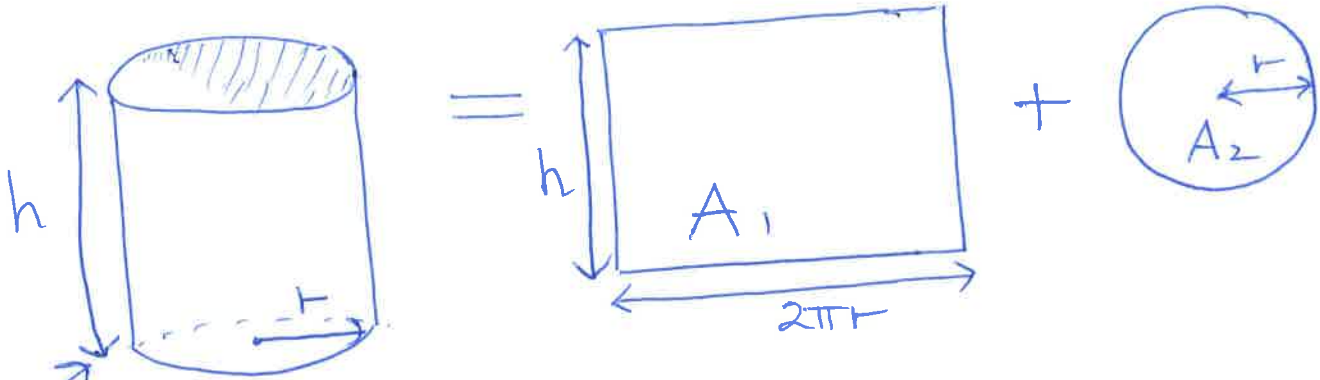
MATH6103 / MATH6500. Exam Style Worked Example.

Differentiation - Real Life Example Application.

Q3 Exam 2007.



Nice drawing!

Volume of cylinder $V = \pi r^2 h$

$V =$ (area of circle) $\times$ (height of cylinder).

Total area $= \quad A = A_1 + A_2$

$A_1 = 2\pi r h \quad , \quad A_2 = \pi r^2$

$\therefore \; A = \pi r^2 + 2\pi r h.$

constraint is $\quad A = 1$,

i.e. $\quad \pi r^2 + 2\pi r h = 1$

Let us write h as function of r

$$\therefore \quad 2\pi r h = 1 - \pi r^2$$

$$h = \frac{1}{2\pi r} - \frac{r}{2}$$

So we can write $\quad V = \pi r^2 \left(\frac{1}{2\pi r} - \frac{r}{2} \right)$

Thus $\quad V = \frac{r}{2} - \frac{r^3 \pi}{2}$

Maximise $V(r)$ ie Solve $\frac{dV}{dr} = 0$ for r.

$$V = \frac{r}{2} - \frac{r^3\pi}{2}$$

$$\frac{dV}{dr} = \frac{1}{2} - \frac{3r^2\pi}{2}.$$

$$\frac{dv}{dr} = 0 \Rightarrow \frac{1}{2} - \frac{3r^2\pi}{2} = 0.$$

$$\frac{1}{2} = \frac{3r^2\pi}{2}$$

$$\frac{\cancel{2}}{\cancel{2} \times 3\pi} = r^2$$

$$\therefore \pm\frac{1}{\sqrt{3\pi}} = r.$$

Maximum when $\frac{d^2V}{dr^2} < 0$.

So $\frac{d^2V}{dr^2} = -3r\pi$.

