Speed, Distance & Accelbration
(v) (s) (a).

In Physics, distance travelled by an object can be modelled by an algebraic function of time. i.e. $s = s(t)$.

Recall
$$v = \frac{ds}{dt} = \dot{s}(t)$$
$$a = \frac{dv}{dt} = \dot{v}(t).$$

The dot means "differentiate with respect to time".

Since Integration is opposite to differentiation, then: given $a = a(t)$

$$v = \int_{t_0}^{t} a(t)\, dt$$

$$s = \int_{t_0}^{t} v(t)\, dt.$$

Typically $t_0 = 0$ and we gain $v(t)$ and $s(t)$.

Lets do an example:

Q: A shuttle is re-entering the atmosphere and will take $t = 5$ minutes to land. The speed of the shuttle, from the point of re-entry, obeys the function

$$V(t) = (5-t)e^{-3t},$$

before coming to an abrupt stop!

Calculate the distance travelled by the shuttle from the point of re-entry to Landing.

since $\dot{S}(t) = V(t)$

then $$S(t) = \int_{t_0}^{t} V(t)\, dt.$$

we take re-entry time $t_0 = 0$ and overall time $\bar{t}$ is t.

$$\therefore \quad S(t) = \int_0^{t} (5-\bar{t})e^{-3\bar{t}}\, d\bar{t}.$$

$$= \int_0^{t} 5e^{-3\bar{t}} - \bar{t}e^{-3\bar{t}}\, d\bar{t}.$$

$$= 5\int_0^{t} e^{-3\bar{t}}\, d\bar{t} - \int_0^{t} \bar{t}e^{-3\bar{t}}\, d\bar{t}.$$

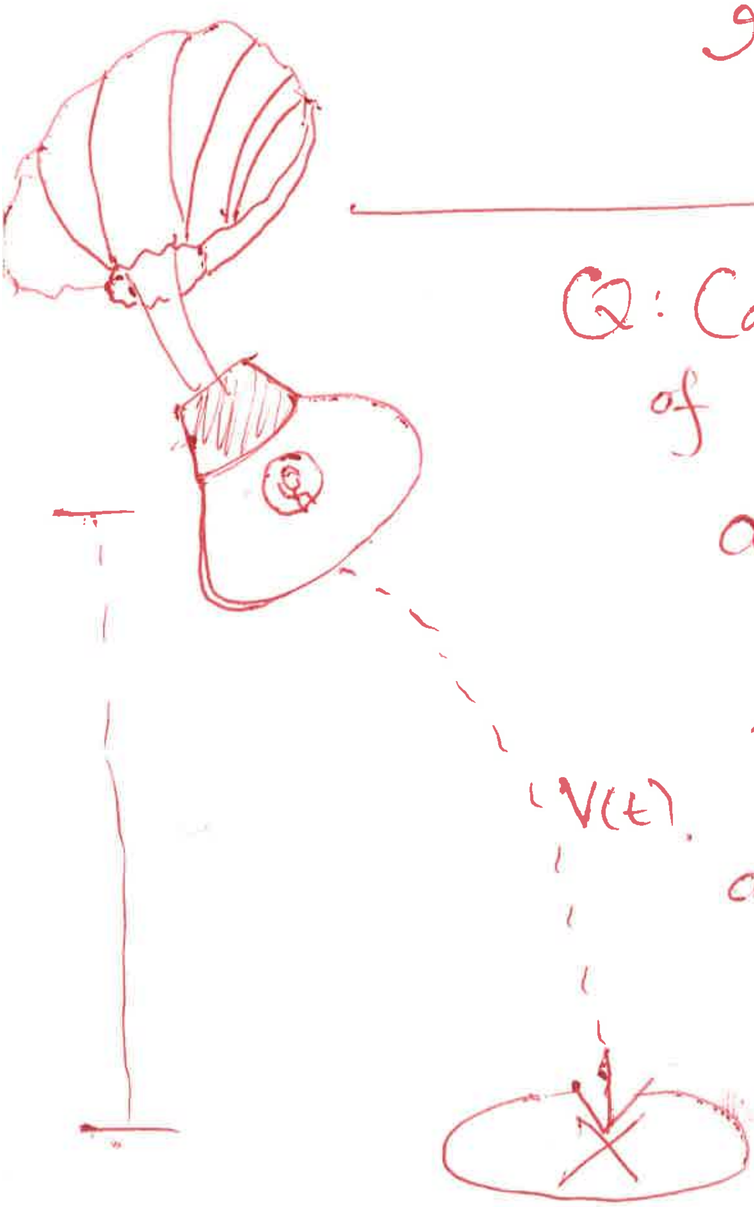
So i) $\int_0^t e^{-3\bar{t}}\, d\bar{t} = \left[-\frac{e^{-3\bar{t}}}{3}\right]_0^t = -\frac{e^{-3t}}{3} + \frac{1}{3}$

ii) $\int_0^t \bar{t} e^{-3\bar{t}}\, d\bar{t} = \left[-\frac{\bar{t} e^{-3\bar{t}}}{3}\right]_0^t - \int_0^t -\frac{e^{-3\bar{t}}}{3}\, d\bar{t} = -\frac{te^{-3t}}{3} - \frac{e^{-3t}}{9} + \frac{1}{9}$

by parts.

$$\therefore S(t) = -\frac{5e^{-3t}}{3} + \frac{5}{3} + \frac{te^{-3t}}{3} + \frac{e^{-3t}}{9} - \frac{1}{9}$$

$$S(t) = \frac{14}{9} + \frac{te^{-3t}}{3} - \frac{14e^{-3t}}{9}$$



Q: Calculate the accelleration of the shuttle.

$$a(t) = \dot{v}(t) = \frac{dv}{dt}$$

$$\therefore a(t) = \frac{d}{dt}\left[(5-t)e^{-3t}\right]$$

$$a(t) = -1e^{-3t} + (5-t)(-3e^{-3t})$$

$$a(t) = (-16+3t)e^{-3t}$$