

**UNIVERSITY COLLEGE LONDON**

**EXAMINATION FOR INTERNAL STUDENTS**

**MODULE CODE : MATH6103**

**ASSESSMENT : MATH6103A**  
**PATTERN**

**MODULE NAME : Differential and Integral Calculus**

**DATE : 14-May-12**

**TIME : 14:30**

**TIME ALLOWED : 2 Hours 0 Minutes**

All questions may be attempted but only marks obtained on the best **five** solutions will count.

The use of an electronic calculator is **not** permitted in this examination.

1. Differentiate the following functions with respect to  $x$ , using any rule of your choice:

(i)

$$f(x) = \cos(e^x),$$

(ii)

$$f(x) = (2x + 3x^2)^{1/2},$$

(iii)

$$f(x) = -\frac{\sqrt{1 + \sqrt{x}}}{2x},$$

(iv)

$$f(x) = \ln(\tan x),$$

(v)

$$f(x) = x^2 e^{\sin x}.$$

2. (i) Given a function  $f(x)$ , state a condition on the derivative  $f'(x)$ , when  $f(x)$  has a *stationary* point (*turning* point) at  $x = c$ .

(ii) Factorise the following polynomial:

$$f(x) = x^3 - 3x^2 + 4.$$

(iii) Find all *stationary* points (*turning* points) of  $f(x)$ , stating whether they are maximum, minimum or points of inflection.

(iv) Sketch the curve  $y = f(x)$ .

3. You are required to design an open top box using the least material possible, subject to the following criteria:

- the box must have a square base,
- the box must have volume of 4 cubic metres.

- (i) Write down expressions for the total surface area  $S$  and volume  $V$  of the box if the square base has side  $a$  metres and the box has height  $b$  metres.
- (ii) Using the criteria, write  $S$  in terms of  $a$  **or**  $b$  alone.
- (iii) Find values for  $a$  and  $b$ , such that the least amount of material is used.

4. (a) Calculate the following indefinite integrals:

(i)

$$\int x \sin(x^2) dx,$$

(ii)

$$\int \tan^2 x dx,$$

(iii)

$$\int x^2 e^x dx.$$

(b) Calculate the following definite integrals:

(i)

$$\int_2^3 \ln(x) dx,$$

(ii)

$$\int_3^4 \frac{2x}{x^2 + 3x + 2} dx.$$

5. Using the *trapezium method* with 4 *equal intervals*, estimate the following integral:

$$\int_1^5 \frac{1}{x} dx.$$

What is the exact value of the above integral? By means of a sketch, explain whether your approximation is an over estimate or underestimate.

6. Solve the following initial value problems:

(i)

$$y' + 2y^2 = 0, \quad y(1) = 1,$$

(ii)

$$y' + y = e^{2x}, \quad y(\ln 2) = 1.$$

7. (i) Find the general solution to the following second order differential equation:

$$y'' - 4y = \sin x.$$

(ii) Solve the following initial value problem:

$$y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$