MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 2

Date: 18/10/2011

Due Date: 25/10/2011

Answer all questions marked with an asterisk (*).

1. Evaluate the following limits, if they exist:

   (i)
   $$\lim_{x \to 2} \left[\frac{2x}{x+3} \right]$$

   (ii)
   $$\lim_{x \to 5} \left[\frac{x+5}{\sqrt{x^2 - 25}} \right]$$

2. * Factorize $f(x) = x^3 + 4x^2 - 7x - 10$. Hence, find the limit
   $$\lim_{x \to 2} \left[\frac{x^3 + 4x^2 - 7x - 10}{x - 2} \right].$$

3. * By drawing the graph of $\cos(x)$, find all the points x where
   $$\frac{d}{dx}(\sin(x)) = 0.$$
   Hint: try find points on the domain $[-4\pi, 4\pi]$, then try find a pattern to cover the domain over the entire real axis.

4. Differentiate the following functions with respect to x

   (i) $f(x) = 3x^6$
   (ii) * $f(x) = \sin x \cos x$
   (iii) $f(x) = (x+1)(x-1)(x^2+1)$
   (iv) $f(x) = \sin^2 x + \cos^2 x$
   (v) * $f(x) = \cos((1 - \sqrt{x})^{2/5})$
   (vi) $f(x) = -(\sqrt{1 + \sqrt{x}})/2x$

5. * Consider the function $f(x) = x + \frac{1}{x}$, for $x > 0$. Find the tangent to f at $x = 2$ in the form $y = ax + b$, where $a, b \in \mathbb{R}$.

6. * Show that the derivative of a differentiable even function is an odd function and the derivative of a differentiable odd function is even. Hint: recall the definition of an odd and even function, apply the chain rule.