MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 1

Date: 11/10/2011

Due Date: 18/10/2011

Answer all questions marked with an asterisk (*).

1. Which of the following $f(x)$, such that $f : D \to \mathbb{R}$ define a function on the given domain D. Give reasoning and constraints on the range.

   (i) *
   $$f(x) = \frac{1}{x^2 - 9}, \quad D = \mathbb{R}$$

   (ii)
   $$f(x) = x + \frac{1}{x}, \quad D = \{x \in \mathbb{R} : x \geq 1\}$$

   (iii)
   $$f(x) = \sqrt{x^3 + 2x}, \quad D = \mathbb{R}$$

   (iv) *
   $$f(x) = \frac{x}{(x^2 + 5)^{1/2}}, \quad D = \mathbb{R}$$

   Furthermore, give domains D for which (i)-(iv) are well defined functions.

2. * Which of the functions in Q1 are odd, which are even and which are neither?

3. Factorise the following expressions for $f(x)$ as far as possible.

   (i)
   $$f(x) = x^3 - 2x^2 - 5x + 6$$

   (ii) *
   $$f(x) = x^4 - 1$$

   (iii) *
   $$f(x) = (x^3 - 3x + 2)(x^3 + 5x^2 + 8x + 4)^{-1}$$

   (iv)
   $$f(x) = x^3 - 7x - 6$$

   Find all *real* solutions to $f(x) = 0$ for (i)-(iv).

4. * Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \frac{1}{1 + 2x}.$$

Find the first derivative $f'(x)$ from fist principles, i.e. from the definition

$$f'(x) = \lim_{h \to 0} \left\{ \frac{f(x + h) - f(x)}{h} \right\}.$$

5. * Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = x|x| = \begin{cases} x^2 & \text{if} \quad x \geq 0, \\ -x^2 & \text{if} \quad x < 0. \end{cases}$$

Sketch the graph of f and find the derivative $f'(x)$ when

(i) $x > 0$,

(ii) $x < 0$.

Is f differentiable at $x = 0$ and if so what is $f'(0)$? Also, does $f''(0)$ exist?