Limits Example:

Q.

Calculate, $L = \lim\limits_{x \to -2} \left\{ \dfrac{x^3+5x^2+8x+4}{x^2+4x+4} \right\}$ if it exists.

Ans: First let us factorise. put

$P(x) = x^3+5x^2+8x+4$. and note $P(-1) = 0$

then $(x+1)$ is a factor so $p(x) = (x+1)(ax^2+bx+c)$

expanding $p(x) = ax^3 + (a+b)x^2 + (b+c)x + c$.

comparing coefficients:

$$\left.\begin{aligned} a &= 1 \\ a+b &= 5 \\ b+c &= 8 \\ c &= 4 \end{aligned}\right\} \Rightarrow \begin{aligned} a &= 1 \\ c &= 4. \\ b &= 4. \end{aligned}$$

so $L = \lim\limits_{x \to -2} \left\{ \dfrac{(x+1)\cancel{(x^2+4x+4)}}{\cancel{(x^2+4x+4)}} \right\}$

$= \lim\limits_{x \to -2} \{ x+1 \}$

$= -2+1 = -1$. (it does exists!)

Q. Calculate $L_2 = \lim\limits_{x \to -2} \left\{ \dfrac{5x+10}{x^2+4x+4} \right\}$ if it exists.

Ans: factorise $q(x) = x^2+4x+4 = (x+2)(x+2)$.

$\therefore L_2 = \lim\limits_{x \to -2} \left\{ \dfrac{5x+10}{(x+2)(x+2)} \right\} = \lim\limits_{x \to -2} \left\{ \dfrac{5\cancel{(x+2)}}{(x+2)\cancel{(x+2)}} \right\} = \lim\limits_{x \to -2} \left\{ \dfrac{5}{x+2} \right\}$

If we put $x=-2$ into expression we get $L_2 = \dfrac{5}{-2+2} = \dfrac{5}{0}$

we say $\dfrac{5}{0}$ is not well defined $\Rightarrow L_2$ does not exist!

or $5/0 = \infty$! $\equiv$ BAD!