

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : MATH6103

ASSESSMENT : MATH6103A
PATTERN

MODULE NAME : Differential And Integral Calculus

DATE : 09-May-08

TIME : 10:00

TIME ALLOWED : 2 Hours 0 Minutes

2007/08-MATH6103A-001-EXAM-22

©2007 *University College London*

TURN OVER

All questions may be attempted but only marks obtained on the best five solutions will count.

The use of an electronic calculator is **not** permitted in this examination.

1. (a) State the definition of the derivative of a function $f(x)$.
(b) Calculate the derivatives of the following functions from first principles. (That is, find the derivatives directly from the definition.)
 - (i) $\frac{1}{x}$
 - (ii) x^6
(c) State the product rule, the chain rule, and the quotient rule.
Prove the quotient rule, using the product rule, the chain rule, and part (b).

2. Differentiate the following, with respect to x .

- (a) 2^x
- (b) $\sin(\sin x)$
- (c) $(7 + 4x^9)^{1/3}$
- (d) $\frac{9x+2}{4x^2+8x+7}$
- (e) $\ln(x^{10})$

3. Calculate the following integrals.

- (a) $\int \cos^2 x \, dx$
- (b) $\int \frac{10x+7}{5x^2+7x+2} dx$
- (c) $\int \ln x \, dx$
- (d) $\int \tan^5 x \sec^2 x \, dx$
- (e) $\int \frac{3x}{x^2-3x+2} dx$
- (f) $\int \frac{4}{\sqrt{4-x^2}} dx$

4. Suppose that a smooth function, F , of a variable, x , takes the following values:

x	0	1	2	3	4	5	6	7	8	9	10
F	4	6	6	7	6	3	1	1	2	4	5

- (a) Use the trapezium method with 5 equal intervals to find an approximation to $\int_0^{10} F(x)dx$.
- (b) Use the trapezium method with 10 equal intervals to find an approximation to $\int_0^{10} F(x)dx$.
- (c) Is it necessarily true that (b) provides a better approximation to the given integral than (a)?

5. A certain sample of a radioactive isotope is known to decay exponentially such that after one thousand years, 80 per cent of the original sample will remain.

- (a) Find a formula for the fraction of the original sample that will remain after t years. Find the constant of decay.
- (b) Find the half-life of the isotope.

You may find the following approximations useful:

$$\ln(0.2) \approx -1.6, \ln(0.5) \approx -0.7, \ln(0.8) \approx -0.2$$

6. A five-sided box (with an open top) is to be constructed to satisfy the following conditions:

- The bottom side (the side opposite the missing one) must be three times as long as it is wide.
 - The volume of the box must be 18 cubic metres.
- (a) Find a formula for the surface area of the box in terms of the length, width, and height.
- (b) Find the length, width, and height which will minimise the surface area, subject to the conditions given.

7. For the following differential equations, for y as a function of x ,
- (a) State whether each is separable and/or linear.
 - (b) Find all solutions to each differential equation, subject to any conditions given.
 - $y' + ye^x = 0$, with the condition $y(0) = e$
 - $y'' + 8y' + 12y = 0$, with the condition $y'(0) = 0$
 - $y'' + 4y' + 4y = 0$