$\frac{d^2V}{dr^2} < 0$ if $r = \frac{+1}{\sqrt{3\pi}}$ (obvious length +ve)

at max $r = 1/\sqrt{3\pi}$ and $h = \frac{1}{2\pi r} - \frac{r}{2}$

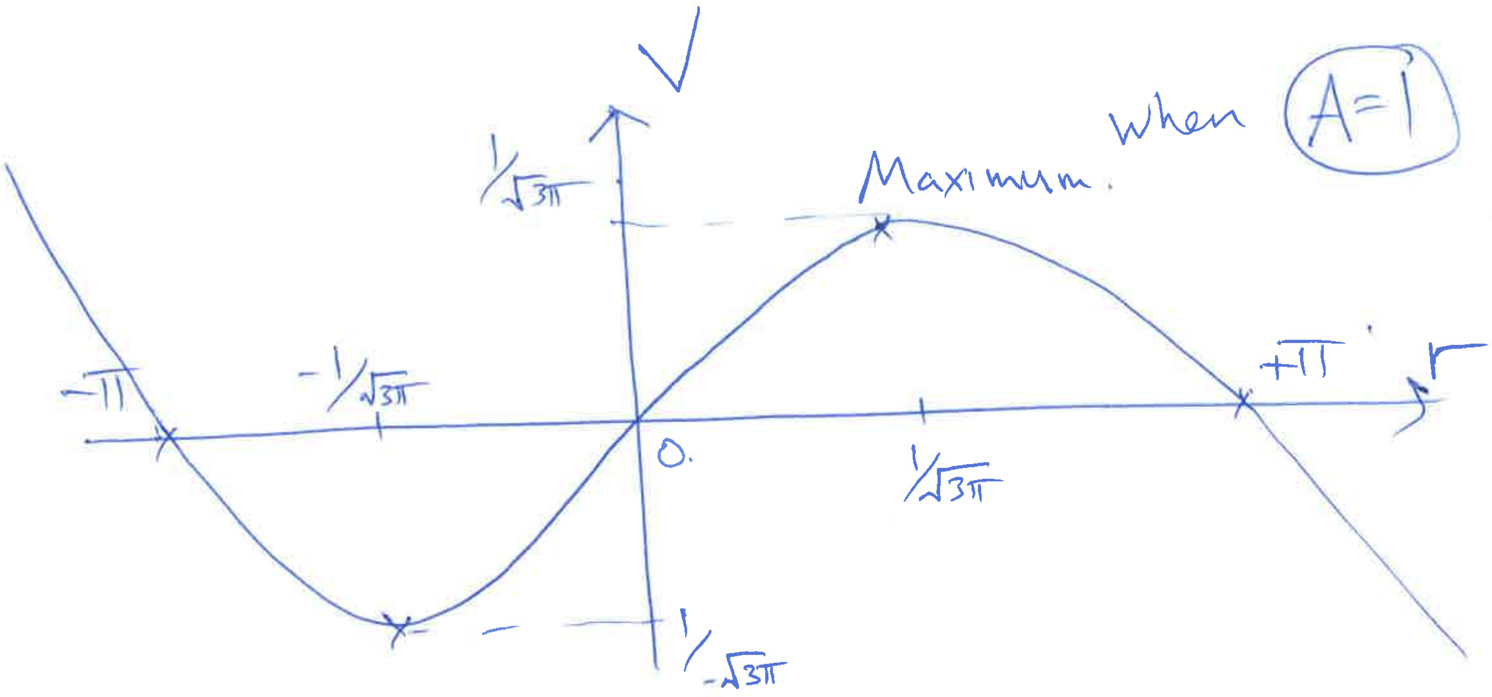
$$h = \frac{1}{2\pi\frac{1}{\sqrt{3\pi}}} - \frac{1}{2\sqrt{3\pi}}$$

$$h = \frac{\sqrt{3\pi}}{2\pi} - \frac{1}{2\sqrt{3\pi}}$$

$$h = 1/\sqrt{3\pi}$$

$$\boxed{\text{So ratio } \frac{r}{h} = \frac{1}{\sqrt{3\pi}} \Big/ \left(\frac{1}{\sqrt{3\pi}}\right) = 1}$$

Sketch $V = \frac{r}{2} - r^3 \pi/2$.



Strictly should be $r \geq 0$.