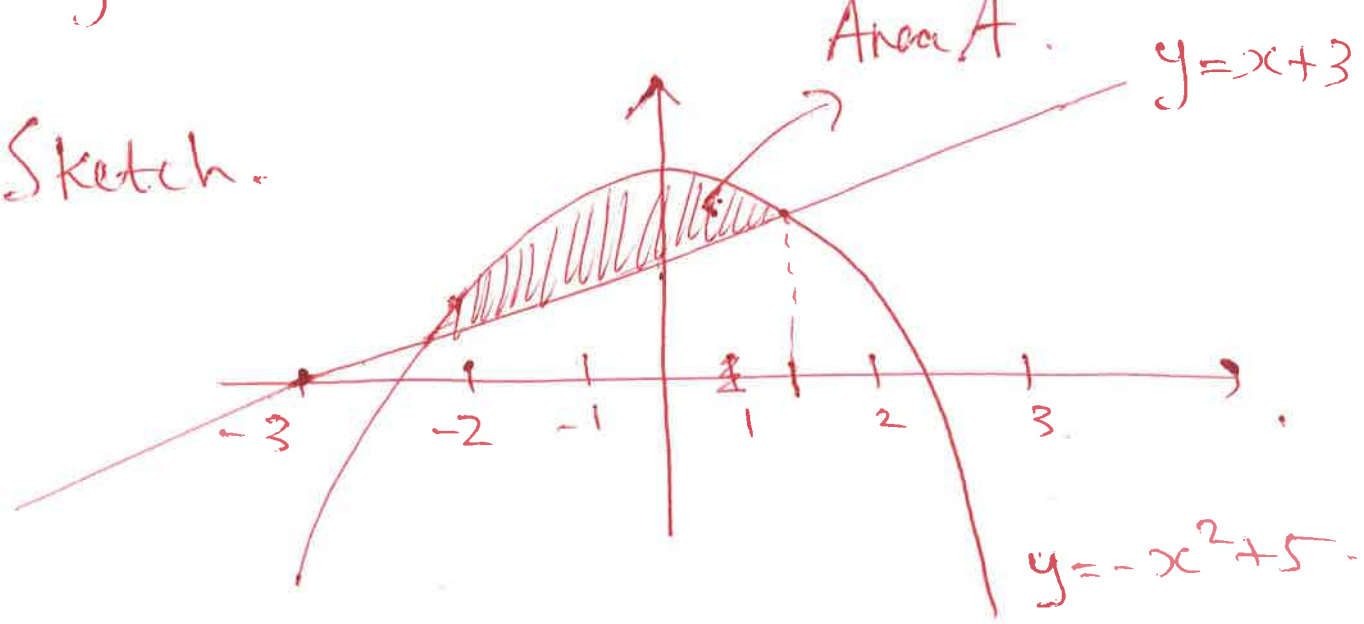
Area Bounded by curves

Q: Find the area bounded by the two curves $f_1(x) = -x^2+5$ and $f_2(x) = x+3$.

Sketch.



Need to find where f_1 and f_2 intersect.

put $f_1 = f_2$ i.e.

$$-x^2+5 = x+3$$

$$\therefore \quad x^2+x-2 = 0.$$

$$(x+2)(x-1) = 0.$$

$$\therefore \quad x = 1, \quad x = -2.$$ (Need to improve sketch!)

So we need to integrate $|f_1(x) - f_2(x)|$ between $x=-2$ and $x=1$. ie.

$$A = \int_{-2}^{1} |-x^2+5-x-3| \, dx$$

since $|f_1 - f_2|$ above x axis in this case and f_1 always above f_2 then

$$A = \int_{-2}^{1} -x^2+5-x-3 \, dx$$

$$A = \int_{-2}^{1} -x^2-x+2 \, dx.$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_{-2}^{1}$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(+\frac{8}{3} - \frac{4}{2} - 4\right)$$

$$= -\frac{9}{3} + \frac{3}{2} + 6 = \frac{27}{6}$$