UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **MATH6103**

ASSESSMENT PATTERN : **MATH6103A**

MODULE NAME : **Differential And Integral Calculus**

DATE : **30-Apr-10**

TIME : **14:30**

TIME ALLOWED : **2 Hours 0 Minutes**

2009/10-MATH6103A-001-EXAM-20

TURN OVER

All questions may be attempted but only marks obtained on the best five *solutions will count.*
The use of an electronic calculator is not *permitted in this examination.*

1. Differentiate the following functions with respect to x

   (a)
   $$\ln\left(\frac{\sin x}{1-x^2}\right)$$

   (b)
   $$x^2 e^{\sin x}$$

   (c)
   $$(1-x+x^3)^{17/3}$$

   (d)
   $$\tan(\cos x - 1/x)$$

   (e)
   $$\sqrt{\ln\sqrt{x}}$$

2. (a) State the definition of the derivative of a function $f(x)$.

   (b) Give a condition for a differentiable function $f(x)$ to have a turning point at $x = x_0$.

   (c) Find all turning points of the real valued function given by

   $$f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 3,$$

   and decide whether each one is a local minimum, a local maximum or a point of inflection.

3. Calculate the following integrals

   (a)
   $$\int_2^3 \frac{x+5}{x^2+x-2}dx$$

   (b)
   $$\int_{\pi/6}^{2\pi/3} \sin(3x)\cos^5(3x)dx$$

   (c)
   $$\int_0^{\sqrt{\pi/2}} x\sin(x^2)dx$$

   (d)
   $$\int_0^1 x^2e^x dx$$

4. Using the trapezium method with 4 intervals of equal length, find the approximate value of the following integral:
   $$\int_{-2}^{2} \frac{1}{1+x^2}dx$$

   By means of a sketch, illustrate the areas that correspond to the above integral and the trapezium method.

   Hence or otherwise find an approximation to $\tan^{-1}(2)(=\arctan(2))$.

5. A population of *E.coli* (a bacterium) in a petri dish has $512 = 8^3$ members on day 0, and $4096 = 8^4$ members on day 2, where the measurements are taken at the same time each day. Assuming a simple exponential growth model, determine the population size at:

   (a) day 1,

   (b) day 6,

   (c) day 60.

   The population was started by a single bacterium landing in the dish (bacteria reproduce asexually). On what day did this occur? How long (in hours) does it take for the population to double? Is the model likely to be valid at day 60?

6. Solve the following initial value problems.

(a)

$$y' + y/x = e^{x^2} \qquad\qquad y(1) = e,$$

(b)

$$y' + y = 3e^{-x} \qquad\qquad y(0) = 1.$$

7. Find the general solution to the following differential equation:

$$y'' - 4y' + 4y = \cos(3x).$$

END OF PAPER