

Limits Example:

By computing Left and Right limits
Find the value of a such that $f(x)$ is
continuous at $x=2$. Sketch $f(x)$ and $f(-x)$

Ans:

$$f(x) = \begin{cases} 3x + 2a & x \geq 2 \\ -x + 5 & x < 2 \end{cases}$$

Right hand limit = $\lim_{x \rightarrow 2^+} f(x) = 3(2) + 2a = 6 + 2a$

Left hand limit = $\lim_{x \rightarrow 2^-} f(x) = -2 + 5 = 3$

If $f(x)$ cont. at $x=2$ then $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x)$

$$\text{i.e. } 6 + 2a = 3 \Rightarrow 2a = -3 \Rightarrow a = -\frac{3}{2}$$

$$\Rightarrow f(x) = \begin{cases} 3x - 3 & x \geq 2 \\ -x + 5 & x < 2 \end{cases}, \quad f(-x) = \begin{cases} -3x - 3 & x \geq 2 \\ x + 5 & x < 2 \end{cases}$$

