x^2)$
   (ii) $1/(\sqrt{x^3 - 4x + 1})$
   (iii) $\cos(x^2)/\tan^3 x$

4. * Find all points x where f achieves a local maximum or minimum for the following functions (i.e. all the turning points). State whether f has local maximum or local minimum at each point.
   (i) $f(x) = \frac{x^4}{4} + 4\frac{x^3}{3} - 7\frac{x^2}{2} - 10x$
   (ii) $f(x) = \frac{x^2+1}{x^2-1}$

5. A particle moves on a two-dimensional path in such a way that its position as a function of time t given by $(x, y) = (t\cos 2t, t\sin 2t)$. Determine the slope of the path at $t = \pi/4$. Hint:
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}.$$