

MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Department of Mathematics,
University College London

Ali Khalid
web: www.homepages.ucl.ac.uk/akhalid
e-mail: a.khalid@math.ucl.ac.uk

Autumn 2011

Contents

1	Functions	1
1.1	What is a function?	1
1.2	The definition of a function	1
1.3	Representing a function	3
1.3.1	Exponents	4
1.3.2	Polynomials	7
1.3.3	Trigonometric functions	13
2	Differentiation	18
2.1	Rates of change	18
2.1.1	What is the slope of a curve at a point?	19
2.1.2	Rules for differentiation	23
2.2	Differentiation of trigonometric functions	33
2.2.1	The derivatives of inverse trig functions	37
2.3	Maxima, minima and second derivatives	39
2.3.1	Second derivative	42
2.4	Real life examples	47
3	Exponentials and Logarithms	51
3.1	Exponentials	51
3.1.1	Slope of exponentials	52
3.2	The natural logarithm	53

3.2.1	Characteristic properties	54
3.2.2	Logarithms base a	55
3.3	Exponential growth and decay	56
3.3.1	Radioactive decay	56
3.3.2	Carbon dating	57
3.3.3	Population growth	59
3.3.4	Interest rate	59
4	Integration	61
4.1	The basic idea	61
4.2	Fundamental Theorem of Calculus	65
4.3	Indefinite integrals	68
4.3.1	Substitution	71
4.3.2	Trigonometric substitution	75
4.3.3	Partial fractions	77
4.3.4	Integration by parts	78
4.4	Definite integrals	80
4.5	Numerical integration	82
4.5.1	Trapezium method	82
4.6	Application of the definite integral	86
4.6.1	Area bounded by curves	86
4.6.2	Finding a distance by the integral of velocity	89
5	Differential Equations	90
5.1	First order differential equations	91
5.1.1	Separation of variables	91
5.1.2	Linear first-order differential equations	94
5.1.3	Second-order linear differential equations with constant coefficients .	101
5.2	Simple Harmonic Motion (SHM)	110

5.3 Solving initial-value problems numerically:	
Euler's method	114