Ex: Integration using Partial Fractions (Definite).

Q:

Calculate the integral

$$I = \int_4^5 \frac{2x^2+2x+2}{x^3+9x^2+26x+24}\,dx.$$

A:

We notice that we can't manipulate top to be differentiation of bottom. Since top is of degree $\leq$ to bottom – use Partial factions:

We can factorise $x^3+9x^2+26x+24 = f(x)$ using methods learn't previously.

$f(-2)=0, f(-3)=0, f(-4)=0$ so

$$f(x) = (x+2)(x+3)(x+4).$$

Now we write (since all factors different)

$$\frac{2x^2+2x+2}{(x+2)(x+3)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+3)} + \frac{C}{(x+4)}$$

$$= \frac{A(x+3)(x+4) + B(x+2)(x+4) + C(x+2)(x+3)}{(x+2)(x+3)(x+4)}$$

(common denominator)

Hence

$$2x^2+2x+2 = A(x+3)(x+4) + B(x+2)(x+4) + C(x+2)(x+3)$$

…t $x = -2$ then

$$4 - 4 + 2 = 2A$$

$$\Rightarrow \; A = 1$$

put $x = -3$ then

$$18 - 6 + 2 = B \cdot (-1) \cdot (1)$$

$$\Rightarrow \; B = -14$$

put $x = -4$ then

$$32 - 8 + 2 = -2C$$

$$\therefore -2C = 22$$

$$\Rightarrow \; C = -11$$

So $$I = \int_4^5 \frac{1}{(x+2)} + \frac{-14}{(x+3)} + \frac{-11}{(x+4)} \, dx$$

$$= \int_4^5 \frac{1}{(x+2)} dx - 14 \int_4^5 \frac{1}{(x+3)} dx - 11 \int_4^5 \frac{1}{(x+4)} dx$$

Now top is differentiation of bottom in all integrands above therefore we have

$$I = \Big[\ln|x+2| - 14 \ln|x+3| - 11 \ln|x+4| \Big]_4^5$$

$$= \big(\ln(7) - 14\ln(8) - 11\ln(9)\big) - \big(\ln(6) - 14\ln(7) - 11\ln(8)\big)$$

$$= -\ln(6) + 15\ln(7) - 3\ln(8) - 11\ln(9)$$

We can manipulate answer further using log rules/laws.