

Integration by parts - Definite integral

Q. Calculate: $I = \int_1^2 x^2 \ln|x| dx$

let us choose $u = \ln|x| \Rightarrow u' = \frac{1}{x}$
 $v' = x^2 \Rightarrow v = \frac{x^3}{3}$

applying: $\int uv' dx = uv - \int v u' dx$

we have: $I = \left[\frac{x^3}{3} \ln|x| \right]_1^2 - \int_1^2 \frac{x^3}{3} \cdot \frac{1}{x} dx$

$$\therefore I = \left[\frac{x^3}{3} \ln|x| \right]_1^2 - \int_1^2 \frac{x^2}{3} dx$$

$$I = \left[\frac{x^3}{3} \ln|x| - \frac{x^3}{9} \right]_1^2$$

$$I = \left(\frac{2^3}{3} \ln(2) - \frac{2^3}{9} \right) - \left(\frac{1^3}{3} \ln(1) - \frac{1^3}{9} \right)$$

recall $\ln(1) = 0$, so

$$I = \frac{8}{3} \ln(2) - \frac{7}{9}$$
