

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : MATH6103

ASSESSMENT : MATH6103A
PATTERN

MODULE NAME : Differential and Integral Calculus

DATE : 19-May-11

TIME : 10:00

TIME ALLOWED : 2 Hours 0 Minutes

All questions may be attempted but only marks obtained on the best five solutions will count.

The use of an electronic calculator is not permitted in this examination.

1. (a) State the definition of the derivative of a function $f(x)$.
(b) (i) State the chain rule for differentiating a composition of functions $f(g(x))$.
(ii) Let f and g be the functions defined by $f(x) = e^x$, $g(x) = \ln(x)$.
Using the fact that g is the inverse function to f and that $f'(x) = e^x$, but **without** using knowledge of the derivative of the logarithm, find $g'(x)$.
2. Differentiate the following functions with respect to x
 - (a) $(x^3 - 4)^{1/4}$
 - (b) $\ln(\tan(x))$
 - (c) $xe^{\sqrt{x}}$
 - (d) $\cos(\cos(x))$
 - (e) $\frac{\sin(x)}{x^2+1}$
3. A five-sided box (with an open top) is to be constructed out of sheet metal to satisfy the following conditions:
 - the bottom side (the one opposite the open side) must be square;
 - the total amount of sheet metal used (surface area) is to be 3 metres².
 - (a) Write down formulae for the surface area and volume of the box in terms of the dimensions of the box.
 - (b) Find the dimensions of the box that will maximize the volume of the box, subject to the conditions given.

4. Calculate the following integrals

(a)

$$\int_0^{\pi} \sin(x)e^x dx$$

(b)

$$\int_0^1 \frac{1}{1+3x^2} dx$$

(c)

$$\int_3^6 \frac{7x+1}{x^2-x-2} dx$$

(d)

$$\int_0^1 3x \sin(3x^2 - 3) dx$$

5. Using the trapezium method with three intervals of equal length, find the approximate value of the following integral:

$$\int_1^4 \frac{1}{x} dx$$

Hence find an approximation to $\ln(2)$. By means of a sketch decide whether your approximation overestimates or underestimates the actual value of $\ln(2)$.

6. A certain radioactive isotope is known to have a half-life of 700 years. Assuming a simple exponential growth model

- (a) Write down a formula for the proportion of a sample of the isotope remaining after t years. Find the decay constant.
- (b) Find the time taken until 90% of a sample decays. Find the time taken for 99% of a sample to decay.

You may use the following approximations: $\ln(2) \approx 0.7$, $\ln(0.1) \approx -2.3$.

7. (a) Solve the following initial value problem

$$y' + y/x = x^3, \quad y(1) = 0$$

- (b) Find the general solution to the following differential equation

$$y'' - 4y' + 3y = 3x^2 + x - 1$$