UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**MATH6103**
ASSESSMENT PATTERN	:	**MATH6103A**
MODULE NAME	:	**Differential And Integral Calculus**
DATE	:	**13-May-09**
TIME	:	**14:30**
TIME ALLOWED	:	**2 Hours 0 Minutes**

2008/09-MATH6103A-001-EXAM-29

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Define the derivative of a function $f(x)$.

   (b) Calculate the derivatives of the following functions from first principles. (That is, find the derivatives directly from the definition.)

   (i) x^3

   (ii) $\frac{1}{x}$

   (c) State the product rule, the chain rule, and the quotient rule.
   Use the chain rule and one of the results above to find the derivative of $\sqrt[3]{x}$.

2. Calculate the following indefinite integrals.

   (a) $\int xe^{(x^2)}dx$

   (b) $\int \tan x \sec^7 x \; dx$

   (c) $\int \frac{3x}{x^2-4x+4}dx$

   (d) $\int \frac{1}{\sqrt{1-x^2}}dx$

   (e) $\int \frac{2x}{\sqrt{1-x^4}}dx$

3. Differentiate the following, with respect to x.

   (a) $3^{(x^2)} - (x^2)^3$

   (b) $\tan(\cos x)$

   (c) $\ln(\sec x + \tan x)$

   (d) $\ln(3x) \times \ln(10x)$

4. Approximate the definite integral $\int_1^3 \frac{1}{x}dx$ by using the trapezium rule with four strips.

   Sketch the graph of the $y = \frac{1}{x}$ in a suitable range, and illustrate the area covered by your four strips.

   Is your answer larger or smaller than $\ln(3)$?

5. Two hours after the start of an experiment, the bacterium population in a sample is recorded as 1200, and it is found to be 3600 after a further two hours. Assuming exponential growth, find

   (a) the number of bacteria when the experiment started,

   (b) the growth constant of the population, and

   (c) the time when the population is 20,000.

   You may find the following approximations useful:

   $\ln(3) \approx 1.099$, $\ln(20) \approx 2.996$, $\ln(50) \approx 3.912$

6. An open water tank of volume 18π cubic metres is to be constructed by joining an open-ended cylinder of radius r and height h to the lower half of a hemisphere of radius r.

   - Find formulae for the volume and surface area of the tank, in terms of r and h.
   - Find the values of r and h that minimise the surface area.

   A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$. An open-ended cylinder of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r h$.

7. For the following differential equations, for y as a function of x,

   (a) state whether each is separable and/or linear.

   (b) find the General Solution to each differential equation.

   - $\sin(x)y' + \cos(x)y = 0$
   - $y'' + 4y' + 4y = 0$
   - $y'' - 6y' + 8y = 3e^x$