Numerical Integration - Trapezium Rule example.

Q: Estimate the integral

$$J = \int_3^7 -(x-5)^2 + 5 \; dx$$

with 4 equal intewal. Is your estimate an over estimate or under estimate. what is the error?

A: $n = 4$.

$$\therefore h = \frac{b-a}{n} = \frac{7-3}{4} = \frac{4}{4} = 1.$$

$$x_i = a + ih = 3 + i.$$

$$f(x_i) = -(x_i - 5)^2 + 5$$

table:

i	x_i	$f(x_i)$
0	3	1
1	4	4
2	5	5
3	6	4.
4	7.	1.

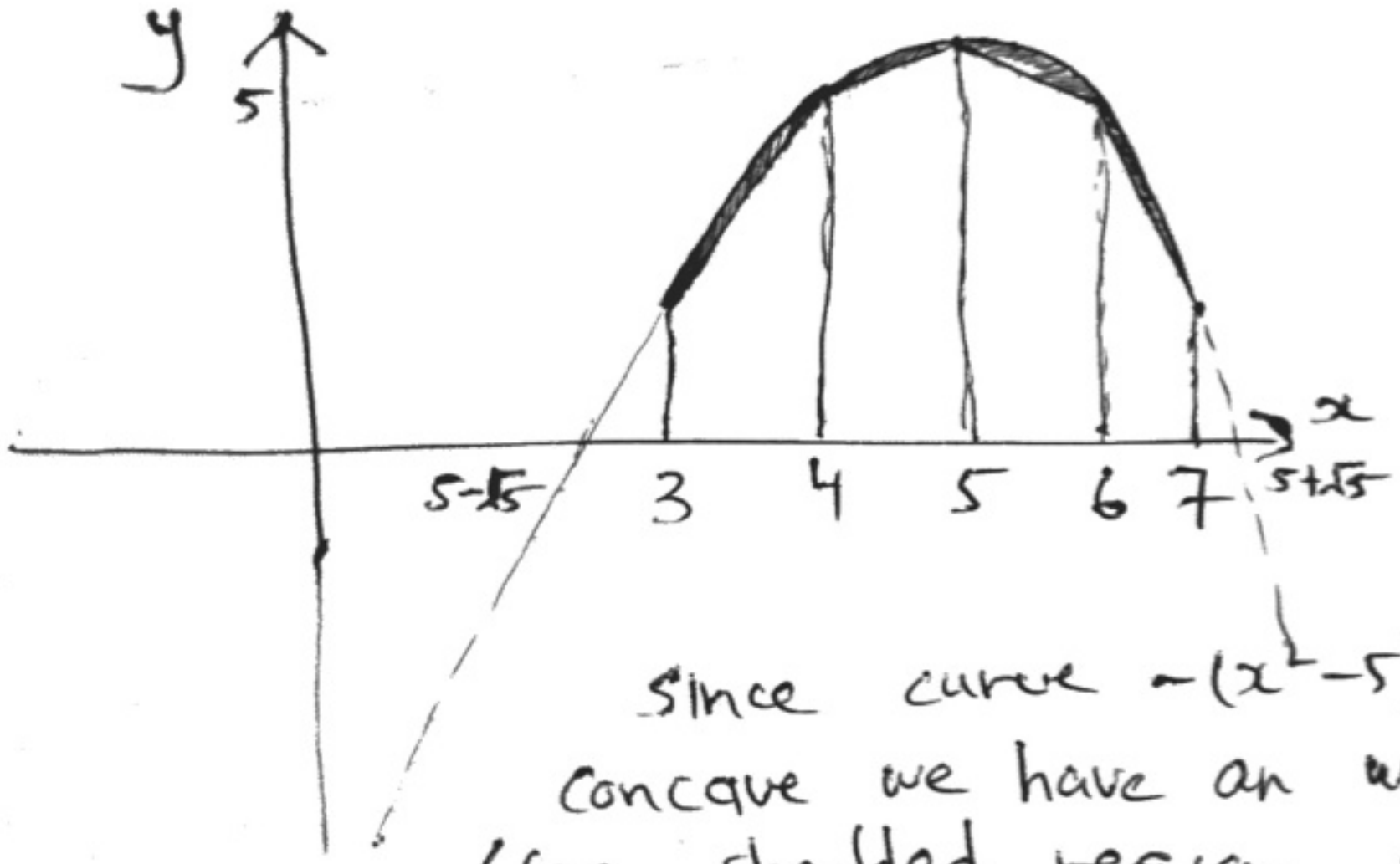
Apply formula $J \approx \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$

i.e $J \approx \frac{1}{2}\left[1 + 2(4+5+4) + 1\right]$

$J \approx \frac{1}{2}[28] = 14.$

Is it over estimate or understimate?

Sketch:



Since curve $-(x^2-5)+5$ is concave we have an underestimate. (see shadded region graph).

Error: Calculate J exactly.

$$J = \int_3^7 -(x-5)^2 + 5\,dx = \int_3^7 -x^2 + 10x - 25 + 5\,dx.$$

$$\therefore J = \int_3^7 -x^2 + 10x - 20\,dx = \left[-\frac{x^3}{3} + \frac{10x^2}{2} - 20x\right]_3^7$$

$$J = \left(-\frac{(7)^3}{3} + 5.(7^2) - 20.(7)\right) - \left(-\frac{(3)^3}{3} + 5(3^2) - 20.(3)\right)$$

$$J = \frac{28}{3} - (-24) = \frac{100}{3} \Rightarrow \text{Error} = \frac{100}{3} - 14 = \frac{58}{3}(!)$$