

MATH 8103/8500 Exam style worked Example
Exponential Growth & Decay.

Q5 Exam 2008.

a) Formula for fraction of original
sample that remains after t
years.

Exp Growth / decay Eq:

$$M = M_0 e^{\lambda t}$$

$$\frac{M}{M_0} = e^{\lambda t}$$

when $t = 1000$, $\frac{M}{M_0} = 0.8$ (Given)

So $0.8 = e^{1000\lambda}$

$$\ln(0.8) = \ln(e^{1000\lambda})$$

$$\therefore \ln(0.8) = 1000\lambda \ln(e)$$

$$\ln(0.8) = 1000\lambda$$

$$\therefore \frac{\ln(0.8)}{1000} = \lambda$$

So after t years

$$\frac{M}{M_0} = e^{\frac{\ln(0.8)}{1000} t}$$

using $\ln(0.8) \approx -0.2$

$$\frac{M}{M_0} = e^{-0.0002 t}$$

b) Half life of the isotope is time when only half original sample remains. i.e. $M = \frac{1}{2} M_0$.

$$\text{So } \frac{\frac{1}{2} M_0}{M_0} = e^{-0.0002 T_{1/2}}$$

$$\frac{1}{2} = e^{-0.0002 T_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-0.0002 T_{1/2}})$$

$$\cancel{\ln(1)} - \ln(2) = -0.0002 T_{1/2} \ln(e)$$

$$-\ln(2) = -0.0002 T_{1/2}$$

$$\therefore T_{1/2} = \frac{\ln(2)}{0.0002}$$

$$\text{or } T_{1/2} = \frac{\ln(2)}{\frac{2}{10000}}$$

$$T_{1/2} = \frac{10000 \ln(2)}{2}$$

$$T_{1/2} = 5000 \ln(2)$$
